Applied Optics
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Lecture: 30
Concept of Coherence - II

Hello everyone, welcome to my class. Today, we will continue with concept of coherence. In the last class, we talked about temporal coherence and the corresponding coherence length. And in this topic, we talked about two experiments, the first one is Young's double slit experiment and the second is Michelson interferometer. Today, we will touch the other category of coherence which is spatial coherence.

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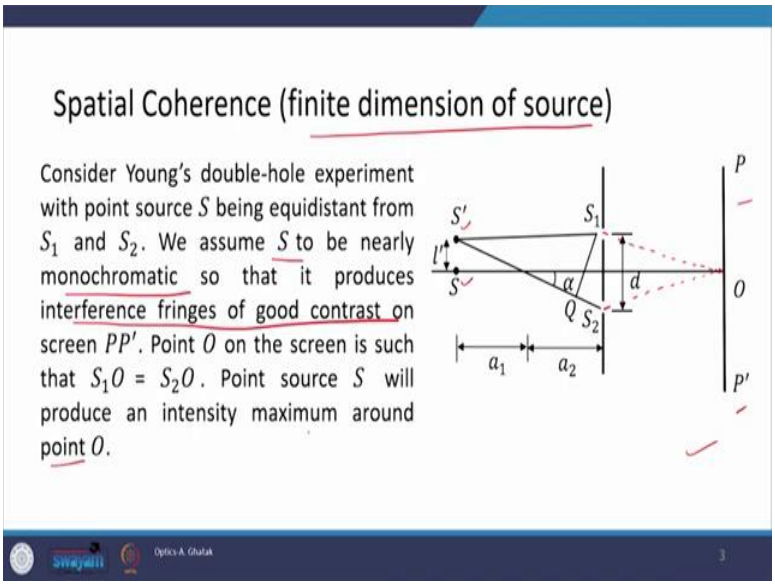

As discussed in the last class, it is related to the finiteness of the source, finite dimension of the source in space. Now, begin with the same experiment which we discussed in the last class for temporal coherence, which is our Young's double slit experiment. Now, here instead of taking one source S from which we generate two sources S1 and S2.

We will take two sources S and S' and S' is assumed to be displaced linearly, linearly means it is displaced vertically by a length l' from S, as shown here in this figure. Now, it is also assumed that S is nearly monochromatic and therefore, it produces interference fringes of good contrast on this screen PP'.

Now, O is a point which is at the center of the screen which is at the axis of the symmetry therefore, S_1O this path length would be equal to S_2O. And therefore, the point source S will produce an intensity maximum around point O.

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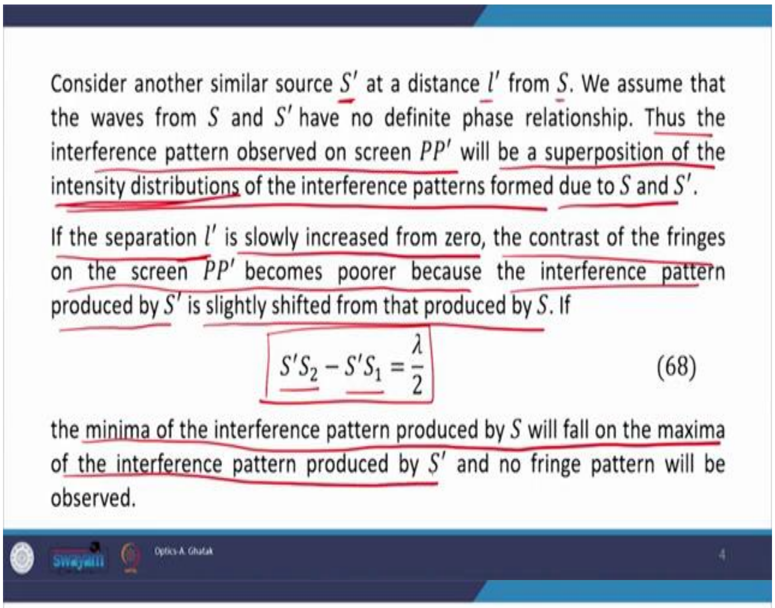

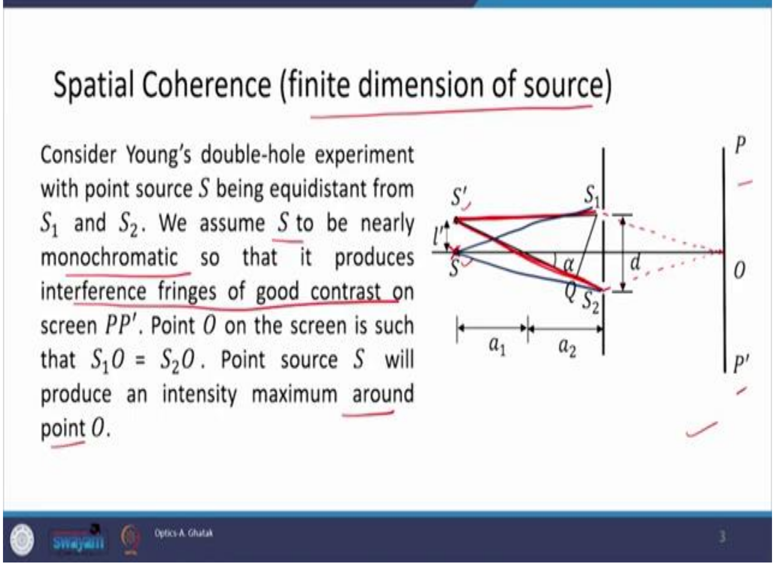

Now, the next source which we name as S' which is at a distance l' from S, for this particular source we assume that it does not have any definite phase relationship with source S, it means S and S' sources are incoherent. Therefore, the interference pattern observed on the screen will be a superposition of the intensity distributions of the interference pattern formed due to S and S' individually.

Now, observe the word, it is superposition of intensity distribution not the amplitude, why, because these sources are incoherent. Therefore, the resultant pattern at the screen would be the sum of irradiances produced by source S and source S'. Now, if the separation l' is slowly increased from 0, initially we assume that l' is 0, it means that S and S' are sitting at the same point.

Therefore, very beautiful fringes are very good contrast would be absorbed, but if we slowly increase l' if we start displacing the sources S' from S then what will happen, the contrast of the fringes on the screen PP' becomes poorer because the interference pattern produced by S' is slightly shifted from that produced by S.

Now, we will as l' would be nonzero then we will have two point sources and these two point sources will produce their independent fringe pattern on the screen and then we will have to add the two radiances and therefore, the contrast of the final pattern would be reduced. Now, if $S'S_2 - S'S_1 = \lambda/2$ it means if these rays are such that the path difference plays half integral multiple of wavelength that is they are creating destructive interference then the minima of the interference pattern produced by S will fall on the maxima of the interference pattern produced by S' which is very much clear. Here S is symmetrically situated.

Therefore, the distance S, let me pick different color the different, the distance $SS_1 = SS_2$ and therefore, the rays which are reaching at point S_1 and S_2 from source S, they are in the same phase, they are phase correlated. Now the second source which is S' since it is not on the axis of symmetry since it is slightly displays the rays which are reaching to point S_1 and S_2 from S', there would be a slight phase difference and if this phase difference is half integral multiple of lambda.

In fact, in this case the minima of the interference pattern produced by S will fall on the maximum of the interference pattern produced by S' and if, this ultimately will give no fringe pattern or uniform illumination on the screen will be visible therefore.

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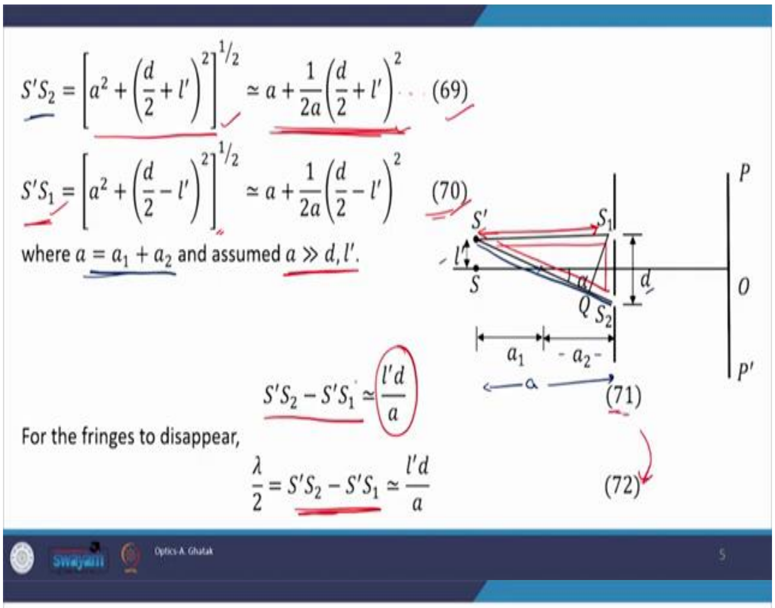

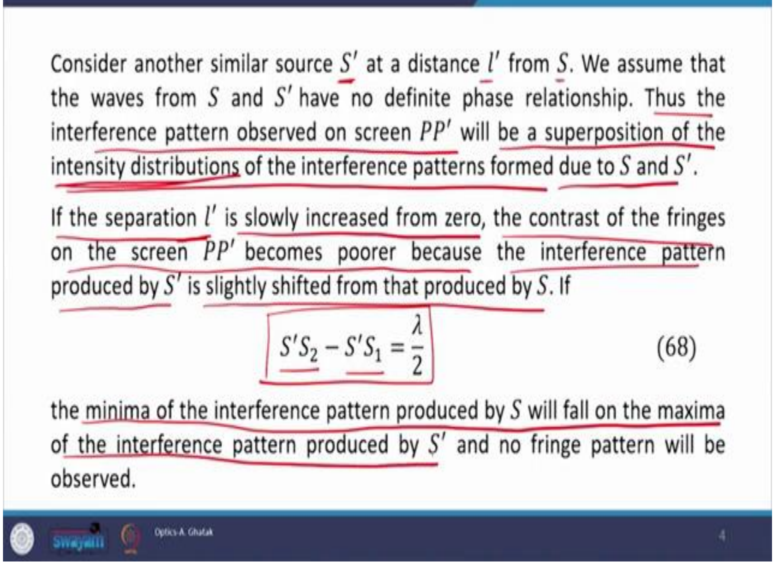

Now, let us do some geometry, let us calculate $S'S_2$ distance, this is $S'S_2$. What would be $S'S_2$? Now, we know that given the distance between S_1 and S_2 is d, S' and S is l', this distance is a_1 where this ray crosses the axis of symmetry is a_1 and this distance is a_2 and $a_1 + a_2$ is a which is given here.

With this condition we can calculate $S'S_2$. Now, if you consider this right angled triangle, then $S'S_2$ is hypotenuse and $S'S_2$ to would be $S'S_1^2 + S_1S_2^2$ and then square root of this sum and this is what is done here. Now, you can expand it using binomial expansion and this reduces to this expression.

We can use binomial expansion and neglect higher order term because a is assumed to be much-much larger than d and l', d is separation between S_1 and S_2, l' is the displacement of S' from

S. Similarly, we can calculate $S'S_1$ which is this length, using the Pythagoras theorem here and this gives equation number 70.

Now, once these two distances are known, then to calculate the path length difference, we will subtract equation number 70 from 69 and then $S'S_2 - S'S_1$ can be calculated and then if you substitute for $S'S_2 - S'S_1$ from 69 and 70 and do a little bit of mathematics then you will get this expression which is $l'd/a$.

The path length difference is $l'd/a$, but, the condition with which we start with is that $S'S_2 - S'S_1 = \lambda/2$ and if this condition holds, we should not get any fringe, let us substitute this expression of $S'S_2 - S'S_1$ which is $\lambda/2$ in equation number 71, this gives equation number 72.

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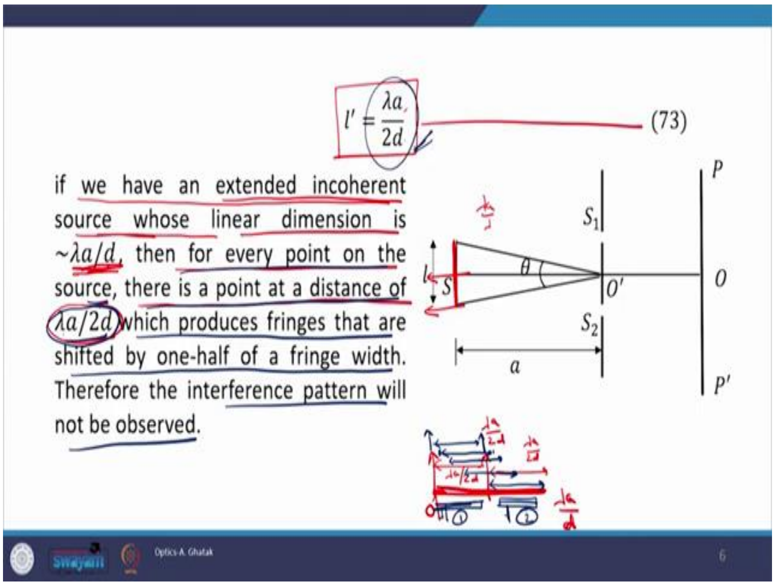

And from here we can calculate the expression for l' which is the separation between the two sources S' and S and this equation number 73 thus gives the requisite separation. It means, if we have two sources and one is on the axis and one is laterally displaced by l' with this distance which is λa/2d then we will not observe any fringe, or if the displacement is larger than this l' then too we will not absorb any fringe, but if this displacement is smaller than this length then the fringes will be observed, we will be able to see the fringes on the screen.

Now, if we have an extended incoherent source, now we are taking a different case, till now, we consider two points sources, now, but now we are considering a line source now, line source means array of point sources.

Now, we have extended incoherent source whose linear dimension is given by λa/d, which is twice of l' because twice of l' is λa/d. Now, we consider an extended source which is extending from here to there, this is the length element now, then from every point on the source, there is a point at a distance of λa/2d.

Now, we are following the same thing, which we did in the last class,, we took a source which with a particular bandwidth and we took two wavelengths out of this bandwidth. One of the extremity and one at the center. The same thing we are doing here, here the source width is λa/d and then we took one point here and other point at the center, these points will be at a distance λa/2d.

Let us do it here, this is the source here it is 0, here it is λa/d and here we pick another point which is at a distance λa/2d from first extremity. Now, this distance is λa/2d and this distance is also λa by twice of d. Now, for each point source on the first band, let us pick another color this is your first band and this is your second band.

Now, for each point on the first band, we can find a corresponding point in the second band this is our first band, this is a second band, for each point in the first band, we can find a corresponding point in the second band which is at a distance 2a/λ, because these two points are at a distance, sorry, which are at a distance λa/2d.

Now, these two points are at a distance λa/2d, similarly, let us shift this point here and this point here this distance is also equal to λa/2d, this distance is also λ a/2d, this distance is also λa/2d, this distance is also λa/2d, it means, for every point on the source.

There is a point at a distance of λa/2d which produces fringes that are shifted by one half of a fringe width. This sentence means that the two fringes would be out of phase by 180 degree, means maxima of one will fall on the minima of other. And therefore, the resultant pattern will not be observed, we will have a uniform background therefore. Now, the same thing we did in the spectral domain for temporal coherence and for spatial coherence we are doing it in space now.

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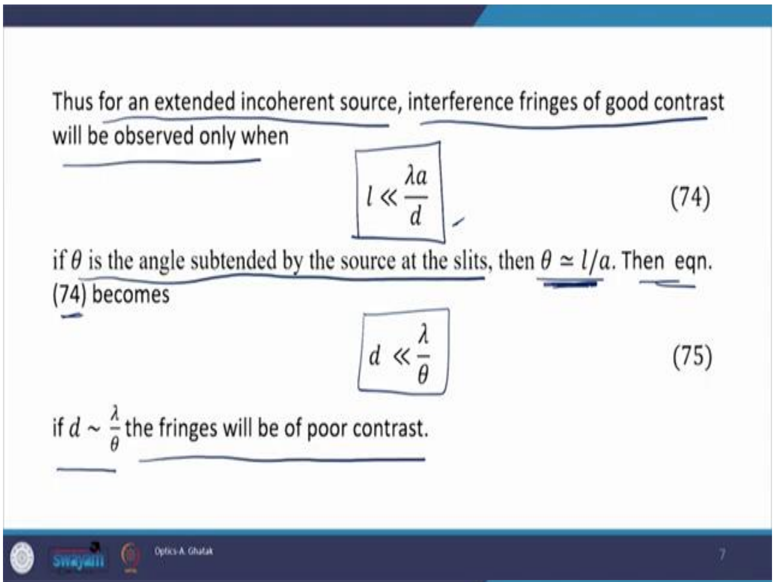

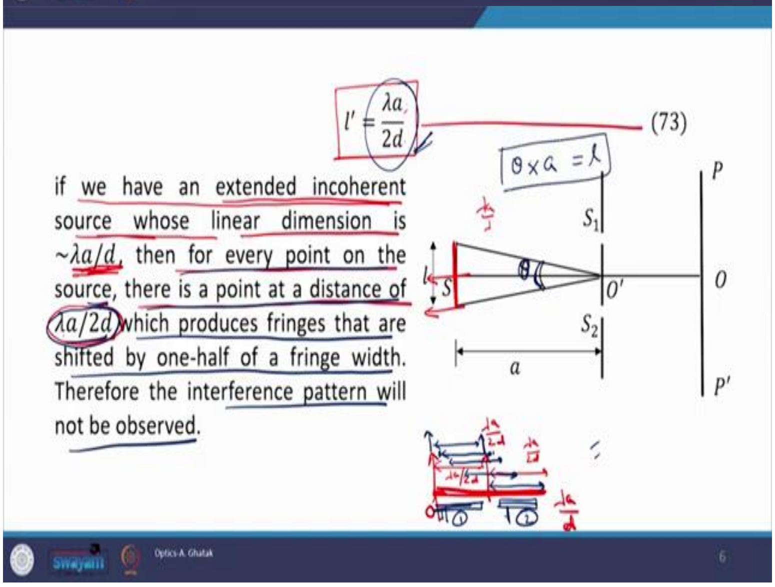

Thus, for an extended incoherent source, extended incoherent source means extended line source which we are considering in this analysis, the interference fringes of good contrast will be observed only when the length element is much-much smaller than λa/d and this is very much obvious from this analysis. Now, if θ is the angle subtended by the source at the slit, now we are considering this figure, this is the angle θ then from the figure we can see that l= θa.

And therefore, from equation 74, l=θa or θ=l/a for very small θ, then from equation 74, let us replace this l/a in equation 74 with θ and then we get d must be much less than λ/θ. And if d is half the order of λ/θ, fringes will be of poor contrast, we will get better fringes if d is much-much smaller than λ/θ, we will get poor contrast fringes if d is equal to almost equal to λ/θ or larger than λ/θ.

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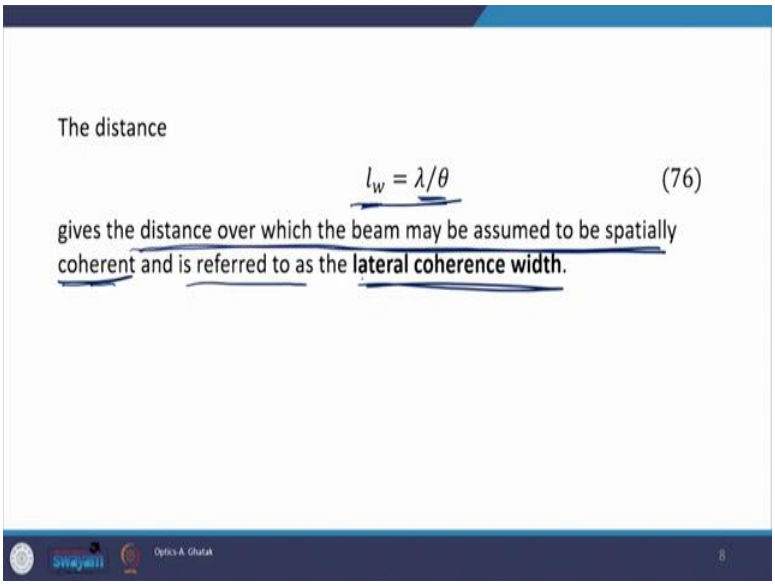

Now, this distance which says that l_w is λ/θ, it gives a distance over which the beam may be assumed to be spatially coherent. Now, we are talking about the extent, the transverse extent of the beam. And this says that if it is equal to λ/θ we will be able to observe, we will see a uniform background no fringes and if it is smaller than λ/θ we will see good contrast fringes and this distance l_w gives a distance over which the beam may be assumed to be spatially coherent and therefore, it is referred to as lateral coherence width, of course it is related to the space, length element. Therefore, it is called lateral coherence width.

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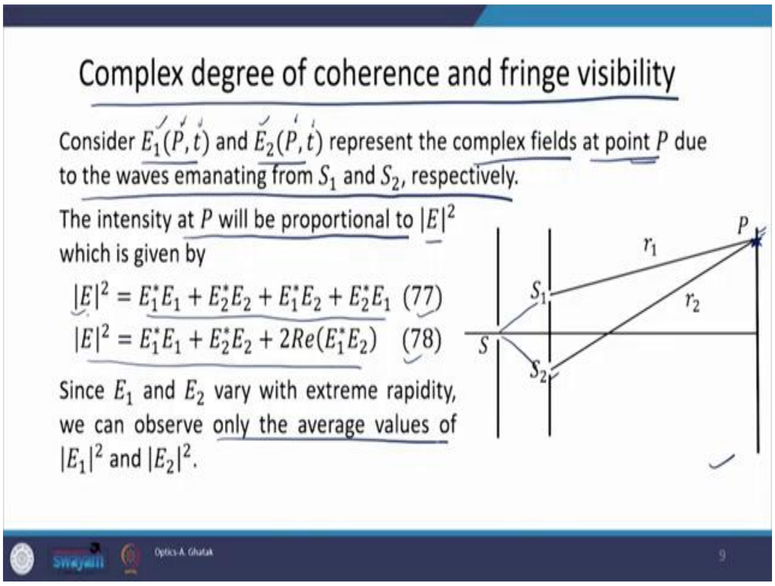

Now, let us dive a bit more deep, let us talk about the degree of coherence, a bit advanced topic. The complex degree of coherence and fringe visibility. This is the title and here we will again consider the conventional Young's double slit setup where we have a source S, out of source S, we are creating two more sources which are completely coherent and the fringes are being observed at a screen at point P which is at a distance r_1 from source S_1 and r_2 from source S_2.

Now, suppose that $E_1(P,t)$ and $E_2(P,t)$ represents the complex fields at point P due to the waves emanating from source S_1 and S_2 respectively, some waves emanates from S_1 and it is reaching to point P and another wave emanate from S_2 and it is reaching point P again and these waves are represented by E_1 function of (P, t), E_2 function of (P, t), P is position, the same point which is at screen and t is the time at which the fields are observed.

Therefore, the intensity at P will be proportional to $|E|^2$ where is superposition of the two complex fields which are E_1 and E_2, how to calculate superposition? E would be $E_1 + E_2$ and then we will take complex conjugate of E and then we multiply the E with complex conjugate of E and this will give the intensity at a point of observation P and this is what is done exactly here and we get equation number 77.

And this can also be written in this form a bit modified which is equation number 78. But we know that fields are rapidly varying quantity particularly in visible range therefore, we observe only average value of this field.

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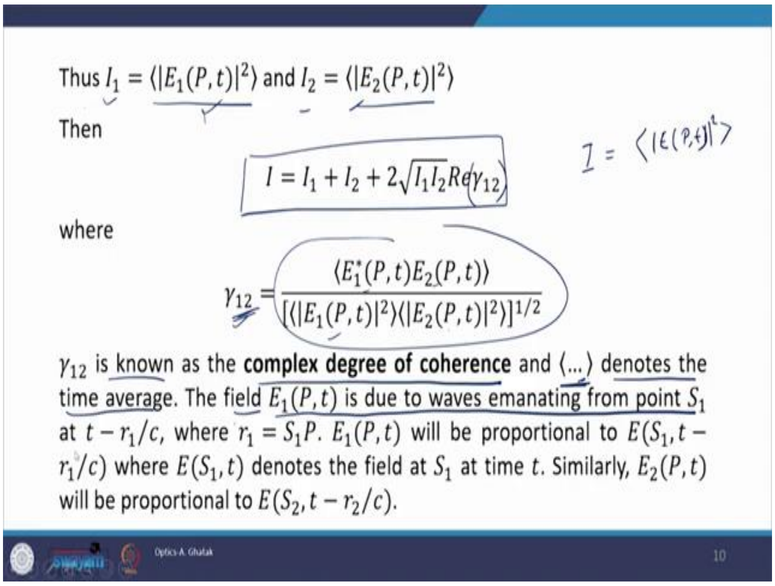

And we define $I_1 = \langle|E_1(P,t)|^2\rangle$, $I_2 = \langle|E_2(P,t)|^2\rangle$ and therefore, this equation 78 reduced to this expression where $I = I_1 + I_2 + 2\sqrt{I_1 I_2} Re(\gamma_{12})$. Therefore, where $I = \langle|E(P,t)|^2\rangle$ and I_1, I_2 expressions are given here, γ_{12} is given by this expression.

It is $\langle E_1^*(P,t)E_2(P,t)\rangle$ and in the denominator we have a $(\langle|E_1(P,t)|^2\rangle\langle|E_2(P,t)|^2\rangle)^{1/2}$ they all are calculated at point P which is on the screen at time t, time of observation of this field at point P, this γ_{12} is known as complex degree of coherence and the terms within these triangles denotes time average, the field $E_1(P,t)$ is due to waves emanating from point source S_1 at some earlier time.

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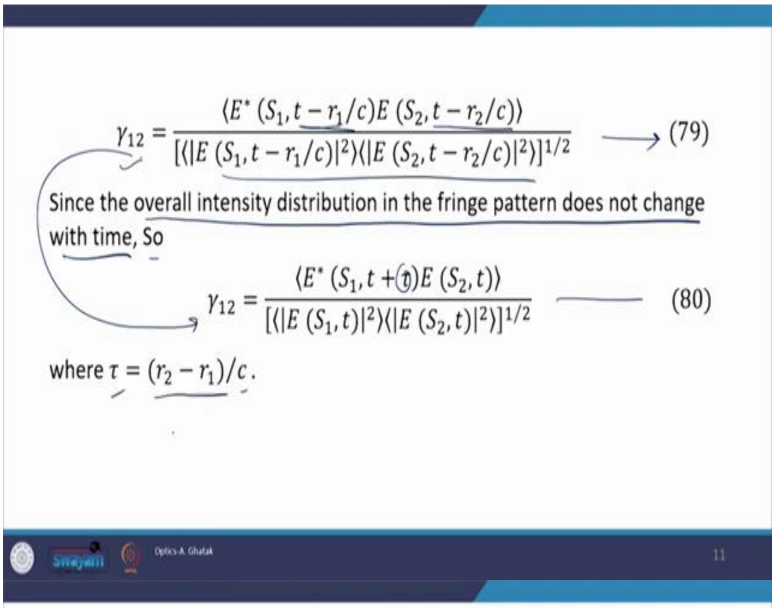

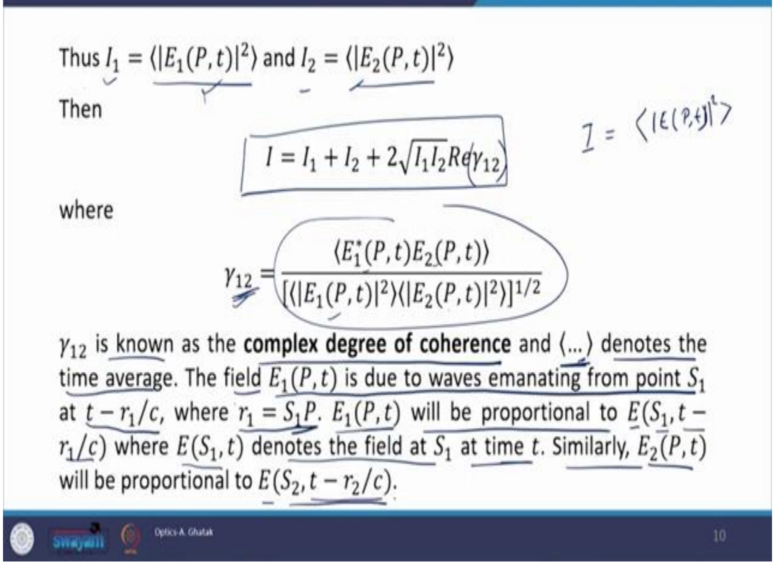

And what is this earlier time, this is $t - r_1/c$, where $r_1 = S_1P$, we have already done it therefore, I am not detailing too much on this. $E_1(P,t)$ will be proportional to $E(S_1, t - r_1/c)$ because the fringe pattern on the screen is invariant in time and E is the field at point P.

Therefore, this field component would be proportional to the field which was at point S_1, the source S_1, at some earlier time and what is that earlier time, earlier time is $t - r_1/c$, where even t denotes the field at S_1 at time t. Similarly, $E_2(P,t)$ would be proportional to $E(S_2, t - r_2/c)$, E_2 is the field contribution at P due to source S_2.

Therefore, the earlier time for this field would be $t - r_2/c$ which is written here. Therefore, we can replace since these E_1 and E_2 are proportional to these fields then we can replace the expression of E_1 E_2 in the expression of γ_{12}, during this we get a bit complicated expression here which is given by equation number 79. Now we are seeing sustained interference fringes on the screen.

And therefore, overall intensity distribution in fringe pattern does not change with time and therefore, instead of writing this complex time, we will just take into account the time difference, we will subtract these two times and get a time difference τ and therefore, this equation of γ_{12} get modified here in expression number 80, where τ is the difference in time.

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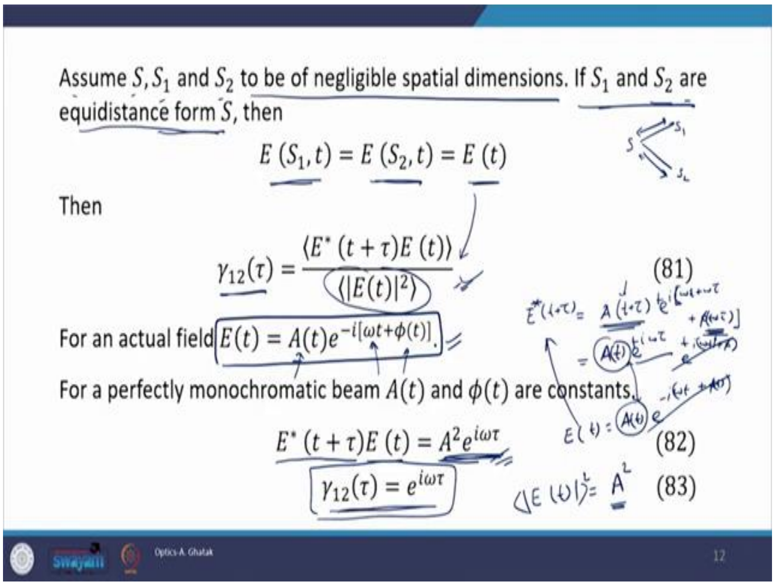

Now, assume that S S_1 and S_2 to be of negligible spatial dimension, we are assuming that they are point sources, if S_1 and S_2 are equidistance from S means S is here and from S, S_1 and S_2 are being generated and if these distances are same then E(S_1, t) would be equal to E(S_2, t) because the source S_1 and S_2 which are being generated from S, they will receive the same field, same amplitude, same magnitude of the field.

Therefore, E (S_1, t) would be equal to E (S_2, t) and let us assume that this is equal to E (t) therefore, the previous expression of γ_{12} again get simplified and this is the expression here we have dropped S_1, S_2 or E_1, E_2 with much simpler expression. But actually the field is represented by this quantity $E(t) = A(t)e^{-i(\omega t+\varphi(t))}$.

Where A(t) is amplitude, φ is, which are slowly varying function of time and the rapidly varying function of time is here in the phase part ωt for perfectly monochromatic beam A(t) and $\varphi(t)$ are constant, they does not vary, they do not vary A(t) and $\varphi(t)$, they do not vary and therefore $E^*(t+\tau)E(\tau)$ if you want to calculate they would be equal to $A^2 e^{i\omega t}$, how, the

$E^*(t+\tau)$ from this expression can be written as $A(t+\tau)e^{i[\omega(t+\tau)+\varphi(t+\tau)]}$, but we know that it is slowly varying function of time A and φ they will not vary they are constant distance it is of course in this case A and φ will not vary because they are constant it is some monochromatic wave.

Therefore, it would be A(t) and in the phase part we have $e^{i\omega\tau}$ and the rest of the function let us assume that it is $i(\omega t+\varphi)$. Similarly, for E (t) if you want to calculate it would be equal to$A(t)e^{-i[\omega t+\varphi(t)]}$, these are the two functions.

Now, it is the complex conjugate here therefore, it would be plus here and if you just multiply these two functions then what you can see is that this function will go away and we would be left with $A(t)^2$ because there are 2A and $e^{i\omega\tau}$ and this is what is given here in equation number 82.

And therefore, the expression for γ_{12} would be $e^{i\omega\tau}$ because in the denominator also we have we are having $\langle|E(t)|^2\rangle$ which is again equal to A^2 and therefore, A^2, A^2 will go away and the final expression of γ_{12} would be $e^{i\omega\tau}$.

What I am seeing is this $\langle|E(t)|^2\rangle = A^2$ which is in the denominator and in the numerator we have $A^2e^{i\omega\tau}$ and therefore the resultant expression of γ_{12} would be equal to $e^{i\omega\tau}$.

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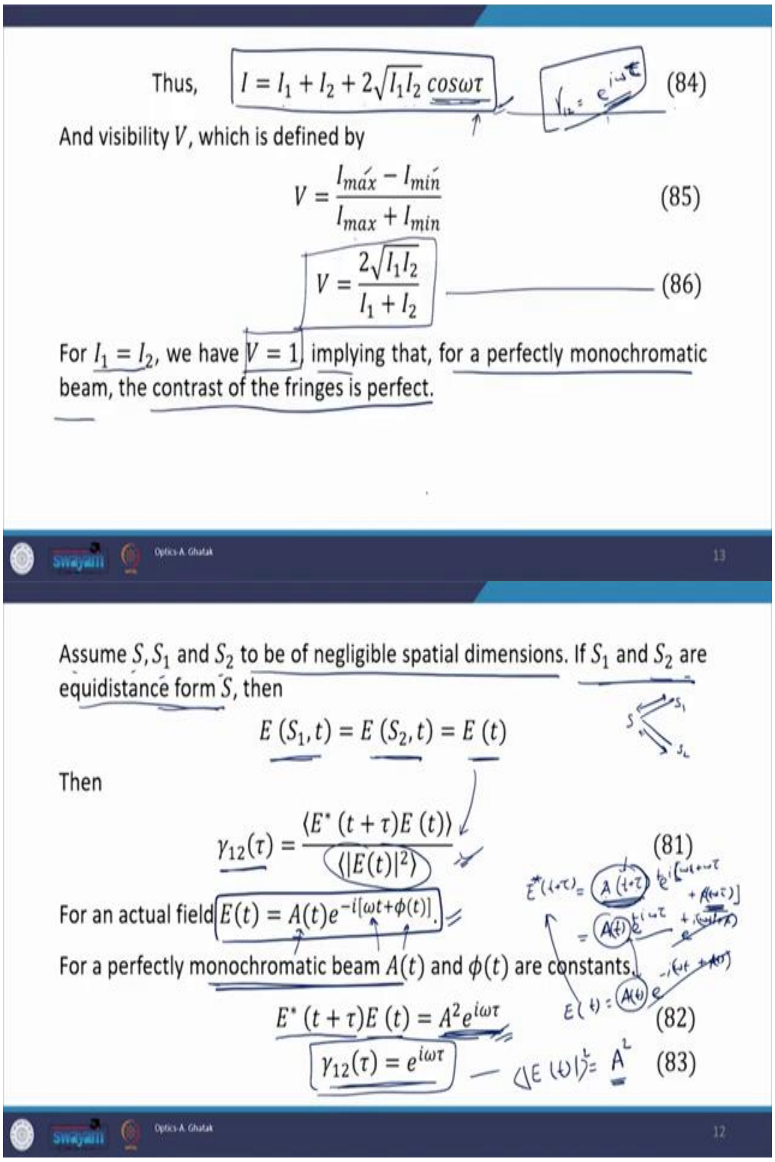

And therefore, the final irradiance expression can be written like this, where in the expression of $\gamma_{12}(\tau)$ which is equal to $e^{i\omega\tau}$, which is a complex number, but we are behind real stuff here. Therefore, we take real part of this function which is $cos(\omega\tau)$, it is a tau.

And therefore, the final expression of irradiance at point V reduces down to equation number 84 which is $I_1 + I_2 + 2\sqrt{I_1 I_2}cos(\omega\tau)$. Now, we will talk about visibility, we will introduce visibility formly after a few slides but since it is a relevant here in this calculation.

I just quickly introduce visibility as $(I_{max} - I_{min})/(I_{max} + I_{min})$ where I_{max} and I_{min} are irradiance maxima and irradiance minima in an interference pattern. Now, if you substitute for I_{max} and I_{min} from equation number 84, we know I_{max}, you will get I_{max} when $cos(\omega\tau)$ is equal to 1.

We will get I_{min} when $cos(\omega\tau)$ is equal to -1, with this substitution the expression of V is given by equation number 86. Now, if the two sources are of equal irradiance when $I_1 = I_2$.

Then we have maximum value of visibility V and when we maximize it, it implies that for a perfectly monochromatic beam the contrast of the fringes is perfect, visibility as its name suggests it tells the contrast and when it is 1 then the contrast is perfect. Now, this derivation was for a monochromatic beam.

For monochromatic beam only A and φ is constant and there we can write A function of (t+τ) is equal to A (t) because A(t) and $\varphi(t)$ they does not depend on any other parameter because they are constantly in itself, but the general expression for γ would not be of this form $e^{i\omega\tau}$, this is for only monochromatic wave.

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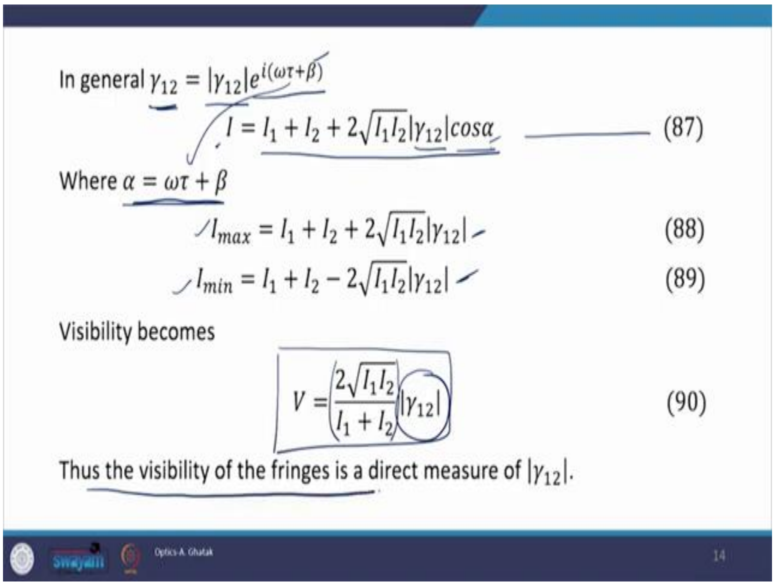

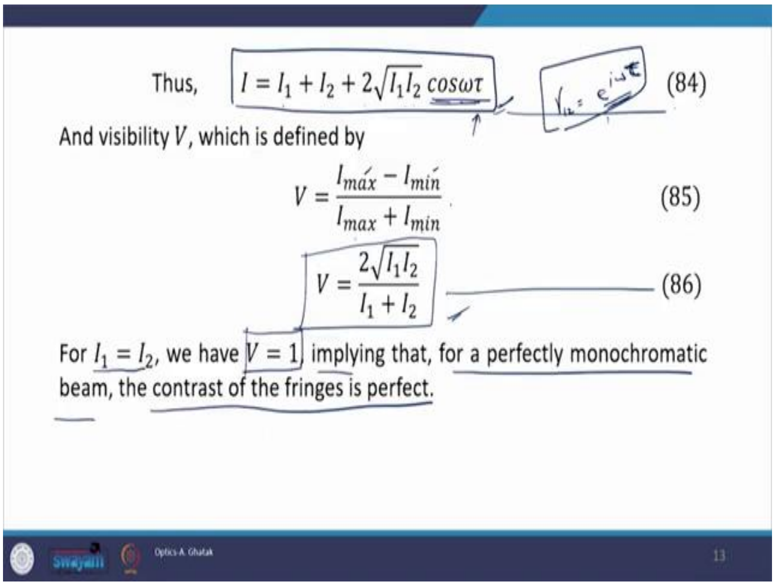

But for the general case γ_{12} can be written as $|\gamma_{12}|$ and $e^{i\omega\tau+\beta}$. And therefore, the expression for the irradiance, total irradiance, at point of observation modifiers by this $|\gamma_{12}|cos\alpha$,, where α is given by $\omega\tau + \beta$, which is here.

Here in this equation number 87 if you calculate I_{max} and I_{min} then they will have these expressions, where of course, when $cos\alpha = 1$.,we have I_{max} and when $cos\alpha = -1$ we have I_{min}. Now, if you calculate visibility then you see that in the expression of visibility apart from this term which was earlier in our previous expression here in equation number 86, we have additional term which is $|\gamma_{12}|$.

It means once you measure the visibility of the fringes, then this will give the information about $|\gamma_{12}|$. And visibility measurement is very simple, just measure the maximum irradiance and minimum irradiance and take their ratio, it is given here $(I_{max} - I_{min})/(I_{max} + I_{min})$, that's it. Once you measure the visibility you have the information of γ_{12}.

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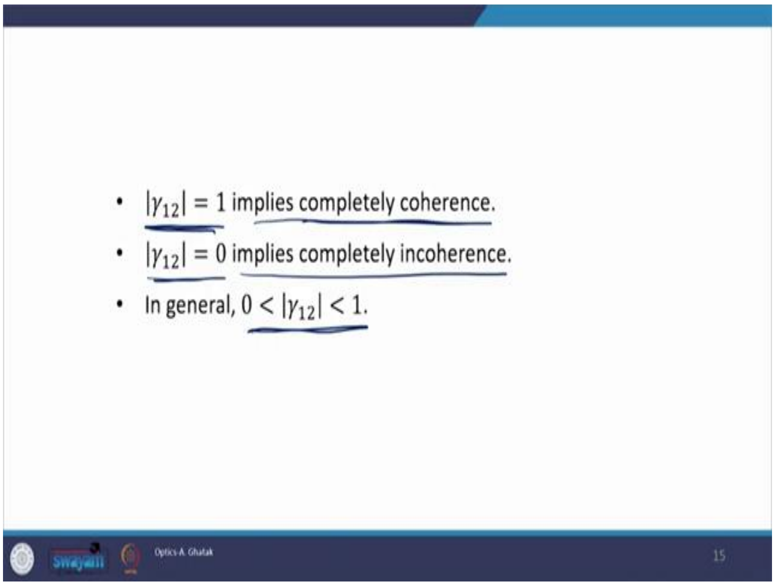

And when $|\gamma_{12}| = 1$, it implies that the sources are completely coherent, when $|\gamma_{12}| = 0$, it means when visibility is equal to 0, it implies that sources are completely incoherent. And in general, for all sources $|\gamma_{12}|$ varies between 0 and 1 and the sources for which $|\gamma_{12}| = 0.86$ are larger than we call those sources as coherent sources.

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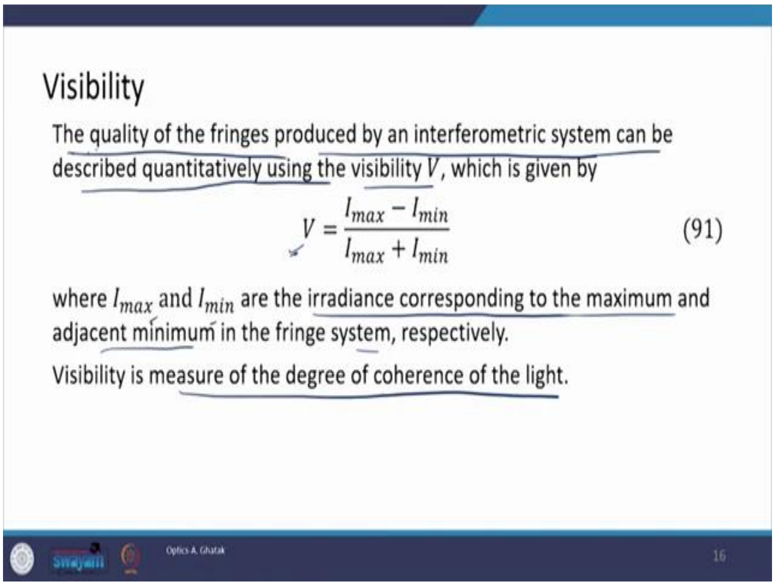

Now, coming back to visibility, the quality of fringes produced by an interferometric system can be described quantitatively using visibility V. And the visibility is defined by this term which we already discussed about. And $I_{max} I_{min}$ are they irradiance corresponding to maxima and adjacent minimum in the fringe pattern respectively. And it measures the degree of coherence of the light, this is already defined. And this concludes everything on coherence and with this I end my lecture, and thank you for listening me.