Applied Optics
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Lecture 39
Grating Spectrum and Resolving Power

Hello everyone, welcome to the class. In last class, I introduced you grating there we talked about grating equations. And we saw that the grating can work in two modes first is called transmission mode, second is reflection mode, the grating which work in transmission mode, we call them transmission grating while the grating which work in reflection mode, we call them reflection grating.

And we also saw the grating equation for the normal and oblique incidences and then we saw that we can also shift the energy from a particular order in a grating to some given order or some other order of grating by just incorporating blaze or some initial tilt in the grating element. Today, we will talk about the most important application of the grating which is in a spectroscopy.

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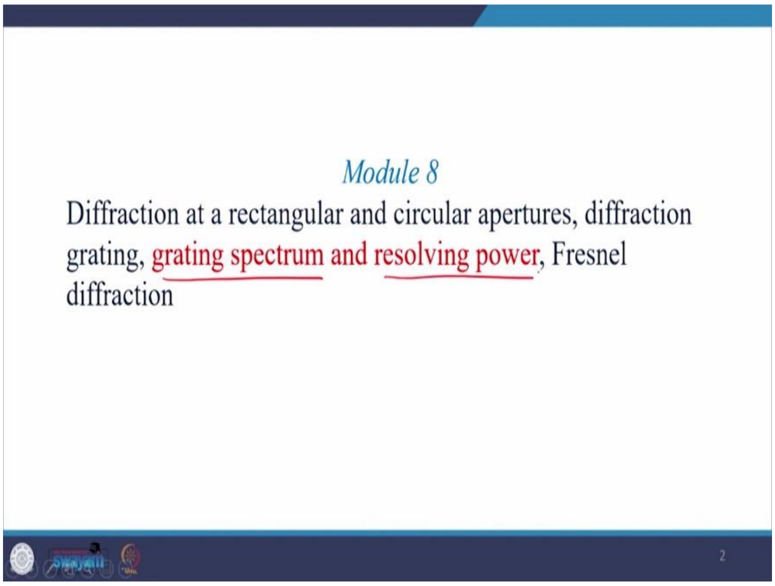

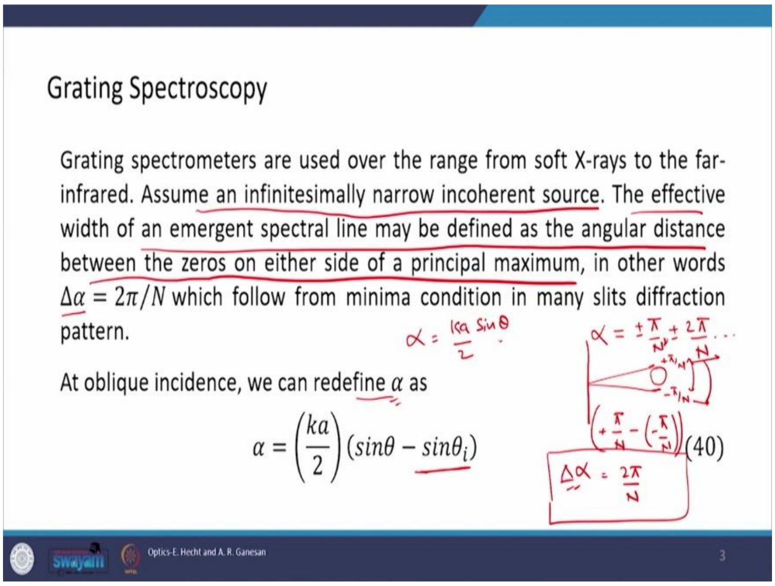

And here we will talk about grating spectrum and its resolving power. Now, grating spectrometers are used over the range from soft X-rays to the far-infrared, it all depends upon the grating element, for different wavelengths, we will have to just tune the size of the grating element. Now, assume and infinitesimally narrow incoherent source. Suppose we have a source which is very narrow, as narrow as possible.

The effective width of an emergent spectral line may be defined as the angular distance between the zeros on either side of the principal maxima, because suppose we have a line and then it is forming a pattern since it is infinitesimally thin then what will happen? It will create a pattern with some central maximum here. And if we want to calculate the width of this principal maximum then we will have to calculate the angular positions of first minima on the either side of principal maxima.

Now, we know that in many slit diffraction patterns, since we are talking about grating therefore, we will always correlate the grating with the multiple slit diffraction. Now, in many slit diffraction pattern, we know that the positions of minima are given by $\pm\pi/N, \pm 2\pi/N$, these were the positions of minima in multiple slit diffraction pattern, $\pm\pi/N, \pm 2\pi/N$.

Now, these positions of principal maxima allow us to calculate the angular separation of minima on either side of principal maxima. How can it be calculated? For the first minima which now we located at position $+\pi/N$ here and $-\pi/N$ here now, if you want to calculate its angular width, just subtract these 2 angles, one angle is given in this direction another angle is given in this direction just take calculate the overall angle.

The overall angle here in this case would be $+\pi/N - (-\pi/N)$ which would be equal to $2\pi/N$ and this will give us the angular distance between the zeros on either side of this maxima and which we designate by $\Delta\alpha$. Now, for oblique incidence this α can be redefined the usual $\alpha = (ka/2)sin\theta$, but this condition is only for normal incidence, for oblique incidence this term would be added $-sin\theta_i$ as we also did the same in grating equation.

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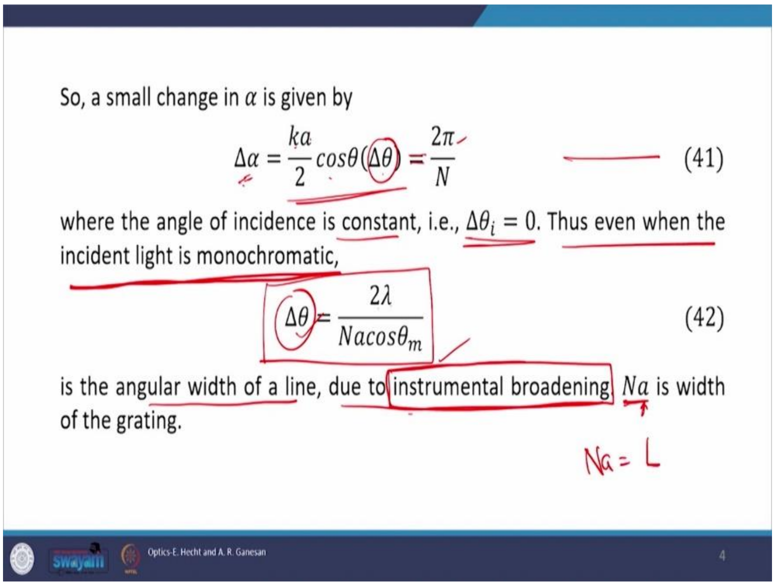

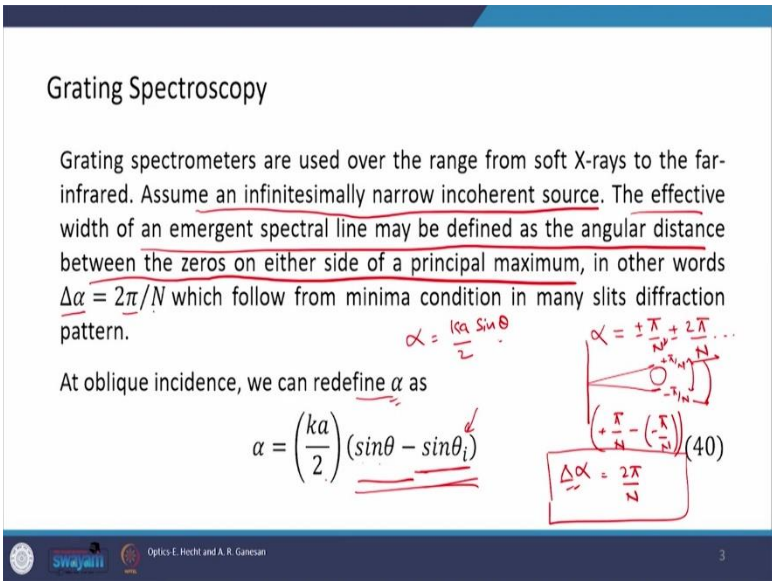

Now, with this modified α we can calculate the change in α or small change in α. How to calculate it? We just differentiate it, now if we differentiate equation number 40 where θ is the angle of incidence which is fixed then you will get $(ka/2)cos\theta\Delta\theta$, this is what is given here, $\Delta\alpha = (ka/2)cos\theta\Delta\theta$ and which is nothing but $2\pi/N$ therefore, we are equating it here with $2\pi/N$.

Now, angle of incidence is constant therefore, we have neglected $\Delta\theta_i$. Now, does even when the incident light is monochromatic here, we are assuming that there is a single wavelength source. We have some spread in θ which is Δθ and this Δθ from equation number 41 can be expressed in form of equation 42 which says that $\Delta\theta = 2\lambda/(Nacos\theta_m)$ and this is the angular width of a line due to instrumental broadening.

The instrumental broadening means that this width has nothing to do with the wavelength spread in the source because even when there is only one wavelength in the source there would be some spread which is here being measured by Δθ and therefore, this spread is named or this spread who owes its origin in instrumental broadening, N is the number of slits and a slit-to-slit separation, a start from center of one slit and it goes to the center of other slit. Therefore, Na would be equal to the length of the grating, Na is the length of the grating L.

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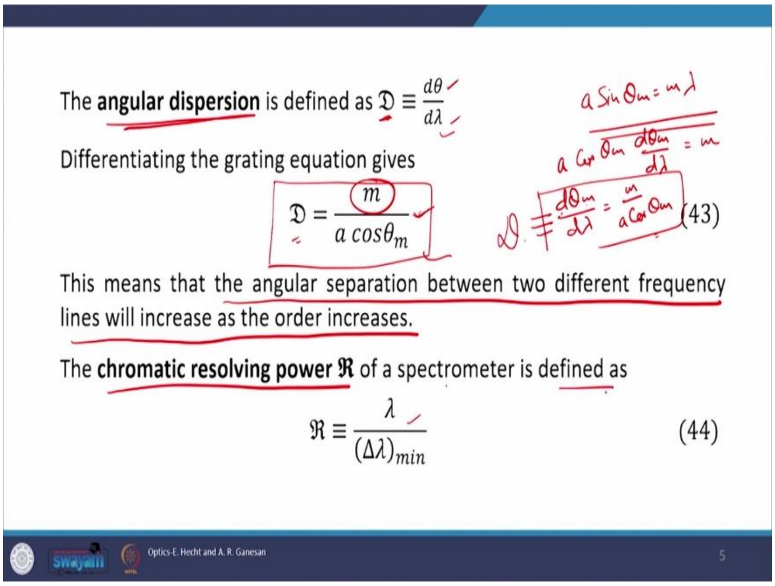

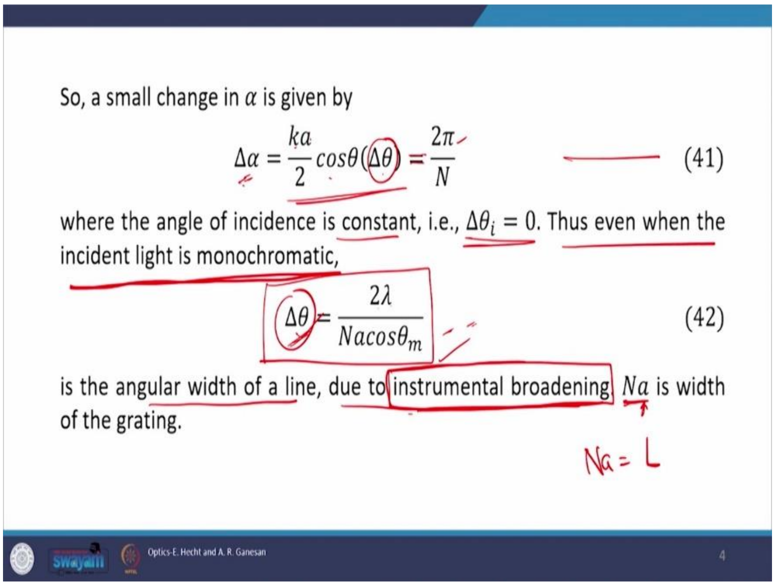

Now, there is another very important parameter which is called angular dispersion and it comes when our source is non-monochromatic, we calculated this angle, the angular spread for sources, which are monochromatic because we cannot have monochromatic sources in real life, all the sources, the real sources, they have certain wavelengths spread. But consider or assume that there are a source which is monochromatic still in that particular case there would be some broadening which we call instrumental broadening, but what if the source has some wavelength spread or bandwidth in that particular case we define angular dispersion which is designated by this italic D and is defined as dθ/dλ.

Now, if you remember the grating equation is given by $asin\theta_m = m\lambda$. Now, if you differentiate this grating equation then you get $acos\theta_m\, d\theta_m/d\lambda = m$. From here $d\theta_m/d\lambda = m/acos\theta_m$ now, this is nothing but your angular dispersion D therefore, this is for grating for normal incidence we can again write this expression for angular dispersion in a modified form and this equation is again valid for oblique incidence too because the angle of incidence is fixed.

Now, this means that angular separation between two different frequency lines will increase as the order increases here, you see that order is sitting here in the numerator and D is nothing but Δθ/Δλ or dθ/dλ. So, as m increases dθ also increases. It means angular separation between two frequency lines will increase with order.

Now, apart from this angular dispersion, there is one more important parameter which we should know and this is known as chromatic resolving power R, the chromatic resolving power of a spectrometer is defined as $\lambda/\Delta\lambda_{min}$, λ is the average wavelength of the source. Suppose, the source has several wavelength that λ is average of that and $\Delta\lambda_{min}$ is the minimum resolvable wavelength separation. The $\Delta\lambda_{min}$ is the least resolvable wavelength difference.

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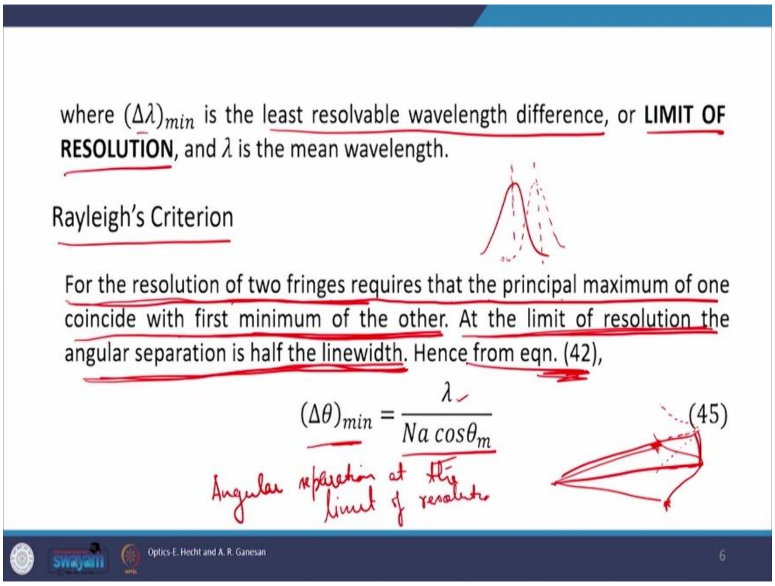

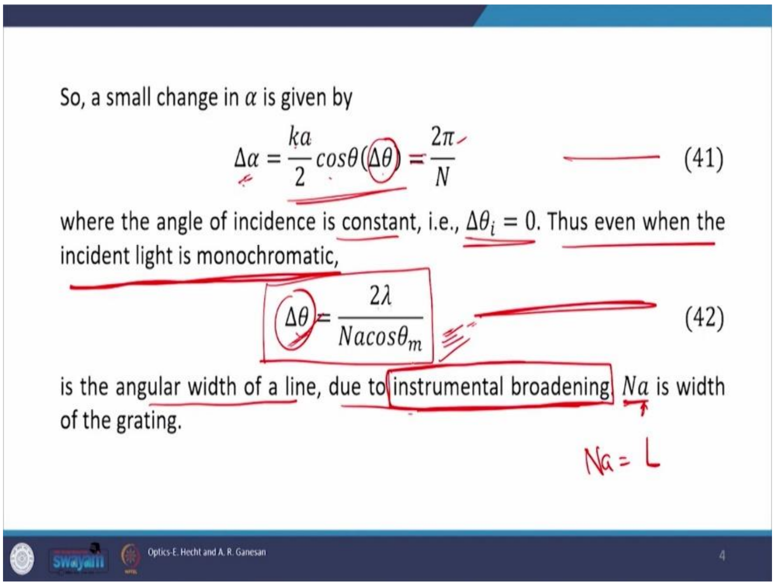

And there is another name of this $\Delta\lambda_{min}$ and which is limit of resolution. Now, once the limit of resolution jumps in into the discussion then we also talk about what is the criterion of refining the resolution then again Rayleigh comes into the picture and Rayleigh gave a criterion which is the same criteria which we discussed in case of multiple slit the criteria says further resolution of two fringes requires that the principal maximum of one coincide with the first minimum of the other.

At the limit of resolution, the angular separation is half the line width, I will reread it, for the resolution of two fringes requires that the principal maximum of one coincide with the first minimum. Suppose there is a fringe pattern which has maxima here than the two wavelength which are forming the diffraction pattern at a screen at some angle, if the principal maxima of one fall on the first minimum of the other then only the two wavelengths are set to be resolved or just resolved, if the separation is larger than this then they can easily be resolved if the separation is smaller than this than the instrument cannot resolve it.

$\Delta\lambda_{min}$ is the minimum the least resolvable difference and it happens when the maxima of one falls at the first minimum of the other and this is called the limit of resolution. And at the limit of resolution, the angular separation is the half the line width because this is our source and the source generator principal maximum and this is the first minimum of the principal maximum which is on the either side of the principal maximum.

Now, on this first minimum, the principal maximum of the other wavelength is falling therefore, this angle, this would be half of $\Delta\theta$ which we measure here in equation number 42. Therefore, at limit of resolution the angular separation would be $\Delta\theta/2$ which we call as $\Delta\theta_{min}$ and this is what is defined here, at limit of resolution the angular separation is half the line width.

$\Delta\theta$ is your line width and hence from equation 42 the equation which I showed you a minute before $\Delta\theta_{min} = \lambda/(\Delta\lambda)_{min}$, this is coming from equation number 42 just, we divided 42 by 2 and then we have this equation number 45 which defines the angular separation at the limit of resolution, this is angular separation at the limit of resolution.

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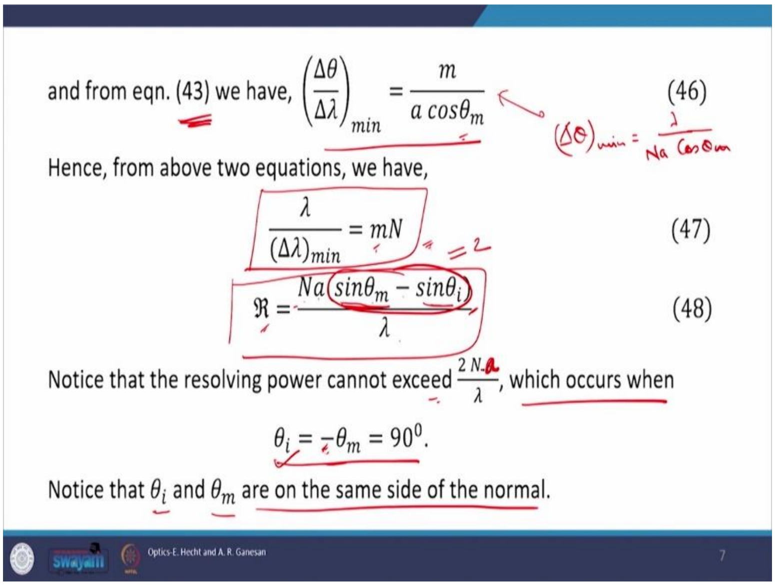

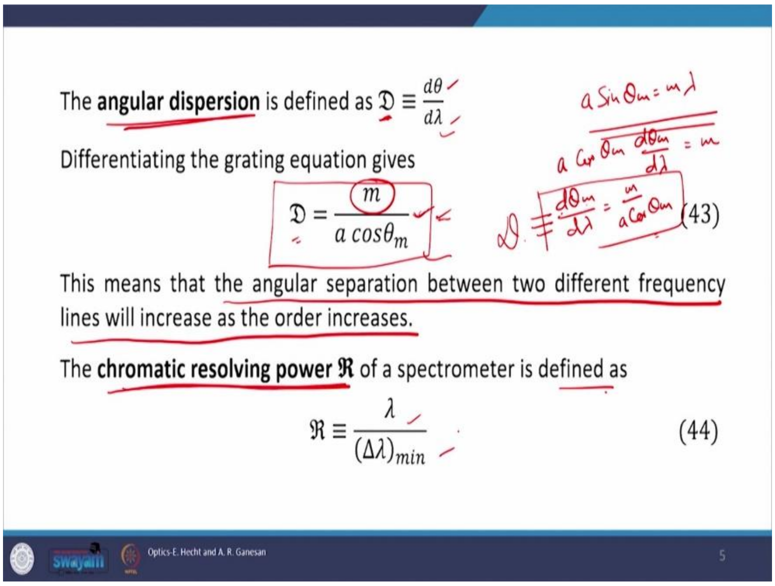

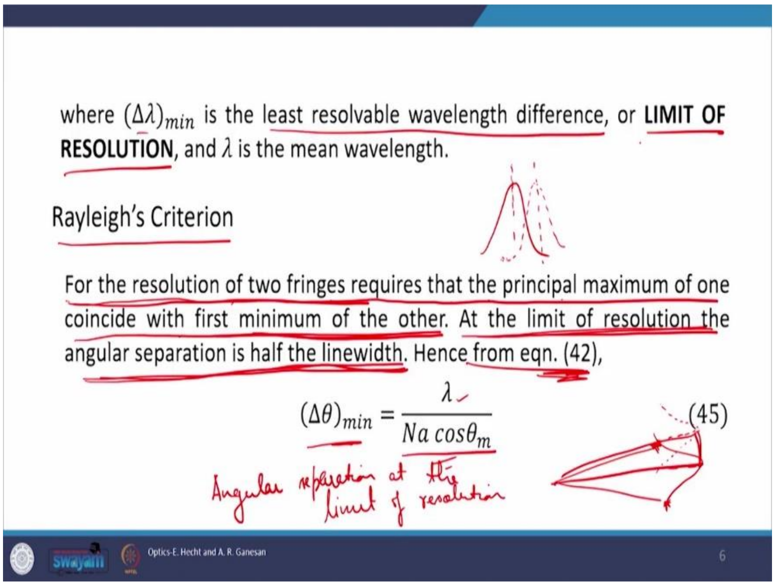

Now, once this is known from again equation 43, we can calculate $\Delta\theta/\Delta\lambda$ 43 is here, which is angular dispersion. Now, $\Delta\theta/\Delta\lambda = m/acos\theta_m$ we have this expression and we have this expression $\Delta\theta_{min}$which is nothing but $\lambda/Nacos\theta_m$, if you divide these two expressions then you get $\lambda/(\Delta\lambda)_{min}$ which is this chromatic resolving power, $\lambda/(\Delta\lambda)_{min}$.

Therefore, chromatic resolving power it is given by mN, N is the number of slits in the grating and m is the diffraction order, the chromatic dispersion power is therefore, equal to mN and it can equivalently be given by this relation which is $Na(sin\theta_m - sin\theta_i)/\lambda$ where we have used grating equation to replace m.

Now, what is the maximum possible value of 48, the maximum possible value of resolving power would be 2 Na/λ here, it is a here, 2 Na/ λ. How to calculate it? This will occur when angle of incidence is 90 degree and when it is equal to angle of diffraction. With this we will have value of $sin\theta_m = 1$ while the value of $sin\theta_i = -1$ or vice versa, we will have this bracket would be equal to 2 and from here we get the maximum value of this resolving power as 2 Na/λ. The minus sign which is here appearing here it is only due to the fact that θ_i, θ_m are on the same side of the normal. Thus, what we saw is that analogous to multi slit experiment we can define resolving power, we can define $\Delta\theta_{min}$.

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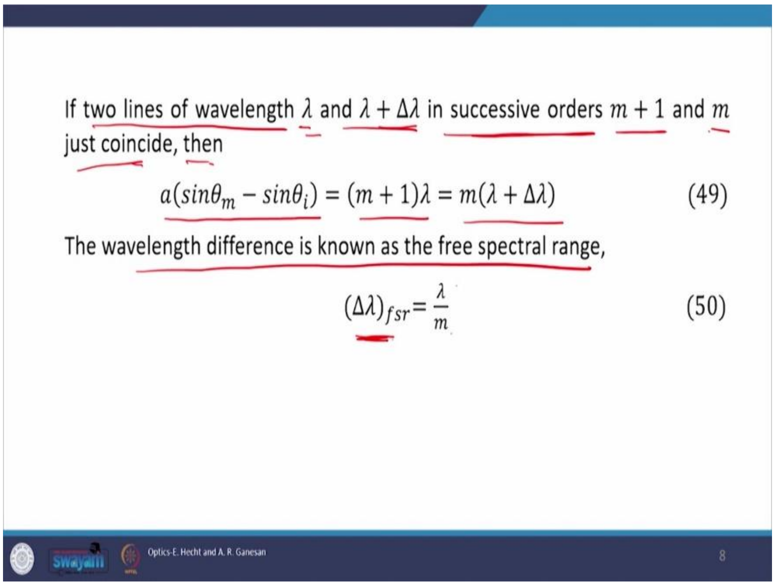

Now, suppose we are given two lines of wavelength λ and λ+$\Delta\lambda$ which are very closely spaced and their successive orders of course would be (m+1) and these successive orders we assume that they are coinciding. Now, if two lines of wavelength λ, $\lambda + \Delta\lambda$ in successive orders (m+1) and m just coincide then from our grating equation what we can write is that $a(sin\theta_m - sin\theta_i) = (m+1)\lambda$ while for the other wavelength this would be equal to $m(\lambda + \Delta\lambda)$.

Now, if you solve this then you get the expression of $\Delta\lambda$, the wavelength difference is known as free spectral range and this is given by λ/m, it means that if we have two wavelength and they are successive order (m+1) and m coincide then the separation between the two wavelength is called free spectral range and which is given by equation number 50. And, numerically you can calculate it by taking the ratio between λ and m.

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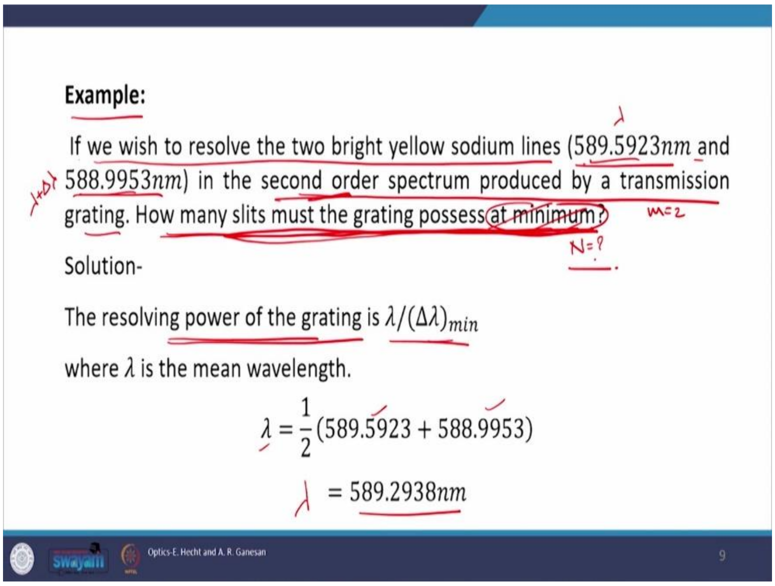

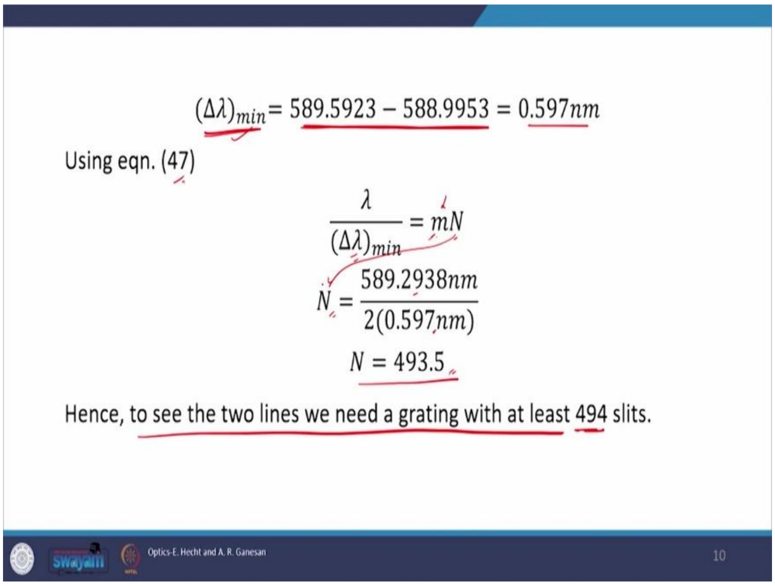

Once this all things are known, we can now solve an example here the problem says if we wish to solve the two bright yellow sodium lines with a wavelength given by this and this in the second order spectrum produced by the transmission grating how many slits must the grating possess at the minimum? What are given? The two wavelengths are given here λ is given and $(\lambda + \Delta\lambda)$ is given, the second order (m) is also given and what is being asked, how many slits the grating possess, at minimum the N is being asked? Two wavelengths are given let us calculate the mean wavelength than.

The mean wavelength is given by the (wavelength 1 + wavelength 2)/2, this is now the mean wavelength, why do we want to calculate the mean wavelength because we want to calculate the resolving power of the grating which is a $\lambda/\Delta\lambda_{min}$. If we want the number of slits to be minimum, then these wavelengths which are given here they must be the minimum wavelength separation $\Delta\lambda_{min}$.

Now, two wavelengths are given from there we can calculate $\Delta\lambda_{min}$. Now, we will use the chromatic resolving power expression which is $\lambda/\Delta\lambda_{min} = mN$. Since, m=2 and $\Delta\lambda_{min}$ is also calculated, λ is also calculated from here we can calculate N which is 493.5. Hence, to see the two lines we need a grating with at least 494 slits which is the nearest integer to the calculated N. I would like to repeat again if we relax this condition which is at minimum then we will have several solutions here. But we want the minimum number of slit to reduce this spectrum.

As the question says that we want to resolve two bright yellow sodium line. Whenever we say resolve means we are seeking $\Delta\lambda_{min}$. Now, if we want to find $\Delta\lambda_{min}$, it means that we are behind resolving power of the grating, resolving power is given by this. Then we can easily calculate mean wavelength, $\Delta\lambda_{min}$ is known, m is known from here we can easily calculate N.

This is all for today. I end my lecture here. Thank you for listening me. See you in the next class.