Applied Optics
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Lecture: 41
Fresnel Half Period Zones

Hello everyone, welcome to the class, today we will start module number 9.

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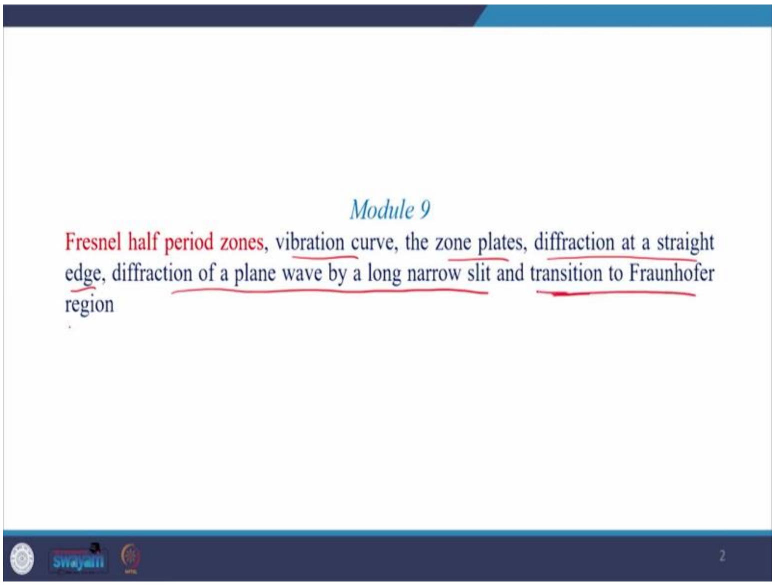

In this module we have several topics, the first one is Fresnel half period zone and thereafter we will talk about vibration curves that would be followed by zone plates. And then we will discuss about diffraction at a straight edge then diffraction of a plane wave by a long narrow slit and at last we will see how the Fresnel diffraction return make transit to the Fraunhofer region. Since this is the first lecture on the diffraction therefore, we must revisit what we studied or what are the conditions in Fraunhofer region.

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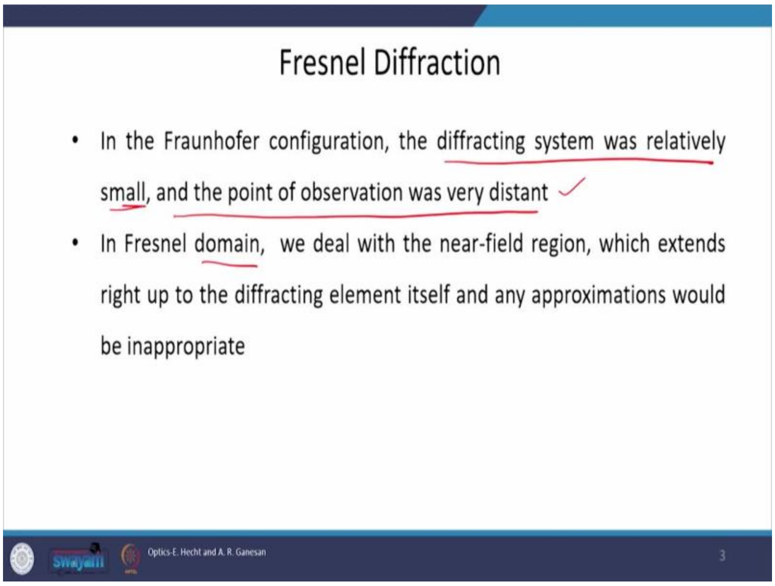

In Fraunhofer domain, the diffracting system was relatively small, relatively small means its size was smaller than the distance between screen plane and source plane and the point of observation was very distant, this is what is the point or let me say it like this, in Fraunhofer configuration, the diffracting system was relatively small and the point of observation was very distant. Because in the very first class of diffraction, where we introduce phenol and Fraunhofer categories, we discussed that in Fraunhofer domain both the source and the point of observation on observation plane they must be at infinity from the diffracting plane or from the diffracting element, this was the condition which was imposed by Fraunhofer diffraction.

While in Fresnel domain, we deal with the near field region which extend right up to the diffracting element itself and any approximation would therefore be inappropriate. The approximation which we considered in Fraunhofer diffraction was that the distances of the source and the screen from the aperture pin must be infinity, but in Fresnel domain, we will not rely on this approximation, we may even go much closer to the diffracting element. Now, with this we will also have to revisit the concept introduced by Huygens Fresnel principle.

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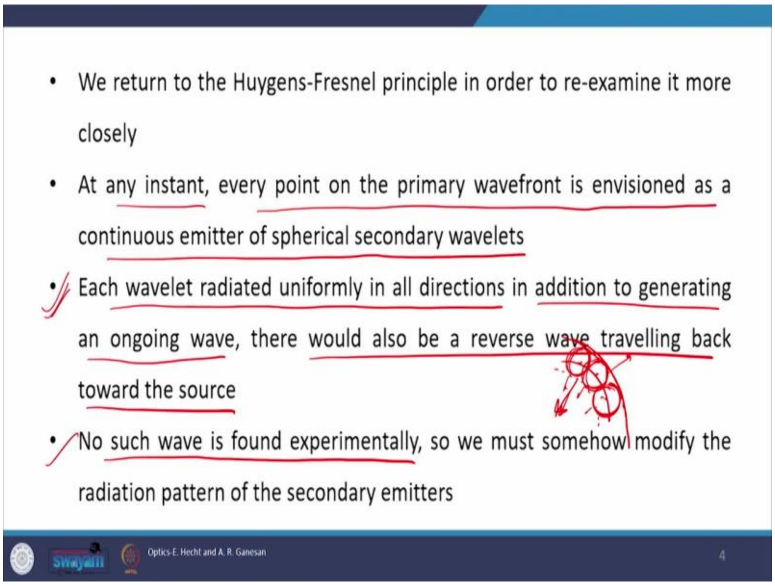

Now, what are the points which need to be reexamined? At any stand every point on the primary wavefront is envisioned as a continuous emitter of spherical secondary wavelets, that was the basic principle or basic statement proposed by Huygens. Now, Huygens again said that each wavelet radiated uniformly in all directions in addition to generating an ongoing wave, there would also be a reverse wave travelling back towards the source. And this is something which we do not see in our daily life here, because suppose, we have a wavefront and each point on the wavefront it emits a spherical wavefront.

Now, having said that, we just draw an envelope on the secondary wavefront and this envelop will give a new position of the wavefront. But since the wave emitted by this point sources on the primary wavefront, they are also propagating in the backward direction and therefore, if you draw an envelop here and you will see that the back the wave is propagating in forward direction as well as in the backward direction, this is quite contrary to the usual observation because we know that the waves propagate in the forward direction.

Therefore, this point is a point of concern because no such wave is found experimentally. So, we must somehow modify of the radiation pattern after secondary emitter and this modification was introduced by Fresnel and the new principle is called Huygens-Fresnel principle, where Fresnel introduce that this emitting wave they propagate in the forward direction only and he also said that they interfere and therefore, the Huygens-Fresnel principle is now can explain diffraction. But exact mathematical formulation came quite later and it was given by Kirchhoff’s.

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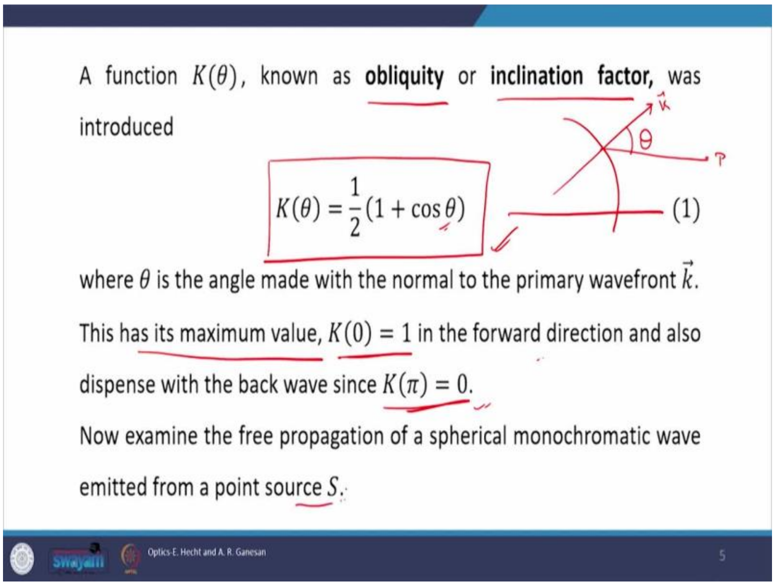

Now, Kirchhoff's, what he said is that he mathematically formulated a function which is called obliquity or inclination factor, K (θ) is called obliquity or inclination factor and it is defined as 1/2(1+cosθ) or (1+cosθ)/2 as is given by equation number 1. And this obliquity factor dictates the direction of propagation of wave. Now, here the angle θ is the angle made with the normal to the primary wavefront $\vec{k}$. Suppose this is our wavefront and this is the $\vec{k}$ vector direction and this is the point of observation P, then this angle is θ.

And of course, from expression 1 we can easily see that this has a maximum value when θ= 0 and when θ= 0, we have K (0) = 1, it means all the energy will propagate in the forward direction. But if you see the propagation in backward direction then what you find is that when θ=π then K (π)= 0, it means nothing is propagating in the backward direction this was the corrections which were introduced later in the Huygens principle and limitations which the Huygens principle had it was removed just by introduction of this obliquity or inclination factor. Now, we will examine the free propagation of a spherical monochromatic wave emitted from a point source.

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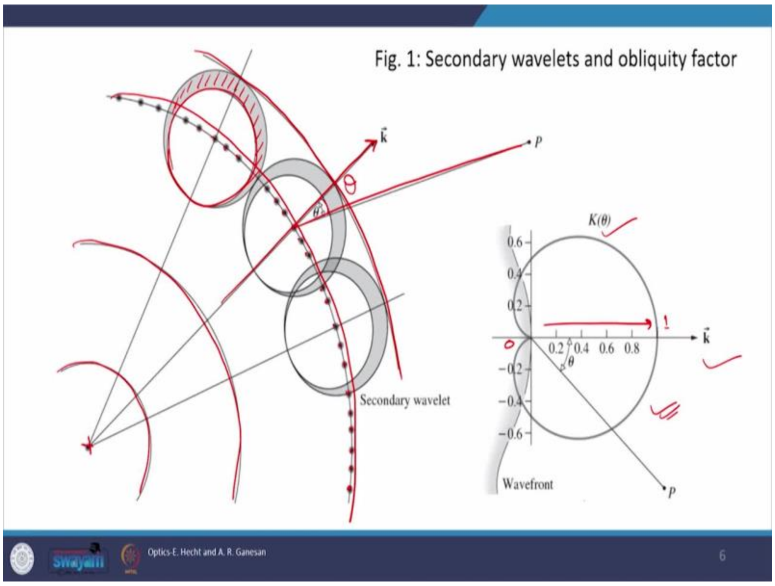

But before that, let us see how does this obliquity factor look like? Here in this figure you see that we have a point source and then these are the spherical wave fronts which are being emitted through this point source. Now to decide the shape or the direction of propagation of the wave and the shape of the new wavefront following Huygens principle we take large number of secondary point sources on a wavefront and this from these secondary point sources, we assumed that new wavelets are being generated. And then we draw common envelop on these all secondary wavelets and this is how the waves propagate.

Now, this is the direction of $\vec{k}$ which is perpendicular to the wavefront and P is the point of observation and the angle between this line and the $\vec{k}$ direction is θ angle and this shaded region shows the part of the energy which will go in different direction which will go in different θ directions. Now, if you plot K (θ) with respect to θ then you see this kind of variation and then obliquity factor show this type of variation. And now you see that in the forward direction its value is 1 while in the backward direction its value is 0, it means that most of the energy will propagate in the forward direction, a part will go in other θ direction but in the backward direction no energy will propagate, the wave will not propagate in the backward direction.

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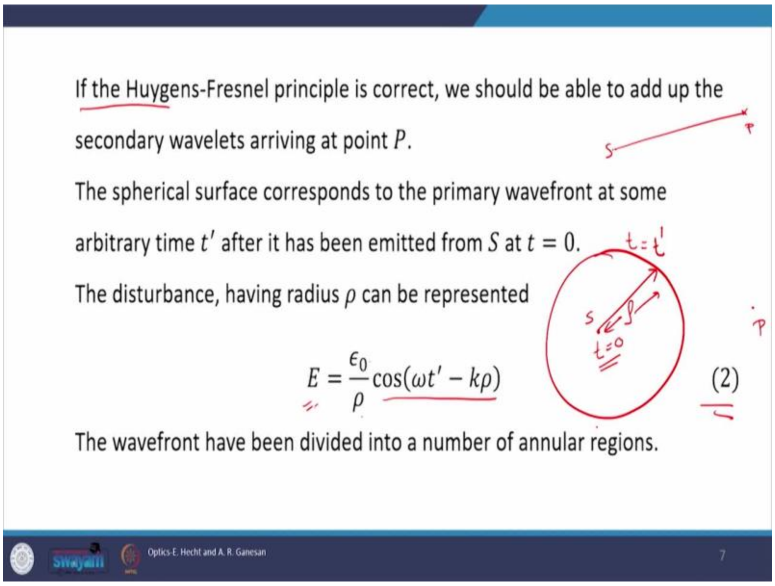

Now, we will consider a wave propagation in free space. Now, if the Huygens-Fresnel principle is correct, we should be able to add up the secondary wavelets arriving at point P. We have a source here and from this source wave is being emitted and at the point of observation P it is being recorded, it is being observed. Now, the principle says that we should be able to add up the secondary wavelets arriving at point of observation P, the spherical surface corresponds to the primary wavefront at some arbitrary time t' after it has been emitted from S at time t is equal to 0.

It means that suppose we have a point source which is emitting a spherical wavefront and this wavefront is recorded at time t' and since it takes some time for wave to reach at this position therefore, it must have started at some earlier time and we assume the wave start from point source S at time t=0 and then it reaches to this position at time t=t'. With this, the disturbance having radius ρ, say this radius of the spherical wavefront is ρ this can be represented by equation number 2, which says that field $E = \epsilon_0/\rho(cos\omega t' - K\rho)$, ρ is the radius of this wavefront.

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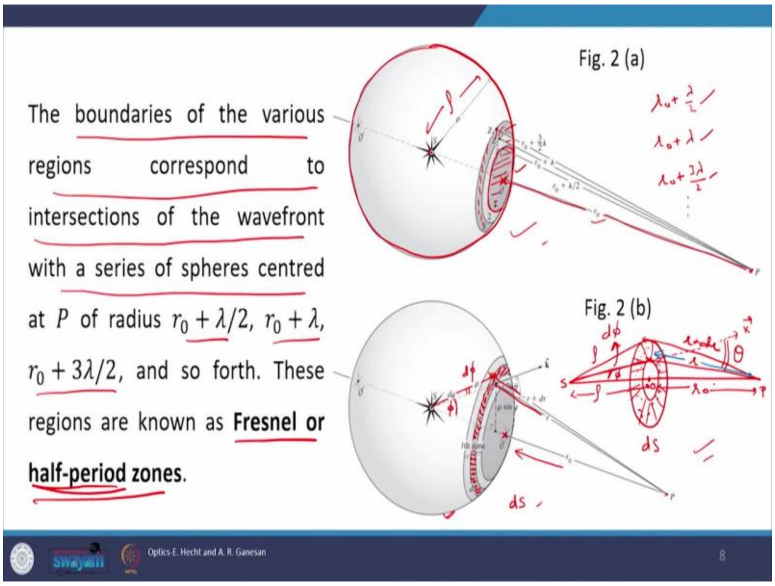

Now what we do is that, suppose we have a point of observation P here and we divide the wavefront in a number of annular regions, how to do this? This is clearly depicted in this figure. Now, in this figure you see we have a point source S and at a time t' the point source has this wavefront and the radius of this sphere is ρ, this is the radius. Now what we do is that from the point of observation P we draw various spheres of radius $r_0 + \lambda/2$ and then $r_0 + \lambda$ and then $r_0 + 3\lambda/2$ and so on. From point P we now draw various spheres of these radius and these radii are varying by λ/2 at each step. The constitutive or at the adjacent sphere which are centered at P they vary by λ/2 in their radii.

Now, when we draw the spheres they will cut this spherical wavefront in different annular regions which are depicted here. The first sphere will generate this shaded region which we name as Z_1, the second sphere will generate this less shaded region which we name as Z_2 the third sphere will generate this dark shaded region which we call Z_3 , and this point is the nearest point on the wavefront from point of observation P which is at a distance r_0. The boundaries of the various reason correspond to intersections of the wavefront with a series of spheres centered at P of radius $r_0 + \lambda/2$, $r_0 + \lambda$, $r_0 + 3\lambda/2$, and so forth.

And these regions are known as Fresnel or half period zone, the word half period comes from the $\lambda/2$ difference in the radii of adjacent sphere, because $\lambda/2$ is half of the period, it is the full period the full wavelength is λ but half period is $\lambda/2$ therefore, these zones are called half period zone or Fresnel zones. Now, what we do is that, we pick some l-th zone, say there is n number of zones starting from 1 and we pick one of these zones and say this zone is a l-th zone.

Now, l^{th} zone is situated between $(l-1)^{th}$ zone and $(l+1)^{th}$ zone, it is between l - 1 and l + 1 zone, this is in between these 2 zones. Now, we assume that this zone has boundaries which are, let me explain it here. Suppose this is the zone which we are interested in, I am shading this zone and the area of this zone is dS, as this point, which is point O which is the nearest point from the point of observation is at a distance r_0 , we assume that this annular ring, the lower portion of this annular ring at this point particularly is at a distance r from point of observation P while the upper point here is at a distance r + Δ r from point of observation P.

Now, this distance of course would be ρ, this distance would be ρ, this angle say is φ and this angle is dφ. Now, I can redraw this figure because it is too much now in this figure, let me redraw it. Now say this is our point S and this is our point P and annular region is somewhere here, this is the annular region which we are interested in and this annular reason has area dS. Of course, this distance is ρ which is the center O and this distance is r_0 as depicted in the figure. Now say this distance is r and from the upper portion of this annular region it is r + dr and this is ρ because this whole of these annular region is on the surface of the sphere therefore, all points of the annular region would be a distance ρ from the source because source is the center of this sphere.

Now say this angle is φ and this angle is dφ, with this we can easily calculate the area element dS. And one more point this is wave vector $\vec{k}$ direction therefore, this angle would be the angle θ. Now, hope this picture is quite clear to you all now in this picture you can see that the source S is effectively at a distance $\rho + r_0$ from the point of observation P. Therefore, what we did is that we defined a ring shape differential area element dS and all the points sources within dS are coherent, since the area element dS is very thin therefore, we assume that all the points sources which are situated on this area element they are coherent.

And we also assume that each of these points sources radiates in phase with the primary wave, this is our primary wave and the sources are here in this small area element and it is also here, and since the thickness is very small we are imposing 2 conditions or we are assuming that they are coherently emitting, whatever they are emitting everything is in phase with the primary wave. The secondary wavelets, secondary wavelengths emits the emission which are coming from the point sources sitting on this area element. The secondary wavelets travelling a distance r to reach at P which is very much clear from this figure you see that, let me pick a different color you see that this distance is r and therefore, the secondary wavelets which are being emitted from point sources sitting on this area element they travelled a distance a r to reach at point P.

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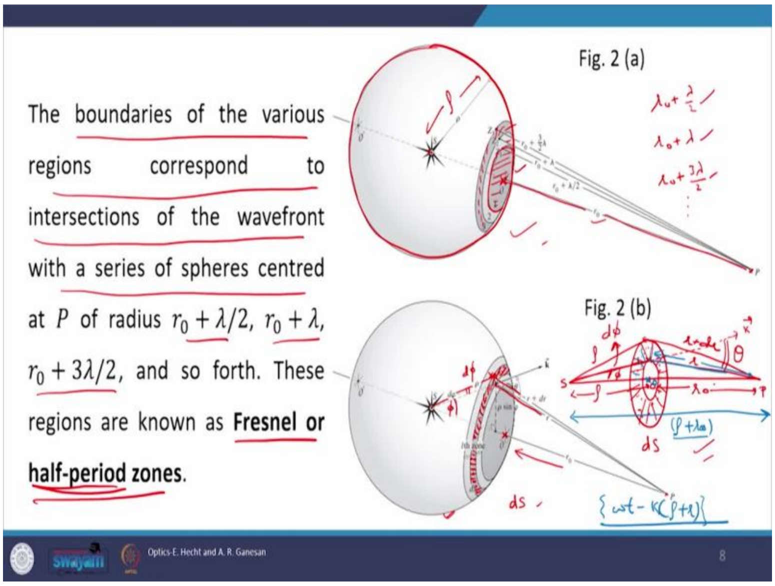

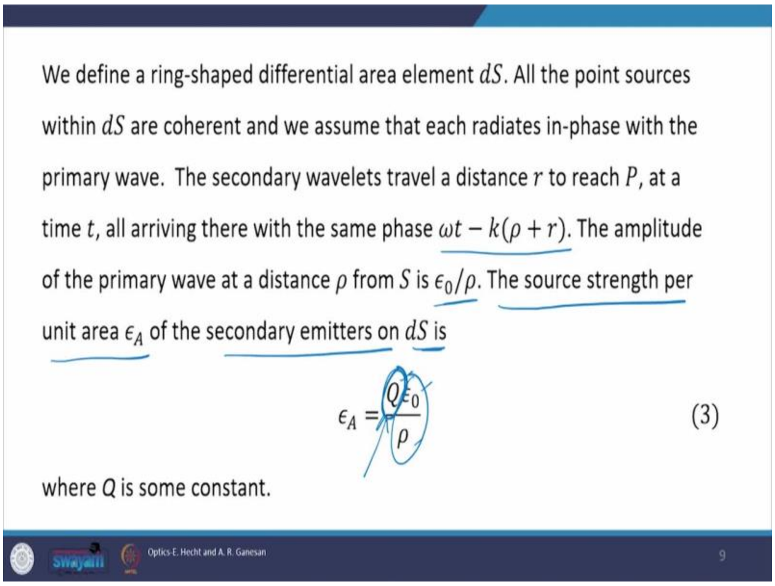

Now, we also assume by the time they reach there it is time t, made this as statement that all the point sources on the area element is coherent and they are radiating in phase. Therefore, all the wave or all the light which reach out point P from this area element would be in same phase. And how to calculate this phase? This phase would be $(\omega t - k) \times$ total distance travelled, and what is the distance travel here? Distance travel would be this distance. And what is this distance? it is $\rho + r_0$, is it but this distance is the horizontal distance but our area element is a bit shifted it is a annular ring which is away from point O which is sitting at the center therefore, the new distance is r and the overall phases therefore would be $\omega t - k(\rho + r)$, this should be the phases of all the wave reaching at point of observation P.

And this is what is written here, therefore, for all these wavelets arrive at point P with phase $\omega t - k(\rho + r)$, where r will vary, if you keep varying the annular ring, r will vary. And the amplitude of the primary wave at a distance ρ from S is given by ϵ_0/ρ, this we have already studied because in case of spherical wave the distance comes in the denominator of the amplitude. Therefore, we define source strength per unit area here too and the source of strength per unit area of the secondary emitters on dS therefore, will be equal to Q ϵ_0/ρ.

But ϵ_0/ρ is the amplitude of the primary wave which is at a distance ρ from the source, we are just modifying this amplitude by some multiplication factor Q and which we do not define at this point, we assume that the amplitude of the primary wave would be modified in case of secondary emitters. But let us assume that this modification is taken care of by this multiplication factor Q, where Q is some constant.

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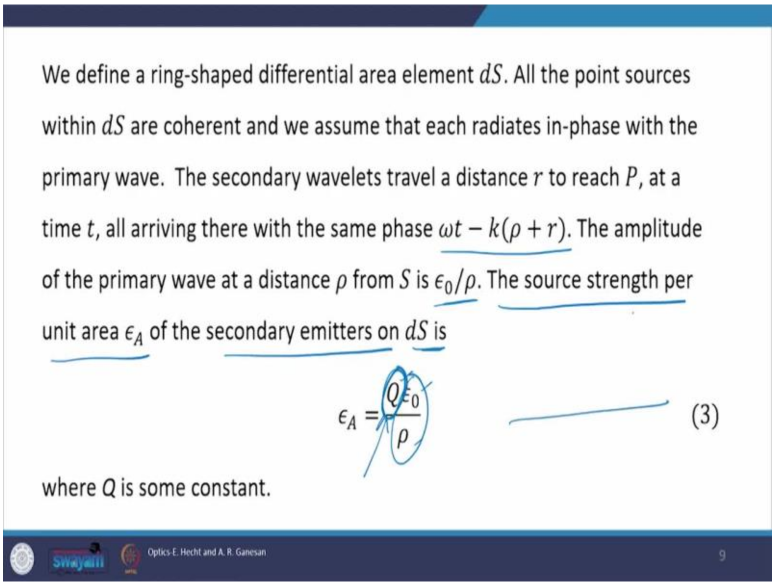

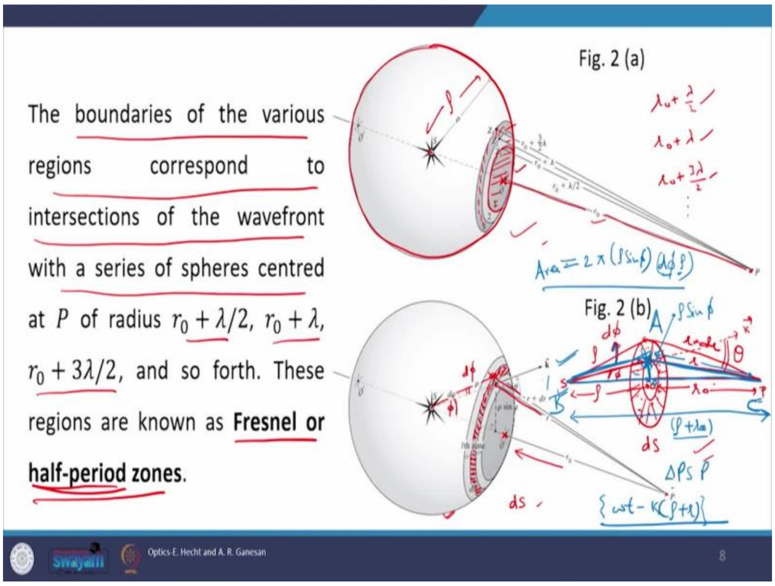

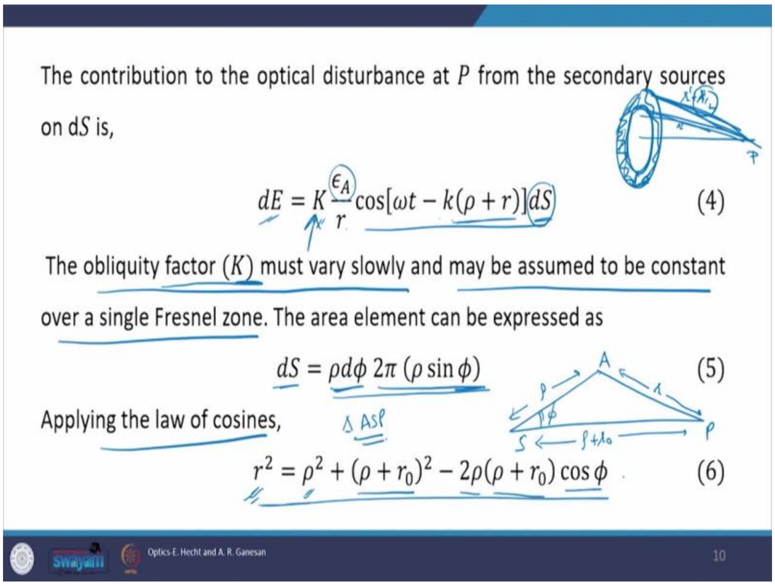

Now, we will move ahead with this, the contribution to the optical disturbance at point P from the secondary source on dS is therefore, be equal to $K\epsilon_A/r$, ϵ_A is defined here in equation number 3 which is source strength per unit area and then the phase part $cos(\omega t - k(\rho + r))dS$, ϵ_A is source of strength per unit area, but for area dS we will have to multiply ϵ_A with dS to get the complete source in strength information. K is obliquity factor here and which dictates the variation of dE with θ, because the wave propagate in all possible direction then it decides what amplitude goes in which direction therefore, we have multiplied it following Huygens-Fresnel principle.

The obliquity factor must vary slowly and may be assumed to be constant over a single Fresnel zone, why? Because Fresnel zone is very thin, each Fresnel zone, the Fresnel zone is defined by some annular ring and this annular ring, the inner circle is at a distance, say r' and outer circle is that distance $r' + \lambda/2$ therefore, the effective difference between the 2 distances is $\lambda/2$. And this says that the width of this ring would be very small and therefore, we may assume that θ does not vary over a single Fresnel zone or over a single annular ring. And if θ does not vary, then K also remains constant, therefore, we can assume that obliquity factor Q is almost constant, its vary-very slowly.

Now we calculate the area element, the area element from this figure can easily be calculated. The periphery is 2π r and r is this distance and this distance is nothing but $\rho sin\varphi$. Once $\rho sin\varphi$ is known, then the area would be equal to 2π r, r is $\rho sin\varphi$ and then the width of this strip the width of this annular region, what would be the width? It would be angle $\times$ radius, angle is here $d\varphi$ and radius is ρ. And this is how the area of the annular region is calculated, this is what exactly is written here. $d\varphi \times \rho$ is the width of the annular portion and $2\pi\rho sin\varphi$ is the periphery of the circle.

Now with this we will also do some mathematics in this triangle. Now, suppose this is a triangle here, as a bigger triangle, say it is A, this is point B and this is point C in triangle ABC let us redefine A, let us pick a smaller triangle here now, we pick a triangle which is starting from here it goes to point source S and then point P and then coming back to this. The tip of the triangle is at the inner circle say this is point A here it is point S and here it is point P. Now in triangle ASP, sorry ASP, from triangle ASP using very basic trigonometry you can write equation number 6, where r^2 which is this AP distance, this is point A, AP is r then this triangle r^2 would be ρ^2.

I will rewrite here, this is A, S, P, this distance is $\rho + r_0$, this distance is ρ, and this distance is your r, and this is our φ angle. Now, in this ASP triangle, we can use basic trigonometry and get this relation between all the sides of the triangle, this triangle all the sides are given and one angle is given. Therefore, r^2 would be $\rho^2 + (\rho + r_0)^2 - 2\rho\ (\rho + r_0)cos\varphi$.

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The contribution to the optical disturbance at P from the secondary sources on dS is,

$$dE = K\frac{\epsilon_A}{r}\cos[\omega t - k(\rho + r)]dS \quad (4)$$

The obliquity factor (K) must vary slowly and may be assumed to be constant over a single Fresnel zone. The area element can be expressed as

$$dS = \rho d\phi\, 2\pi\,(\rho \sin\phi) \quad (5)$$

Applying the law of cosines,

$$r^2 = \rho^2 + (\rho + r_0)^2 - 2\rho(\rho + r_0)\cos\phi \quad (6)$$

This yields,

$$2rdr = 2\rho(\rho + r_0)\sin\phi\, d\phi \quad (7)$$

with ρ and r_0 held constant. Making use of the value of $d\phi$, we find that the area of the element is therefore,

$$dS = 2\pi\frac{\rho}{(\rho + r_0)}r\,dr \quad (8)$$

The disturbance arriving at P from the l^{th} zone is,

$$E_l = K_l 2\pi\frac{\epsilon_A \rho}{(\rho + r_0)}\int_{r_{l-1}}^{r_l}\cos[\omega t - k(\rho + r)]dr \quad (9)$$

Hence,

$$E_l = -\frac{K_l \epsilon_A \rho \lambda}{(\rho + r_0)}[\sin(\omega t - k\rho - kr)]_{r=r_{l-1}}^{r=r_l} \quad (10)$$

Upon the introduction of $r_{l-1} = r_0 + (l-1)\lambda/2$ and $r_l = r_0 + l\lambda/2$ the expression is reduced to

$$E_l = (-1)^{l+1}\frac{2K_l \epsilon_A \rho \lambda}{(\rho + r_0)}\sin[\omega t - k(\rho + r_0)] \quad (11)$$

The amplitude of E_l alternates between positive and negative values, depending on whether l is odd or even.

Now, if you differentiate this relation, this equation number 6, if you differentiate it then you get this $2rdr = 2\rho(\rho + r_0)sin\varphi d\varphi$, where r_0 and ρ are held constant because they are fixed numbers. Now, from here we can calculate the expression of $d\varphi$ and with this $d\varphi$ is then substituted back in equation number 5 here we will substitute for $d\varphi$ which is given in equation number 7 and from the using equation 7, equation 5 takes this form, this is from equation 5 and we substitute this value of dS in the expression of the field.

Now, the expression of the field is given here equation number 4. Now the value of dS is substituted in equation number 4 and equation number 4 is written for some area element dS. Now, we assume this area element as I said before is l^{th} zone l^{th} Fresnel zone, l^{th} half period zone. Then to consider all the points sources in this l^{th} half period zone, we integrate the equation from r_{l-1} to r_l, because if this is the half period zone, the shaded region which we are considering in our discussion and we named this as a l^{th} zone then say this is point of observation P then these distances r_0 and this is a l^{th} zone, for this distance would be $r_0 + \lambda/2 \times (l - 1)$ and this distance would be $r_0 + \lambda/2 \times l$. And we name these distances as r_l and r_{l-1}. With this, we can safely put these 2 limits in this integral in equation number 9.

And then after integration we get equation number 10, where r_{l-1} as I said before is equal to $r_0 + (l - 1)\lambda/2$ and r_l is $r_0 + l\lambda/2$, which is what I wrote here. Now with this substitution, equation number 10 reduces to equation number 11 and here you see that we get $(-1)^{l+1}$. It means if you vary l, the sign of E_l also varies, the amplitude of E_l alternates between positive and negative values and it depends whether l is odd or even, the rest of the term are shown here, K_l is the obliquity factor for l^{th} zone, ϵ_A source strength per unit area, ρ is the radius of the spherical wavefront, λ is the wavelength, all the things you have already defined here.

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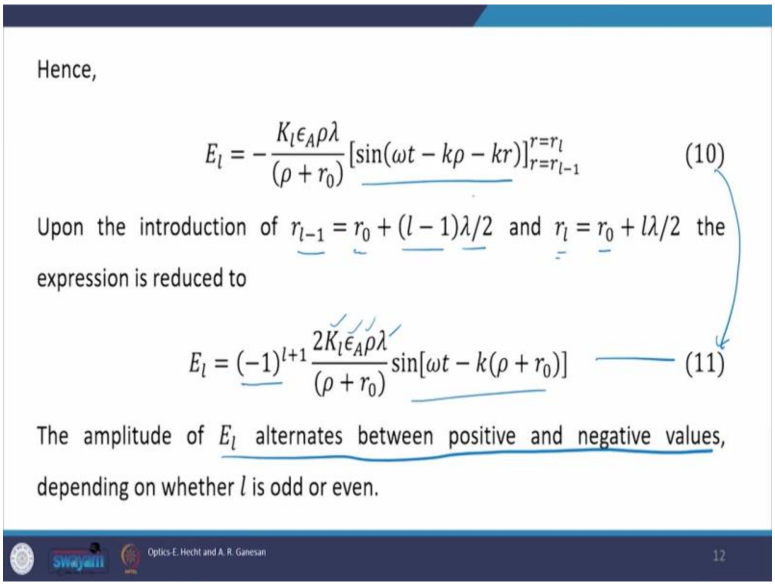

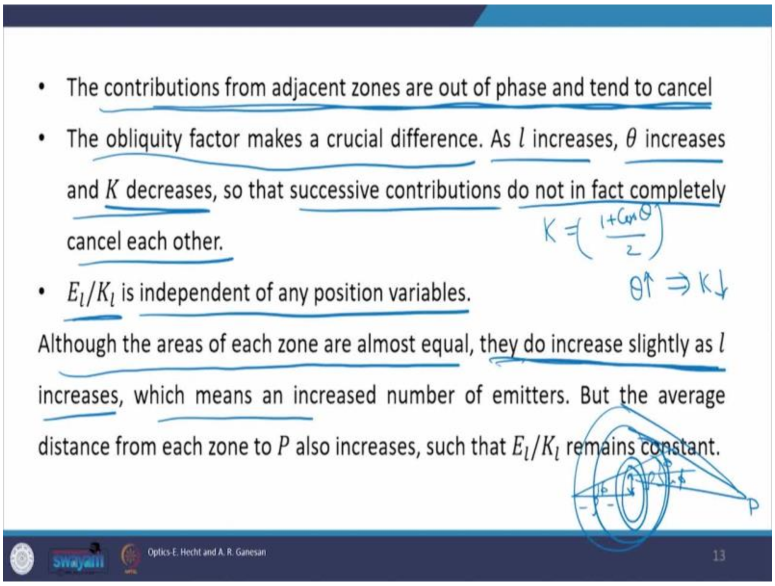

Now, there are few points which we can draw from this expression. The contributions from the adjacent zone are out of phase and tend to cancel, because in equation 11 we see that E_l alternates between positive and negative values, if l =1, then E_l is positive, if $l = 2$ then E_l is negative, it means the adjacent zone are 180 degree out of phase therefore, the contribution from the adjacent zone are out of phase and tend to cancel and each zones are very thin. Therefore, the amplitude of the wave received at point P emitting from point sources sitting on these zones, they are almost equal but opposite in sign therefore, they cancel each other, but they do not cancel perfectly, why? Because as you increase l, the angle θ increases and therefore, the obliquity factor K it decreases.

Therefore, the obliquity factor makes a crucial difference, as l increases, θ increases and K decreases, because $K = (1 + cos\theta)/2$. Therefore, with increasing θ, K decreases therefore, the contribution from each zone as you move up as you go increase l it successively reduces and therefore, the successive contribution do not in fact, completely cancel each other. The contributions weaken as you go up as you increase l, but this ratio E_l/K_l is independent of any position variables, why? Because, although the areas of each zone are almost equal, they do increase slightly as l increases.

Because if you increase l then the radius of these zone would be larger, it is a $\rho sin\theta$ and the radius of these zone are $\rho sin\theta$, we define these zones like this. And this distance was ρ and this was φ then the radius of zone is $\rho sin\varphi$ or if you talk in terms of θ then too this radius is increasing. Therefore, as you increase l, the area of these zones slightly increases and since the area of these zones are increasing, the number of emitters which are sitting on a particular zone it also increases with l.

I repeat although the area of each zone are almost equal, they do increase slightly as l increases, which means an increased number of emitters, but the average distance from each zone to P also increases, because as you increase this ring radius, this distance also increases, the distance from the point of observation P and this increase is such that E_l/K_l remains constant. Number of emitters are increasing therefore, E_l should increase, but alongside θ is also increasing therefore, E_l/K_l remains constant.

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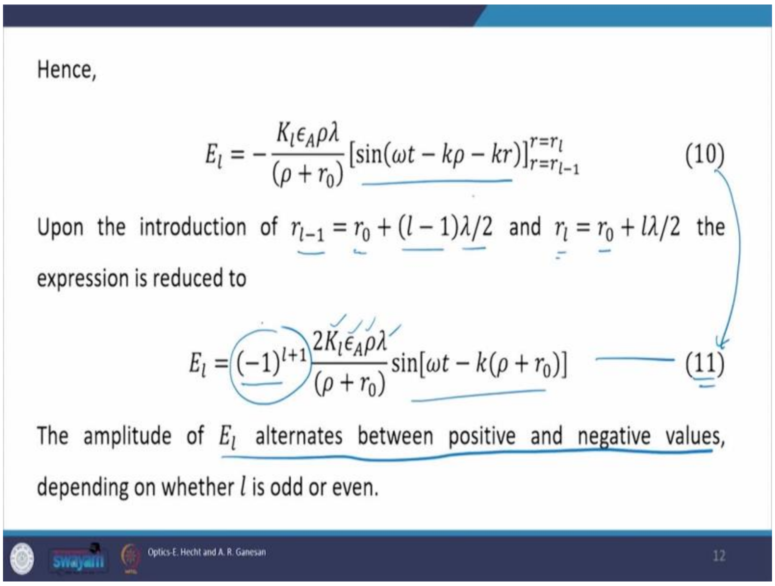

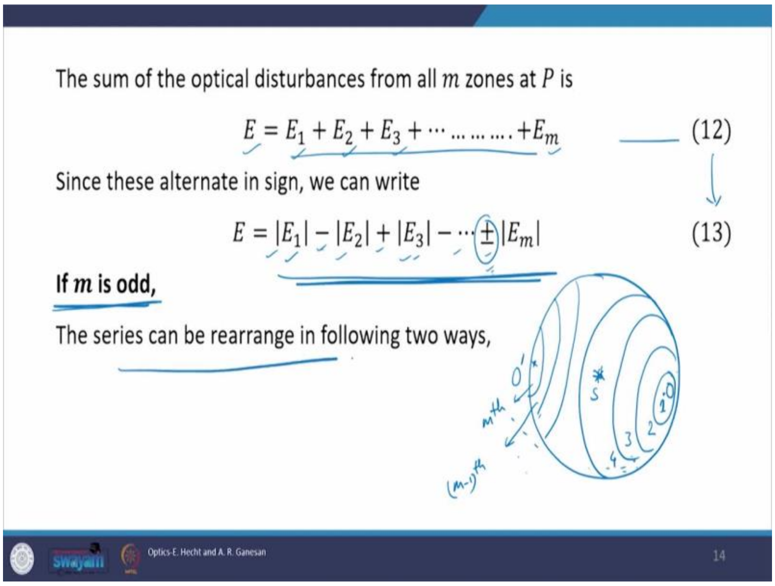

Now, once we have calculated the disturbance at point of observation P due to some arbitrary annular ring and then we can calculate the total contribution at the point of observation P and this total contribution comes from all the annular rings, from all the zone plates. Now, since we have done it for lth zone, we can vary l and then we will get from first zone and second zone and then third zone and then the last zone, the mth zone, we named the last known as mth zone. Now this is the wavefront which was emitted by point source S, this is how we splitted the zone, this is your first zone, this is second zone, this is third zone, this is fourth zone, and at the center here there is a point O.

Similarly, if you keep increasing then these zones will also be here too. And say this point is named as O' and this is our last zone our mth zone, this is your (m - 1)th zone and so on, this is how the number decreases here and it is increasing in this direction, the point source is here. And these zones are very thin, then we calculate the contribution from each zone and sum them up and this would be the resultant optical disturbance at point of observation P.

We also know that in the expression of E_l in this equation number 11 you see that amplitude of E_l alternates between positive and negative value. Therefore, we will also enjoy this knowledge and we will implement it in equation number 12 and therefore, the 12 modifies to 13 which now have alternates signed before all field contributions, all disturbance contribution even as positive value to its negative, either it is positive or is negative and so on and so forth. But we do not know whether the number of zones, the total number of zone is even or odd therefore, we put both plus and minus here it is not decided yet. But now, let us consider a case when m, the total number of zone is odd. Now, this series which is given in equation number 13, this series can be arranged in two ways.

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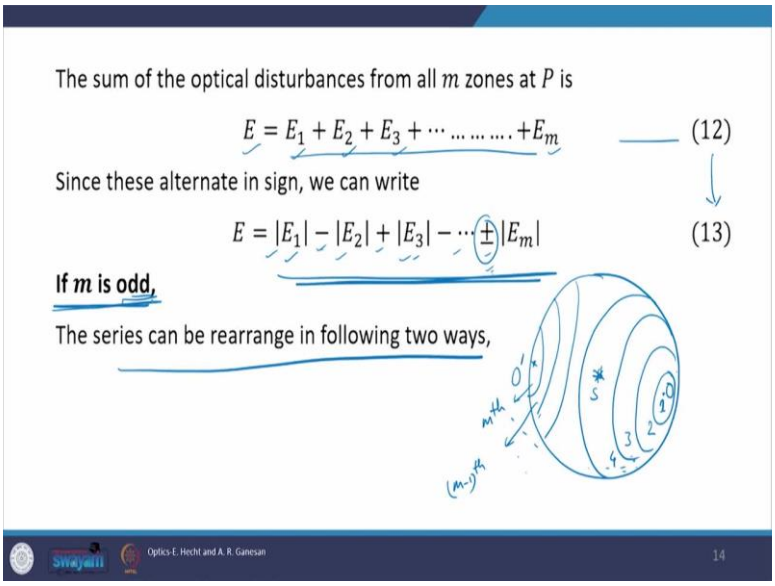

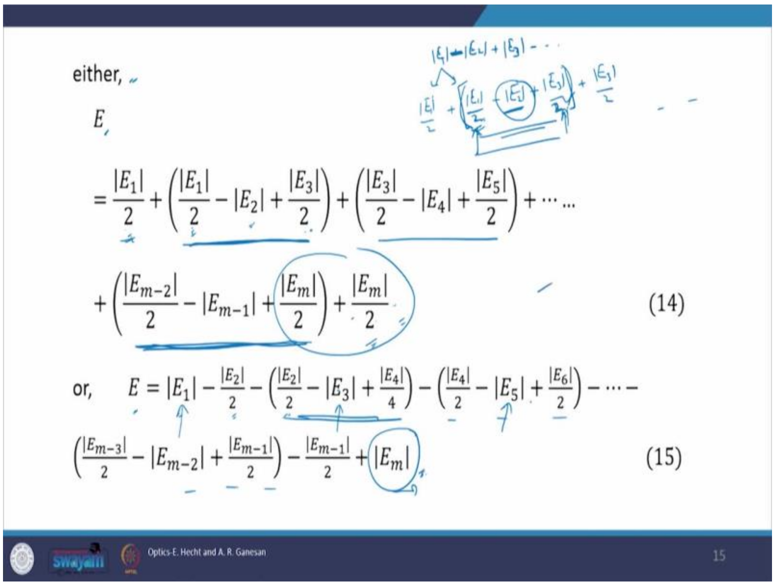

Now, the first way is $E = E_1/2$, like what we did is that we were having E_1 and then E_2, this is how the things were written in equation number 13. But here what we do is that, we just split E in two parts, we write as sorry it is E_1, $E_1/2$ and $E_1/2$. Similarly, E_2 and for E_2 we just write E_2 we do not split it, but for E_3 we again perform this splitting and we write it like this.

Now, we have one term here in this bracket in which we have contribution from first zone, contribution from third zone as well as the contribution from the second zone, the full contribution of second zone E is embedded here in this middle term, but the contribution from the first and third of zone is considered partially here because you are seeing that in the denominator we have a term 2, it means half of the contribution from the first zone and half of the contribution from the third zone is there within this bracketed term and this is performed for all the coming terms.

Now**,** if you write this equation number 13 in this form, then you get first term here in the bracket, the half of the first term, the second term and half of the third, similarly this bracket, this bracket. And since the number of term we assumed as odd, it means we will be left with $E_m/2$, $|E_m|/2$ here, since the number of term is odd, you see that first term is splitted in two, the second term is left as it is, third term is splitted in two, therefore the last term would again be splitted in two, all terms are splitted in two, therefore last term will be again splitted in two, and we will have this bracketed terms before the last term.

Now alternatively we can also write it this way, the field E is first term and now split the second term and perform the same things with a second, third and fourth term and then proceed in a similar manner. In this case, the last term would not be splitted because here the odd terms remain as it is therefore, E_m is kept as it is, it is not splitted because the number of terms in this series is assumed to be odd.

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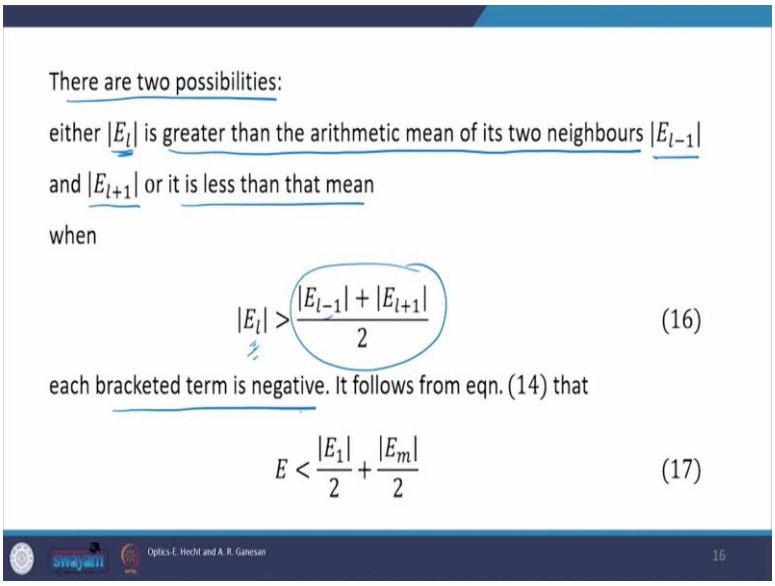

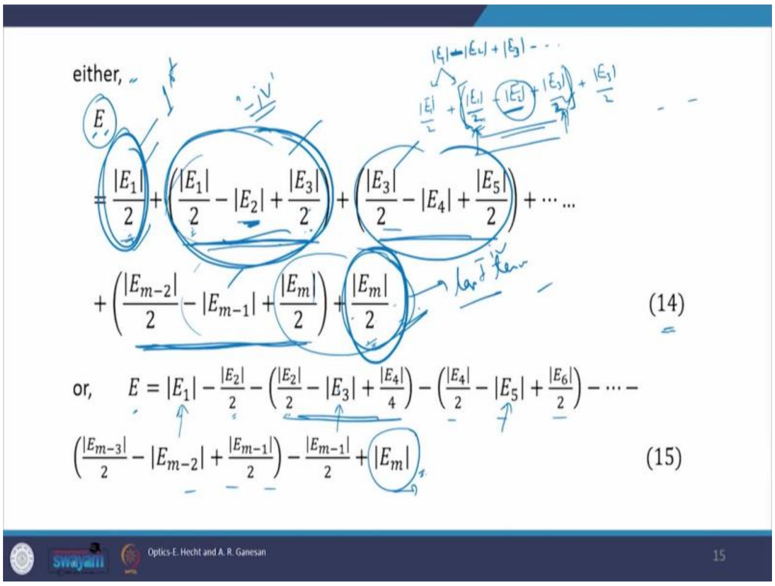

Now, with this again there are two possibilities, with either of the last two series there are two possibilities. Either E_l a particular field is greater than the arithmetic mean of its two nearest neighbour that are E_{l-1} and E_{l+1} or it is less than that mean. It means to say is that either E_l would be larger than the mean of E_{l-1} and E_{l+1} or it would be smaller than mean of E_{l-1} and E_{l+1}. If E_l is larger than the mean of E_{l-1} and E_{l+1} in that particular case each bracketed term is negative.

Let us go back into equation number 14, let us look particularly in this term. Here we assume that E_2 is larger than the average of E_1 and E_3, if E_2 is larger then each bracketed term here will produce a negative quantity, this would again be a negative quantity, similarly the last bracket will also be a negative quantity. Therefore, the resultant disturbance at P would be smaller than this term plus this term because all the terms are bracketed these are only the two free terms, the first one and last one. And since we are in equation 14 in terms $E_1/2$ and $E_m/2$ we are only adding some negative terms, since all the bracketed terms are negative because we have assumed that E_l is larger than the average of the two therefore, E from equation number 14 it would be less than $E_1/2 + E_m/2$.

I repeat we have a series where this is the first term and this is the last term and in this series apart from these 2 terms we have lot many negative terms. And once these negative terms or these terms are subtracted from this first and last term we get E therefore, E must be smaller than $E_1/2 + E_m/2$ and this is what is written here. If you do not add those negative term then E will be smaller than $E_1/2 + E_m/2$.

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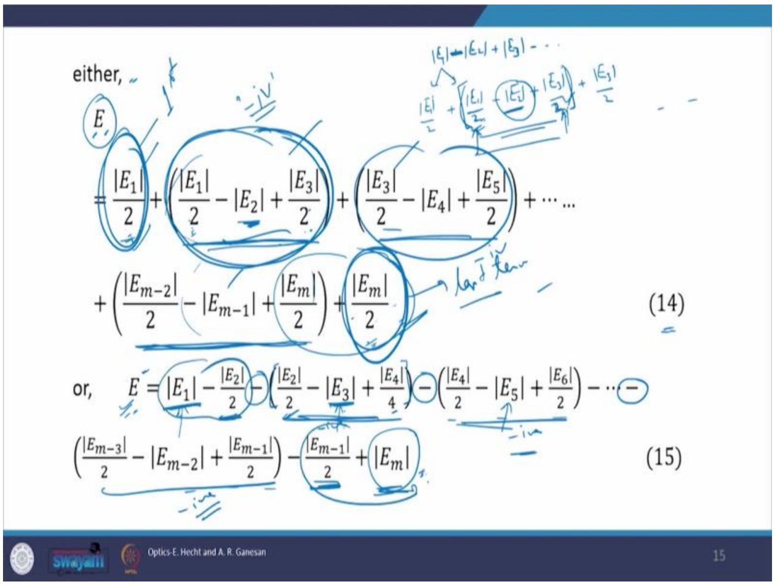

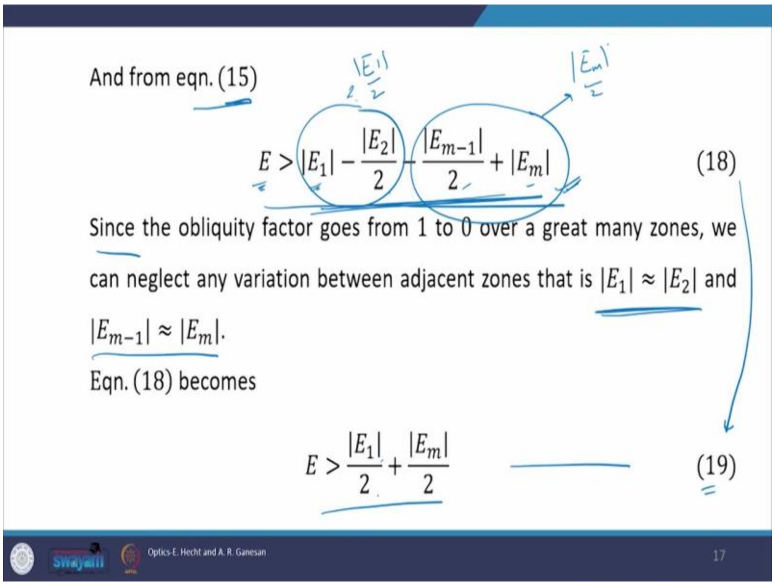

Similarly, from equation number 15, let us go to equation number 15. Now, now, we have assumed that the middle term is larger than the rest of the two in all the bracketed term. Therefore, here we get again negative term, this bracket term will also give a negative term, this bracketed term will also be a negative term, all these terms are negative, but before all these negative term we have minus sign therefore, all these bracketed term would be positive. Now, what are the free terms in the equation number 15 which do not have any bracket $|E_1|, -|E_2|/2, -|E_{m-1}|/2$ and $|E_m|$, these four terms they do not have any bracket and all these bracketed term are positive.

Now, if you develop a relation between or equality between E the total resultant field and these four terms which do not have any bracket then we will get this relation, E is larger than $|E_1|$

$-|E_2|/2 - |E_{m-1}|/2 + |E_m|$, why? Because from the right hand side, we have removed some positive term and therefore, E must be larger than these term. Had we have added those positive terms E would have been equal to this plus those positive term but since those positive terms are gone, we have removed them therefore, E must be larger than these terms.

Once these things are understood, then we can proceed further and we know that the obliquity factor, the maximum value of obliquity factor is 1 and the minimum value is 0 and this obliquity factor goes from 1 to 0 over a large angle, when angle $\theta = 0$ we have K=1 and when θ is very large then we get K=0, it means that obliquity factor goes from 0 to 1 over a great many zone, we have to consider very large number of zones for this to realise this change in K obliquity factor, therefore, we can neglect any variation between adjacent zones.

It means that since the two adjacent zone area is almost same therefore, we can assume the adjacent zone they contribute equally at the point of observation P. And therefore, we can roughly assume that $|E_1| \approx |E_2|$ or $|E_{m-1}| \approx |E_m|$. With this, this equation it reduces to question number 19, because now we assume that $|E_1| = |E_2|$ then from here we will get $|E_1|/2$. Similarly, from here we will get $|E_m|/2$ and with this we get a modified version of equation number 18 which is equation number 19 which says is larger than $|E_1|/2 + |E_m|/2$.

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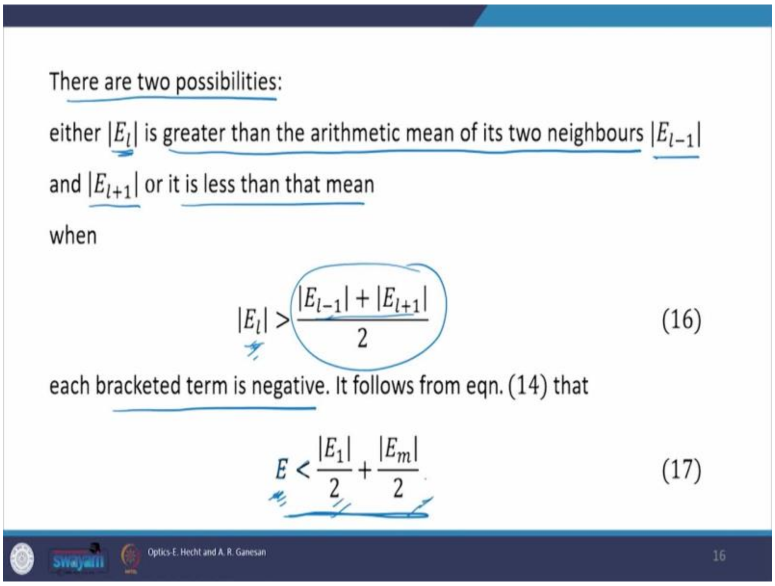


And from eqn. (15)

$$E > |E_1| - \frac{|E_2|}{2} - \frac{|E_{m-1}|}{2} + |E_m| \qquad (18)$$

Since the obliquity factor goes from 1 to 0 over a great many zones, we can neglect any variation between adjacent zones that is $|E_1| \approx |E_2|$ and $|E_{m-1}| \approx |E_m|$.

Eqn. (18) becomes

$$E > \frac{|E_1|}{2} + \frac{|E_m|}{2} \qquad (19)$$

The sum of the optical disturbances from all m zones at P is

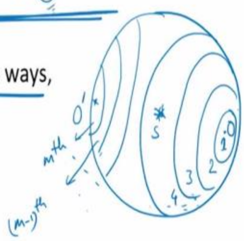
$$E = E_1 + E_2 + E_3 + \cdots \ldots \ldots \ldots . + E_m \qquad (12)$$

Since these alternate in sign, we can write

$$E = |E_1| - |E_2| + |E_3| - \cdots \pm |E_m| \qquad (13)$$

If m is odd,

The series can be rearrange in following two ways,



We can conclude fromeqn. (17) and eqn. (19) that

$$E \approx \frac{|E_1|}{2} + \frac{|E_m|}{2} \qquad (20)$$

The same result is obtained when

$$|E_l| < \frac{(|E_{l-1}| + |E_{l+1}|)}{2} \qquad (21)$$

If the last term m is even in eqn. (13), the same procedure leads to

$$E \approx \frac{|E_1|}{2} - \frac{|E_m|}{2} \qquad (22)$$

Similarly, we can conclude from equation 17 and 19, 17 is here and 19 is here. Now you will see that once E is less than $|E_1|/2 + |E_m|/2$ and once it is larger than $|E_1|/2 + |E_m|/2$ with those 2 equations, we can conclude that E is almost equal to $|E_1|/2 + |E_m|/2$. Now, for arriving at equation number 20, we have assumed that field due to the l^{th} zone is larger than the average of fields due to the adjacent zone.

Now we assume opposite if the field reaching at point P is smaller than the average of the adjacent zone. In this in this case too if we follow the same procedure and if the last term m is even in equation number 13, equation number 13 is this our main equation, this is equation number 13. Now, if we assume that m is even and then follow the same procedure, then we get this. I repeat from equation 17 and 19 we get this relation and the same result would be obtained when we assume that contribution from l^{th} zone is less than the average of the adjacent zone, then too we get the same result.

And on top of this, if we consider our second case, wherein m is even then from equation number 13, after repeating all the procedures, which we did in our last few slides, we get equation number 22. Therefore, equation number 20 and equation number 22 are our final results when m is odd and when m is even respectively, this is for m even and this is for m odd. Let us now talk about obliquity factor and its role in deciding the final expression of the resultant field due to all the half period zone at point of observation P.

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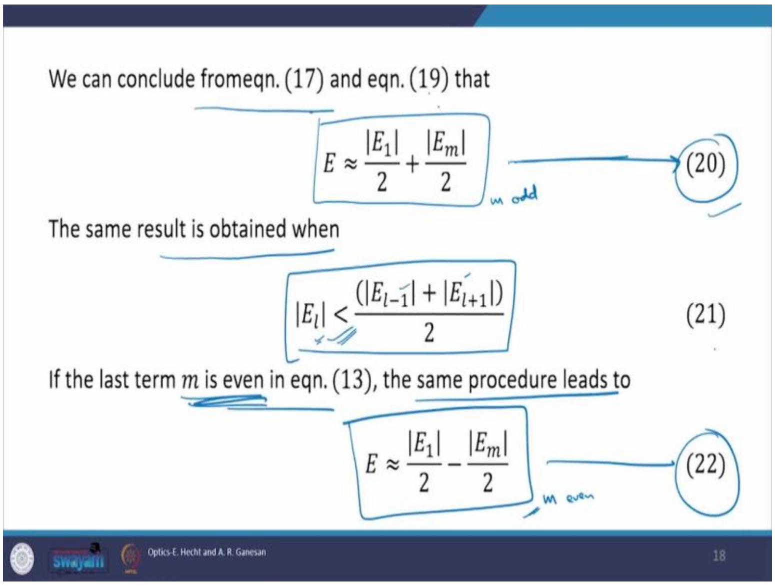

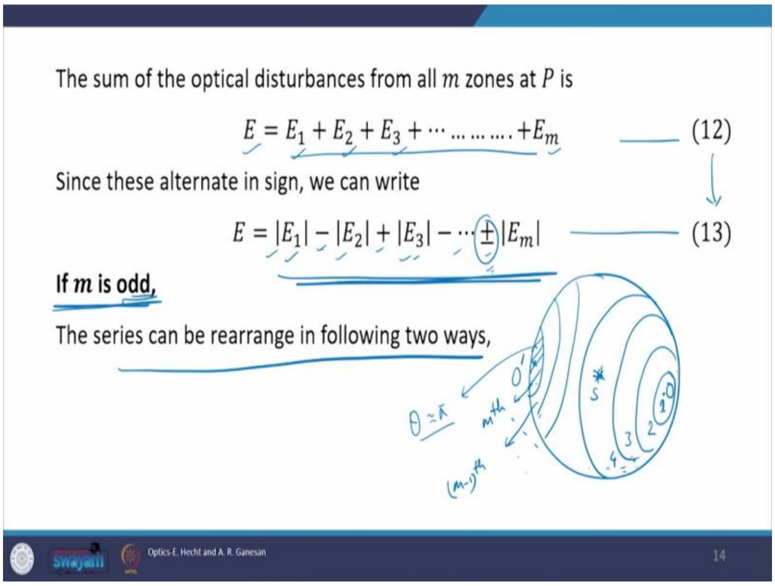

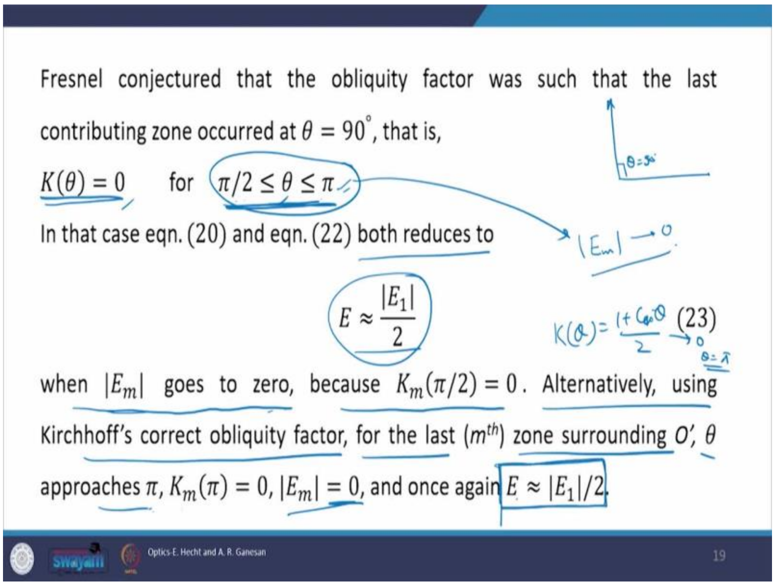

Now, Fresnel conjectured that the obliquity factor was such that the last contributing zone occurred at $\theta = 90$ degree, it started with $\theta = 0$ and we assumed that when $\theta = 90$ degree the last zone appear and therefore for $\theta = 90$ the K becomes 0, for K to be 0 θ should be between or equal to $\pi/2$ and π, this is the range for the θ which was given for K to be 0. With this equation number 20 and 22 which is here they both reduces to E almost equal to $|E_1|/2$ we can neglect this, this reaches to 0.

Why? Because this is the last half period zone and for this θ is between this range for E_m θ lies between $\pi/2$ and π and therefore, the obliquity factor goes to 0 which is a multiplication factor in the expression of E_m and therefore, E_m goes to 0 and this is what exactly is written here, when $|E_1|$ goes to 0 because of K_m $(\pi/2) = 0$ when this happens then $E = |E_1|/2$, but Kirchhoff's corrected the expression of obliquity factor, he said that obliquity factor goes to 0 when $\theta = \pi$. And when will $\theta = \pi$? $\theta = \pi$ for the last half period zone, which is this zone and for this zone you can easily see that $\theta = \pi$ it would be nearer to π, the last period zone will contain O', which is diagonally opposite to O.

With this let us again see equation number 20 and 22. Now, alternatively using Kirchhoff's correct obliquity factor for the last zone surrounding O', θ approaches to π and with θ $\pi/2$ this expression of obliquity factor which is $(1 + cos\theta)/2$ it again goes to 0 for $\theta = \pi$. It means $|E_m| = 0$ either you apply Huygens-Fresnel principle or the Kirchhoff's modification. Then the final resultant disturbance and at the point of observation P is therefore equal to $|E_1|/2$.

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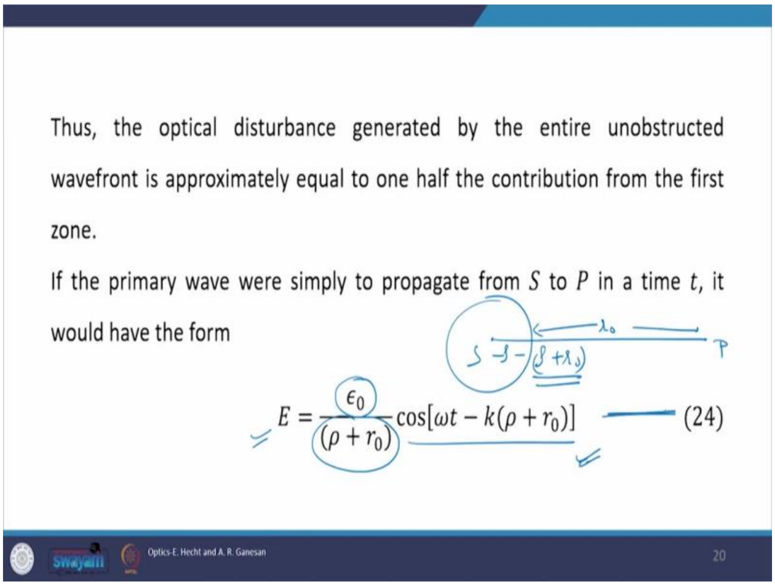

Now, we see that we have a point source which emits a spherical wavefront and then we splitted this spherical wavefront in large number of annular ring, we calculated the contribution from all the rings at the point of observation P. And what we found is that the total contribution at point of observation P is half of the contribution of the first annular ring or the first half period zone. Thus, the optical disturbance generated by the entire unobstructed wavefront is approximately equal to one half the contribution from the first zone, rest got cancelled among themselves, the phases is such that rest do not contribute.

Now, this is a quite cumbersome complicated analysis, but effectively what we did is that we considered a point source and we considered a point of observation P and we will let the wave propagate from this point source to the point of observation P and then following this analysis, we calculated the field at point P, but there is a very straightforward way of calculating the field, we know what would be the field at point of observation P from a point source S which is separated by certain distance and what is this traditional way? This is this expression, the amplitude by the radial distance into phase part $cos[\omega t - k(\rho + r_0)]$, because effectively we have a source and we have a point of observation and this spherical wavefront is start from a point source and this distance is $\rho + r_0$, ρ is the radius of this wavefront and r_0 is this distance.

Now the total distance is $\rho + r_0$ then if you are asked to calculate the field at a point P which is at a $\rho + r_0$ distance use the simple formulation E would be equal to amplitude by radial distance into cosine function and then the phase part $\omega t - k(\rho + r_0)$, this is the expression which we have used multiple times. Now, this equation number 24 must be equal to the expression which we calculated which we derived using Fresnel half period zone but half this from Fresnel half period zone we know that the resultant E would be $E_1/2$.

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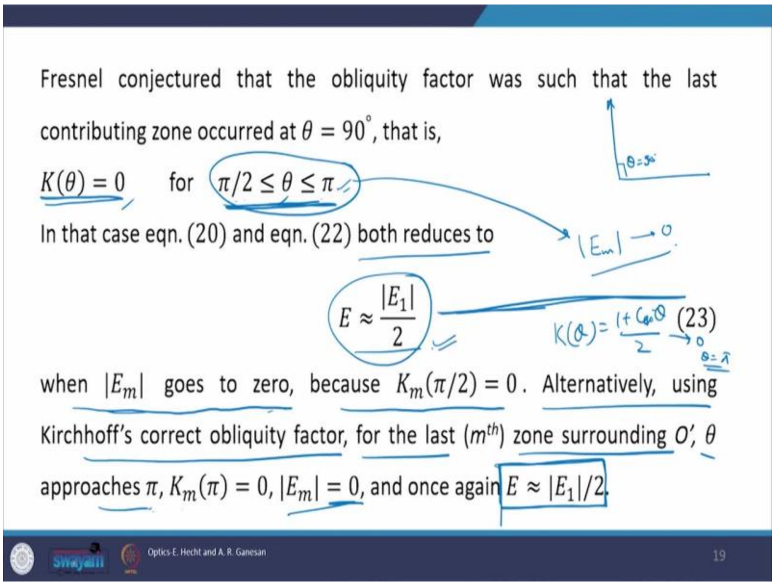

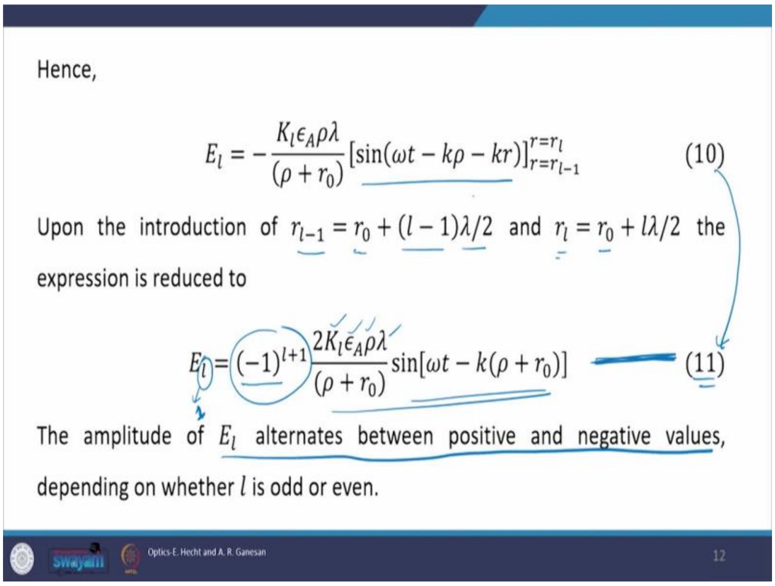

What is E_1 then, the E_1 is given here, let us go to equation number 23 this is the final expression from half period zone calculation. $E = E_1/2$ and what is E_1? To have an expression of E_1 let us go back to the general expression of E_l, this is equation number 11 is the expression here in we will just replace l by 1 in place of l we will write 1 and then we will write rest of the term on the right hand side and this will give the expression of E_1 and then we will half this expression we will just divide it by 2 and this will give us the resultant field at a point of observation P.

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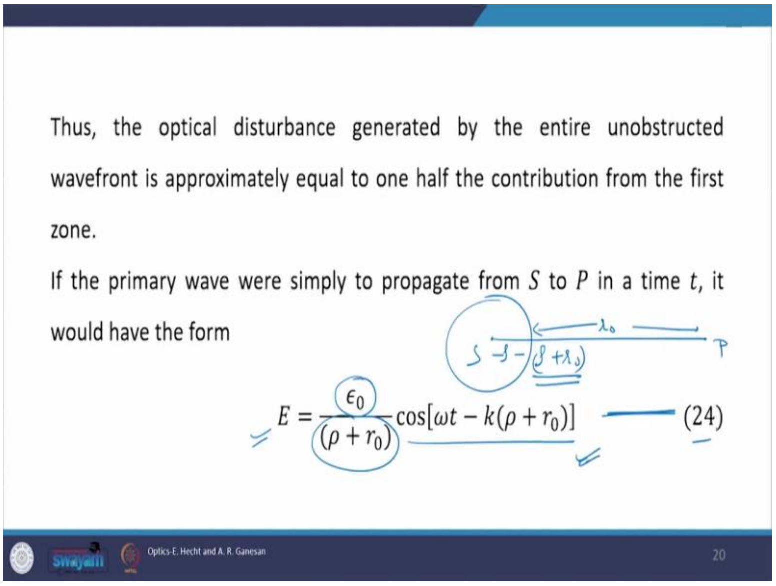

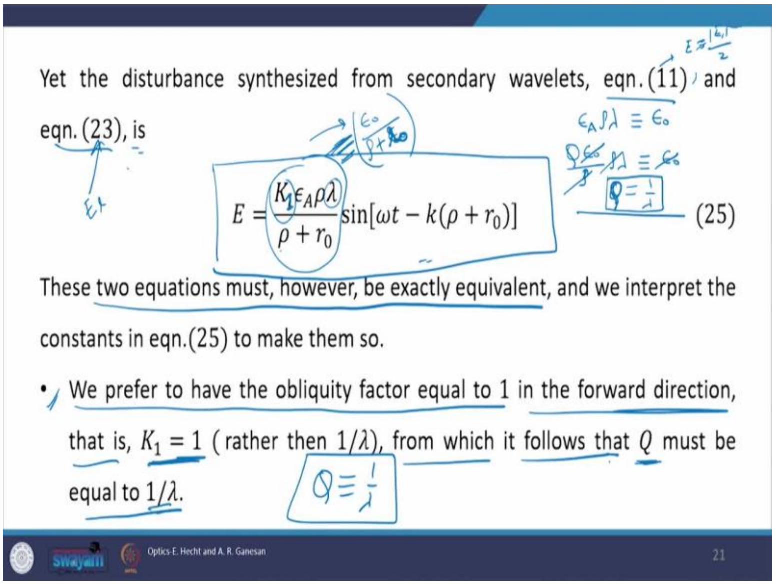

Let us do this and after doing this we get this expression. The disturbance synthesised from secondary wavelets, equation 11 and equation 23 is this equation number 25, equation 23 is the expression of E_l and equation 11 is equal to almost equal to $|E_1|/2$ and with these two equation we can get equation number 25 very easily. Now this equation number 25 must be identical with equation number 24 but let us compare now, we will compare these 2 equations must however be exactly equivalent because they are saying the same thing and we interpret the constant in equation number 25 to make them so.

If you remember while introducing the strength of the source per unit area we introduced some constant Q which is unknown and K is also there, it is K_1. K_1 is also here and we will also calculate the value of K here by just by comparing equation number 24 and 25, let us compare them. Let us compare the amplitude part here amplitude is $\epsilon_0/(\rho + r_0)$ and here amplitude is $K_1\epsilon_A\rho\lambda/(\rho + r_0)$. Let compare them with this, sorry it is r_0, ϵ_0 this is the amplitude here. Now, this amplitude must be equal to the amplitude of equation number 24.

Now, since we are talking about the first half period zone, for first half period zone you know the angle is very small, it is very close to 0 therefore, we prefer to have obliquity factor equal to 1 in the forward direction and that is K_1 would of course be equal to 1. If we put $K_1 = 1/\lambda$ then this λ will go with K and we would be close to the amplitude part of equation number 24, but we know that we are talking about $E_1/2$ and for E_1, θ is very small therefore, $K_1 = 1$, we cannot put $K_1 = 1/\lambda$ just to adhere with this equivalence.

From which it follows that the $Q = 1/\lambda$, why? Because when $K_1 = 1$ then what we are left with is this term, this must be equal to ϵ_0 but what is ϵ_A, $\epsilon_A = Q\epsilon_0$ and it was equal to ρ here in the denominator and then $\rho\lambda$, this must be equal to ϵ_0. Then from here ρ will go away, ϵ_0 will go away and from here $Q = 1/\lambda$.

Therefore, if we want to adhere with this equivalence if we want the two expression 24 and 25 to exactly say the same thing then $Q = 1/\lambda$ and this is how we decided the unknown constant Q, we initially assumed that there is some constant Q which appear in the sources strength now, its values decided with the with this equivalence here and $Q = 1/\lambda$, Q is now no more unknown coefficients or unknown constant this is the first point which came out of this comparison.

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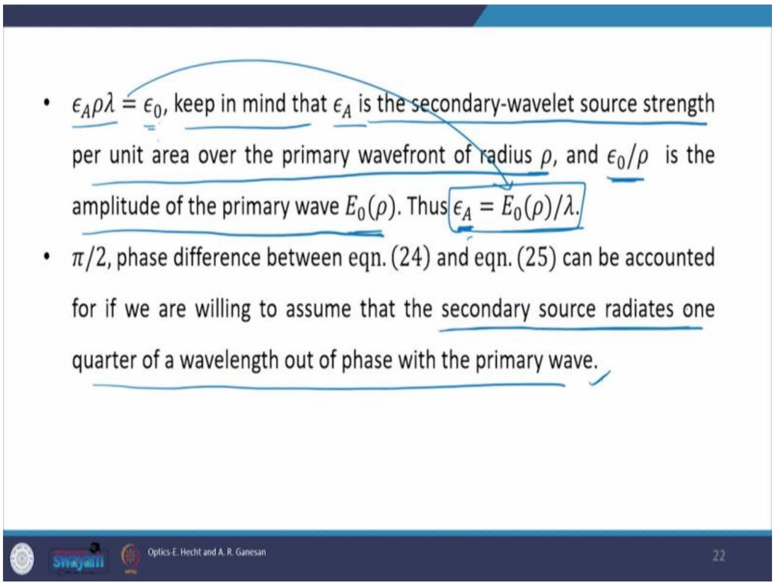

What is second point? Now $\epsilon_A \rho \lambda$ this must be equal to ϵ_0 this is what is written $\epsilon_A \rho \lambda$ this must be equal to ϵ_0. Keep in mind that ϵ_A is the secondary wavelet source is strength per unit area over the primary wavefront of radius ρ. ϵ_A is secondary wavelet sources strength per unit area over the primary wavefront of radius ρ and ϵ_0/ρ is the amplitude of the primary wave $E_0(\rho)$ therefore, the source of strength per unit area ϵ_A would be $E_0(\rho)/\lambda$ which is again very much clear, this came from this relation directly.

$\epsilon_A = \epsilon_0/\rho\lambda$ and we know that $\epsilon_0/\rho = E_0(\rho)$ and with this replacement we get $\epsilon_A = E_0(\rho)/\lambda$. Once we have equated the amplitude part, let us go to the phase part. Now, let us go again to equation number 24 and 25, in equation number 24 we have $cos(\omega t - k(\rho + r_0))$, while in equation number 25 we have $sin(\omega t - k(\rho + r_0))$. In equation number 24 we have cosine term while in equation number 25 we have sine term.

$\pi/2$ phase difference therefore, is there and this $\pi/2$ phase difference between equation number 24 and 25 can be accounted for if we are willing to assume that the secondary source radiates one quarter of a wavelength out of phase with the primary wave, there is something which is very new. Here what is being introduced is that secondary wave source or secondary wavelets are out of phase with the primary one by $\pi/2$ angle, there is a phase difference between the primary and secondary radiations. If we assume that this is correct, then only equation number 24 and 25 they say the same thing then only they will be exactly equivalent.

I will talk more on this phase difference between primary and secondary source in coming lecture, but right now, we will just assume that this is so and if we assume this then we see that this equivalence is correct, the equation 24 and 25, they are exactly equivalent and they are same and everything is well justified, all the analysis what we did using this secondary wavelet theory it is correct till now. This is the first introductory lecture on Fresnel diffraction and the mathematics which be developed it would be used throughout in understanding the diffracting element in Fresnel zone domain. With this I end my lecture, and thank you for joining me, see you all in the next class.