Applied Optics
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Lecture 20
Interference with White Light, Displacement of Fringes, Fresnel's Biprism

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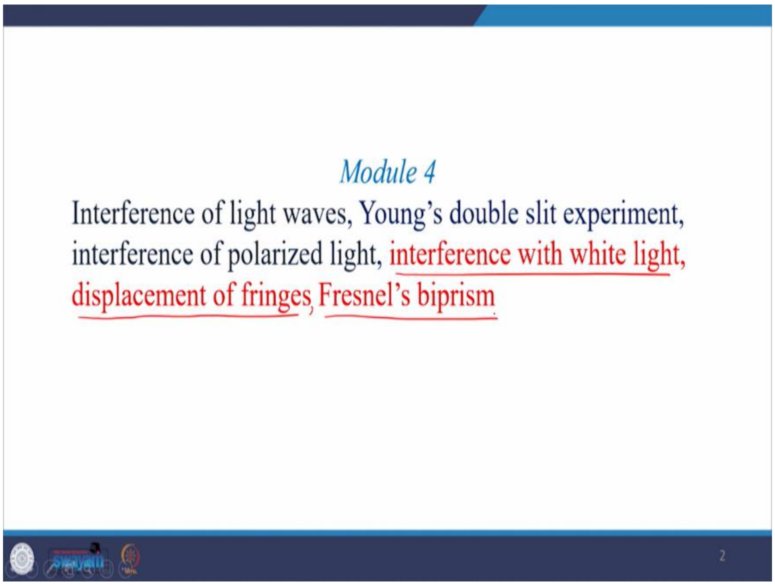

Hello, everyone. Welcome back to my class. Today, we will talk about interference with white light and then we will also talk about displacement of fringes and in last we will study Fresnel's biprism.

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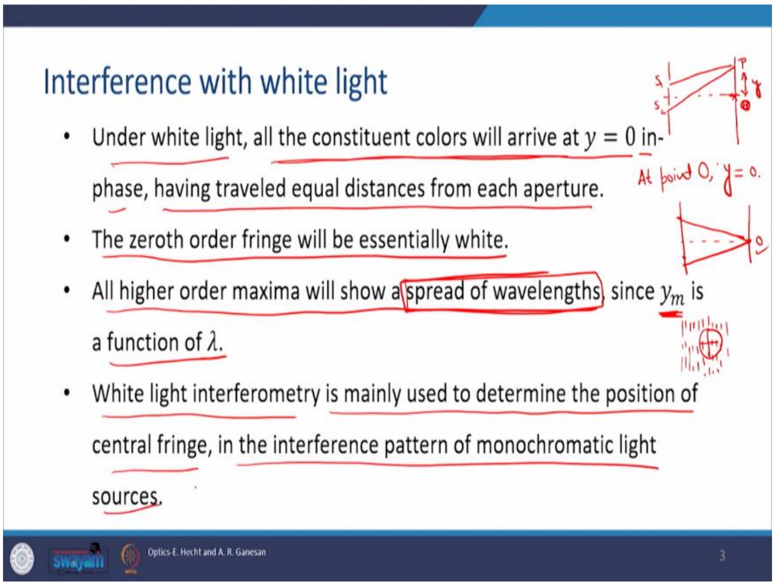

Till now, we have dealt with interference experiment where we use coherent light source. But what will happen if instead of coherent light sources we use white light source? This we will discuss today. Now, under white light illumination, all the constituent colors will arrive at the center of the screen. Let me remind you, we were having a source plane here where we were having S_1 and S_2 in Young's Double Slit experiment and this was our screen and there were some point P where we were observing the fringe pattern and this P was at a distance y from the O, the center of the screen.

Now, at O, y is equal to 0. At point O, we were having y as equal to 0. And at y is equal to 0, under symmetric illumination, we saw that we always find maxima at y equal to 0. And here too, with white light illumination, we will find maxima at y equal to 0 irrespective of the wavelength.

What I mean to say by irrespective of the wavelength is that when we illuminate our sources S_1 and S_2 with white light, then the maxima for red color will be formed at y is equal to 0 as well as the maxima for blue color will be formed at y is equal to 0, as well as the maxima for yellow color will be formed at y is equal to 0.

Since all the light colors are giving maxima conditions at y is equal to 0, they all combined and the resultant color would be white. And this is what is written here. All the constituent colors will arrive at y is equal to 0 in phase and why they will arrive there in phase at y is equal to 0 because they have travelled the same optical distance in the medium.

And this is what it is written here, having travelled equal distances from each aperture because this point O on the screen is symmetrically situated from the two sources, this distance is equal to this distance. And therefore, the total path difference is 0 at O. Therefore, irrespective of the wavelength, irrespective of the color, we will always find maxima at O.

And under white light illumination condition, since all colors are forming maxima at O, all colors will combine and give the resultant as white color. Now therefore, we can say that zeroth order fringe will be essentially white. Zeroth order means the central fringe, the fringe at y is equal to 0, or the fringe at point O. All higher order maxima will show a spread of wavelength. Now, since central maxima is talked about, we will go to the next order of maxima.

Now, for the next order of maxima which would be away from this point O which is symmetrically situated as soon as we go to a point which is asymmetric with respect to the sources S_1 and S_2, then the criteria of maxima or the position of maxima will be different for different colors. The criteria of maxima would be same but within the criteria of maxima, there is λ dependence. Since there is wavelength dependence embedded in the criteria of maxima, the maxima for different color will be formed at different positions or different y positions.

Therefore, we will see a spread of wavelength for higher order maxima. Why? Because the position of maxima is a function of wavelength λ (lambda). Now what is the use of white light interferometry? Why to use white light in Young's Double Slit experiment or in any interferometric experiment?

When we perform an interference experiment with a monochromatic source or with a usual coherent source, then what happens is that we usually find, let us talk about Young's Double Slit experiment. Then in Young's Double Slit experiment, we find straight line fringes and these fringes are almost indistinguishable.

We cannot distinguish one fringe from another and when we just to measure the fringe width in the experimental setup, we have a cross wire, there is a micrometer attached in the screen and there is an IPS, and in that IPS we have a cross wire and then all these fringes which are there in the background, we can focus or we can align this cross wire with one of them and then take the reading and then we go to the next fringe and then take the next reading. And the difference between these two readings give us the fringe width. But all these fringes are indistinguishable. They are totally identical to each other.

Now suppose due to some reason, the fringes got shifted. Now since the fringes got shifted, we must know the amount of shift because the field of view, even after this shift would be identical to the previous one. Fringes are identical. Therefore, even after shift, we would not be able to distinguish the field of view. We will not be able to count how many fringes has shifted due to some sudden change. Then how to get rid of this complex situation? Then white light fringes will help here.

When the setup is illuminated with white light, then colored fringes would be formed where central is white and higher order fringes are colored. Now, if there is a sudden shift, then we can clearly distinguish the field of view because the shift in the white light pattern is visible, it is distinguishable and then just by moving our crosswire with the help of micrometer, we can measure the shift. And this is used in several ways which we will discuss in forthcoming slides and this is what is written here. White light interferometry is mainly used to determine the position of central fringe in the interference pattern of the monochromatic sources.

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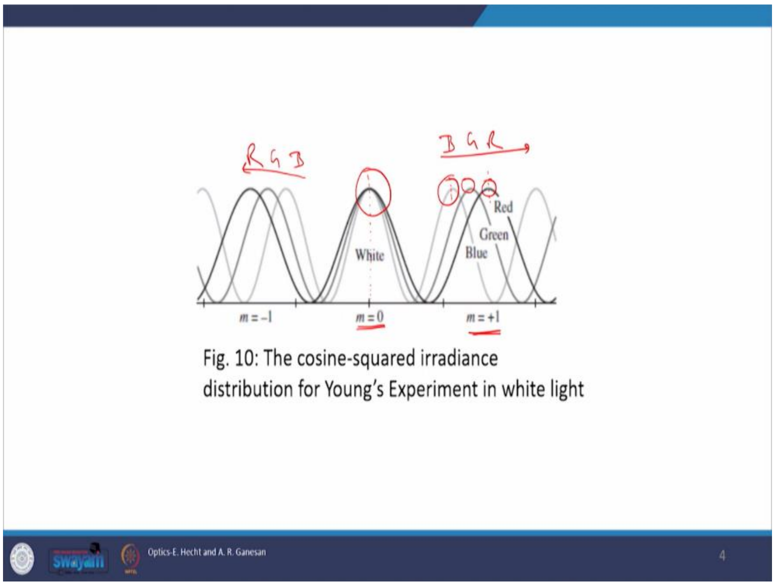

In white light fringe, as discussed in the last slide, the central fringe is white because in the central fringe maxima of all colors coincide. While the higher order maxima for different colors are formed at different places, now you see for m=+1 for next order the maxima of blue is forming earlier or before as compared to that for red color.

Therefore, in white light experiment, interference experiment the central fringe looks white. Next order fringe, first blue up comes and then green and then red. Similarly, on the other side the same things is followed. First blue will appear, then green will appear and then red. Here again blue, green and red.

And as you move away from the center, the separation between different color maxima increases. But if we move further then we will again see a white light effulgence because all these color then get start to mix.

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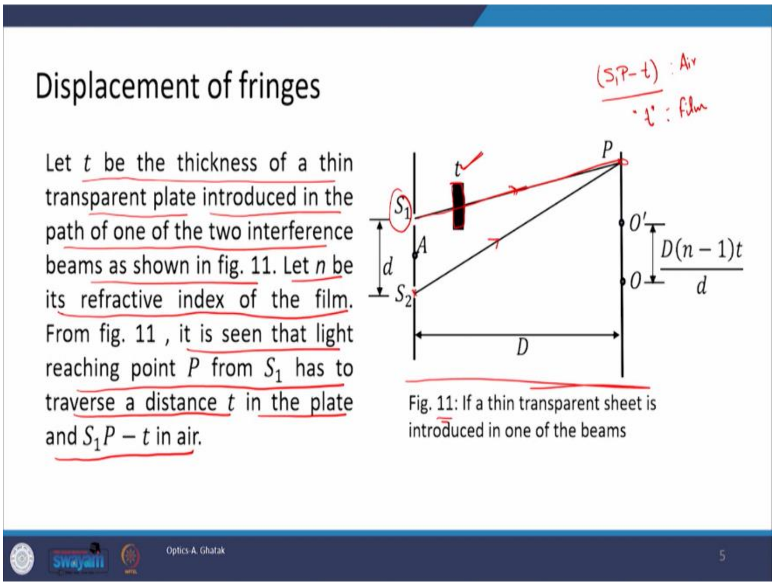

Now we will see one application of Young's Double Slit experiment. And what is this application? This application is finding the refractive index of a thin film if thickness is known or alternatively, finding the thickness of a thin film once refractive index is known. This is the Young's Double Slit experiment set up and in this setup one source say, S_1 is covered with a film of thickness t. This film must be transparent. We must take a transparent film here in this experiment because the ray must pass through this film so as to give us some interference pattern on the screen.

Now let us read the statement. Let t be the thickness of a thin transparent plate or film introduced in the path of one of the two interference beam as shown in figure 11. In this figure we are introducing this thin film in path of one of the ray or we can say that we are covering one of the sources with this thin film and let n be the refractive index of this film, refractive index of this film is n which is larger than the refractive index of air of course. Now, it is seen that light reaching point P from S_1 has to traverse a distance t in the film or plate and S_1P-t in air.

Now let us see the source S_2 emits a ray which reaches to point P and then source S_1 also emits a ray which is reaching to point P which is point of observation on the screen. Now earlier the optical path was S_1P because the setup is kept in air. Therefore, optical path for rays which are emanating from point S_1 and reaching to point P is S_1P. But due to the insertion of thin film of thickness t and of refractive index n, the ray will travel S_1P-t path in air. This would be in air while this path, this thick path would be in film or plate whatever you say it. Now, there are two paths and since the two paths are in different refractive index media therefore they would be treated differently.

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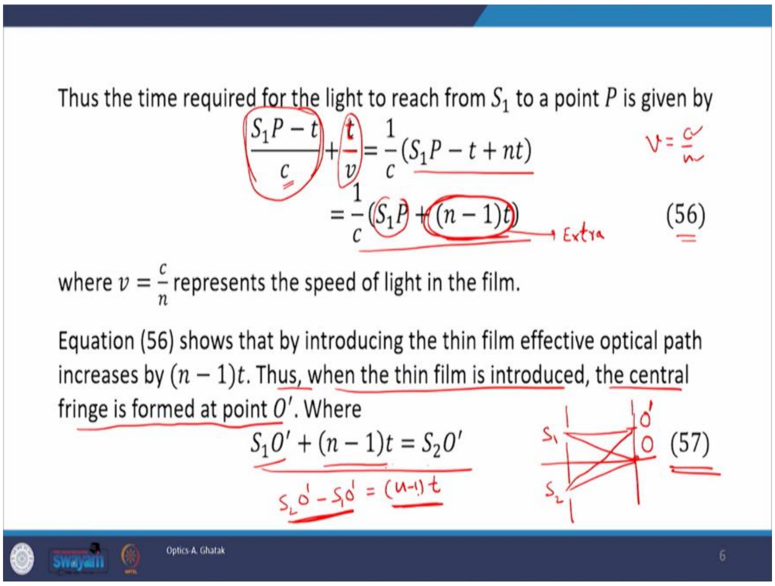

Now, let us calculate the time required to travel this optical path S_1P by a ray which is emanating from point S_1 or source S_1. Now, time which the ray takes in traveling S_1P-t distance is S_1P-t by speed of light in air which is c here. This is the time which the rate takes in traveling the air path but apart from air, we have inserted a film of thickness t. Therefore, there would be some extra time and how to calculate this extra time. Again, thickness of the film by the speed of the light in the film which here is denoted by v. v is the speed of light in the film of refractive index n.

Now, we know v is equal to c/n. We will substitute v by c/n and where c is the speed of light in air or vacuum and n is the index of refraction of the film. Then we can take 1/c out of the bracket and this is what we are left with. Now, equation number 56 gives the expression for time which a ray take in traversing the distance S_1P.

Now, we see that had there been no film then S_1P/c would have been the time which the ray must have taken while traveling the path had there been no film. But due to the insertion of the film, this is the extra time which the ray is taking (n-1)t/c is the extra time which the ray is taking due to insertion of this film or extra path length travelled by the ray is (n_1-1)t, here this is our extra path length and this is only due to the insertion of this thin film.

Now, suppose due to insertion of thin film or due to this extra path length, the central maximum now will be shifted. It is very much obvious. Central maxima is the position which is symmetrically situated from source S_1 and S_2 but now this symmetry is lost due to insertion of a film in the path of one of the ray and to balance this, the central maxima will shift either up or down. Now, it depends upon the relative value of the refractive index of the film.

Now, assume that when the film is introduced, the central fringe is formed at another point which is O'. O' is different from O. Now, in this particular case, earlier you see that O was here and then S_1 was here, S_2 was here and S_1O-S_2O was equal to 0 but now there is a shift. O is shifted here to point O'. Now, in this case, S_1O'-S_2O' would be equal to the extra path length. Okay, this can be represented by equation number 57 which says S_1O'+$(n_1 + 1)t = S_2O'$. Now under this condition, this is nothing but this relation S_2O'-S_1O' = (n-1)t. Now the part difference would be equal to the extra path.

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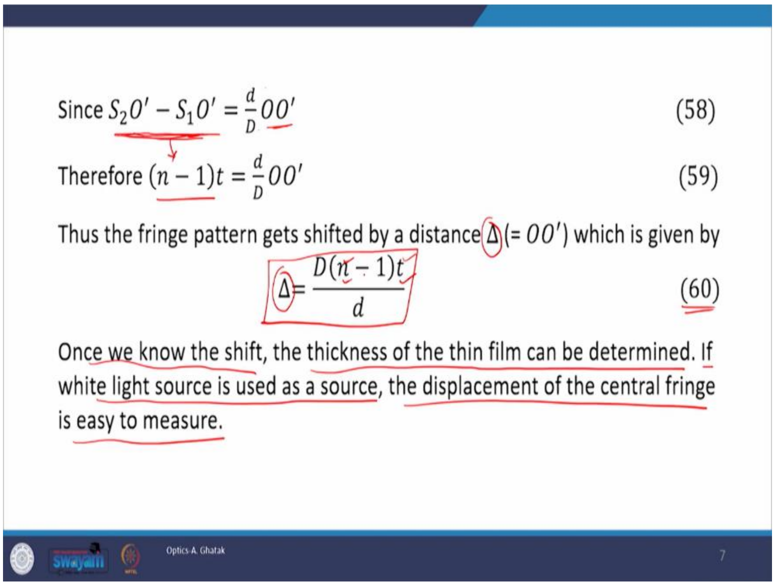

Now once it is done, then we know that S_2O'-S_1O' is equal to nothing but $r_2 - r_1$ in the calculation of Young's Double Slit experiment, if you remember. There $r_2 - r_1 = dy/D$. And what is y here? OO'. Therefore, from our knowledge of Young's Double Slit experiment, we can write S_2O'-S_1O' = (d/D)OO'. S_1O'-S_2O' is nothing but $r_2 - r_1$. Yeah. And therefore, we can safely substitute for S_2O'-S_1O' which is (n- 1)t. We substituted for this and therefore, (n-1)t = (d/D)OO' . What is OO'? OO' is shift in the fringe, shift in the central fringe.

Now let us represent the shift in the central fringe by a symbol Δ. Therefore, OO' or Δ=D (n-1) t/d. This Δ represents shift in the central fringe due to insertion of a thin film of thickness t and refractive index n. Now from equation 60, we can see that if the thickness of the film is known, then the refractive index can be calculated just by observing the shift in the central fringe position or if the refractive index of the thin film is known, then we can calculate the thickness. This is one of the application of Young's Double Slit experiment, the interferometric setup. Therefore, once we know the shift, the thickness of the thin film can be determined.

And if white light source is used as a source, the displacement of the central fringe can easily be measured. Here the white light interference comes handy and since you know that as soon as we are inserting the fringe, the whole fringe pattern is shifting but since the things are indistinguishable, the fringes are indistinguishable, we would not be able to discriminate or we will not be able to measure the shift. How to measure the shift?

Also illuminate the setup with the white light. Once there is illumination with the white light and now if you insert a thin film, the white light maxima would shift and using the micrometer and the crosswire we can measure the shift in the white light pattern and that shift will give the values of OO' or Δ and from here we can calculate things.

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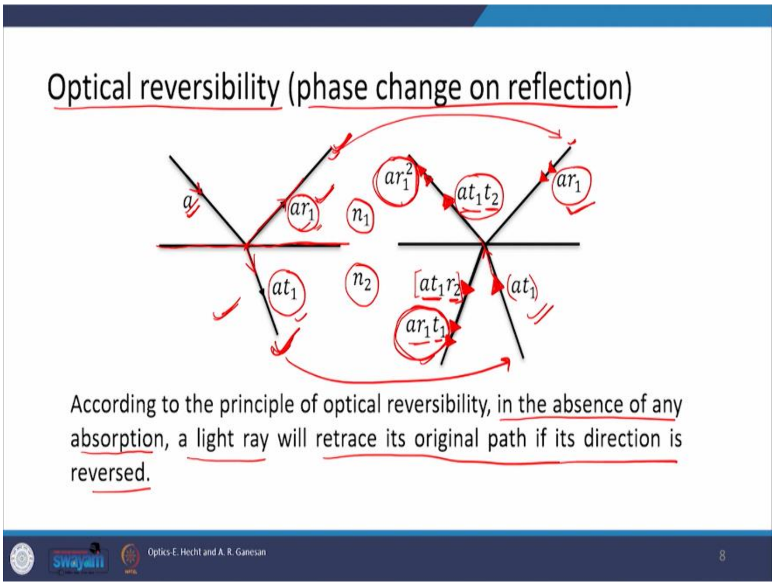

Now, we will move to the next topic which is about optical reversibility. And herein we will also establish that phase changes on reflection. Now what is the principle of optical reversibility? The principle of optical reversibility says that in the absence of any absorption, we are assuming that the system is lossless. In the absence of any absorption, a light ray will retrace its original path if its direction is revered. It is a very simple principle. It says that if the light ray direction is reversed then it will retrace its original path in absence of losses or absorption.

Now to understand it schematically, now suppose we have an interface which is given by this horizontal line and it is dividing two medium. Above this line the medium has refractive index n_1 and below this line the medium has refractive index n_2. Now, suppose a ray of light falls on this interface which is separating the two media of refractive indexes n_1 and n_2 and this incident ray has amplitude a. When it falls at the interface, a part of the light gets reflected and a part gets transmitted.

Now if it is falling from top to bottom, if it is falling from first medium to the second medium, we assume that amplitude reflection coefficient is r_1 and amplitude transmission coefficient is t_1. In this case, the reflected amplitude is given by $r_1 a$ or ar_1. Whereas the transmitted amplitude is given as $t_1 a$ or at_1. We just multiply the reflection coefficient with the magnitude or amplitude. This gives the final amplitude which is getting transmitted or reflected. Yeah, just to repeat it once more, a ray of amplitude a is met to incident on an interface. Now ar_1 amplitude would be reflected and at_1 amplitude would be transmitted where r_1 and t_1 are amplitude reflection and transmission coefficients respectively.

Now in this situation we will exercise the principle of optical reversibility and we will see what happens. Let us reverse now the directions of all the rays here. Now let us start with this r and we will reverse the direction of ray which has amplitude now ar_1. Now let us represent this arm in new figure and this ar_1 is the ray which is now traveling again back towards the interface and this ray is represented by double arrow.

Now this ray is falling on the interface, a part will be reflected. Now, since reflected amplitude to calculate the reflected amplitude, we just multiply it with amplitude reflection coefficient which is r_1 therefore, we will multiply r_1 in ar_1 and this would be equal to ar_1^2. Therefore, the wave of magnitude ar_1 after reflection becomes ar_1^2.

But apart from the reflection, a part will also get transmitted. Therefore, this part of the wave is getting transmitted. And the transmission coefficient is t_1 and therefore, the amplitude of the wave which has transmitted is ar_1t_1. This is the resultant amplitude of the wave which got transmitted. Here again, we are multiplying with transmission amplitude, transmission coefficient t_1, ar_1 was the initial amplitude and ar_1t_1 is the final amplitude of the transmitted wave. We are just following this first system here. a was incident, ar_1 got reflected, at_1 got transmitted and here, ar_1 was incident, ar_1t_1 got transmitted, ar_1r_1 got reflected. And this is how you can calculate the amplitudes of reflected and transmitted wave once amplitude reflection and transmission coefficients are known. This is for this arm only.

Now, let us also reverse this arm which is in the transmitting medium and this arm is shown here. And this is represented by single arrow here and the amplitude of this wave is at_1. Now in the first case, the wave is coming from medium n_1 and then it is going into medium n_2. It is falling from n_1 medium and then it is falling at the interface between the two.

Now, for this wave which is a magnitude at_1, it is starting in medium n_2, it is starting from other medium. Therefore, the reflection and transmission coefficients would be different from the previous reflection and transmission coefficients. r_1 and t_1 will not be the reflection coefficient for this wave which is starting from the second medium, the lower one, lower medium.

Here we assume that the reflection coefficient is r_2 and the transmission coefficient is t_2. Now the wave incidents at the interface and part get reflected and here the reflection coefficient is r_2 and therefore, we will just multiply $r_2$into t_1. Now, you see that at_1 is the initial amplitude and to calculate the reflected amplitude, we just multiplied at_1 with r_2 and this is our final amplitude. This is the part of the amplitude which got reflected.

Similarly, a part of the light, a part of this at_1 magnitude will get transmitted. And this, let us represent again, this transmitted part with a single arrow, at_1 is the incident one to calculate the transmitted amplitude just multiplied with the next new amplitude transmission coefficient which is t_2 here and this gives us the final magnitude of the wave which got transmitted. And this is at_1t_2.

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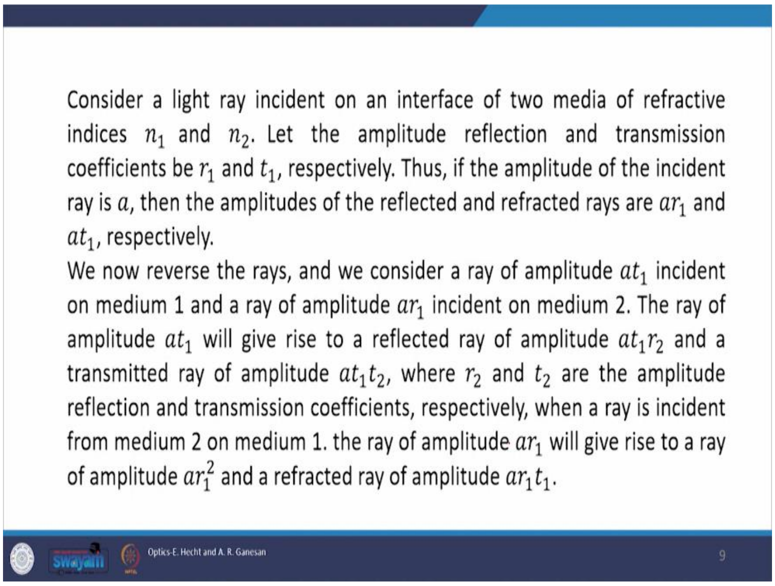

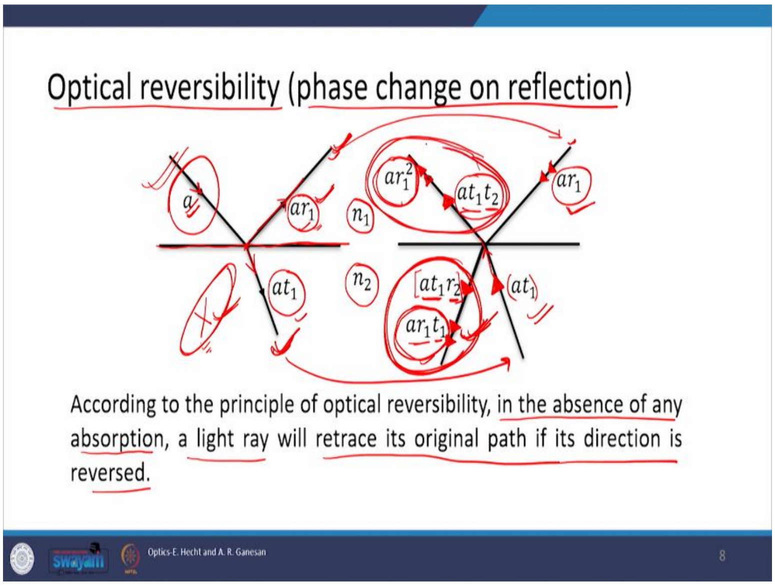

All this exercise is mentioned here. This is written everything in great detail. I am not reading it. I am leaving it all for you, for your concept clarification. Now, once all these things are done, now we will see that what the principle of optical reversibility gives us. Now, we saw that in this part, there was no ray. There was no ray here initially in the first part but when we reversed the optical path, we saw that there is a ray with some magnitude.

But the principle of optical reversibility says that if you reverse the path of the light, it must retrace its original path. Therefore, the sum of amplitudes which we are getting in this arm here, it must be equal to 0. Similarly, here the ray which was initially incident on the interface has magnitude a. While the ray which we are seeing here, it has magnitude $ar_1^2 + at_1t_2$. Therefore, this resultant magnitude must be equal to a.

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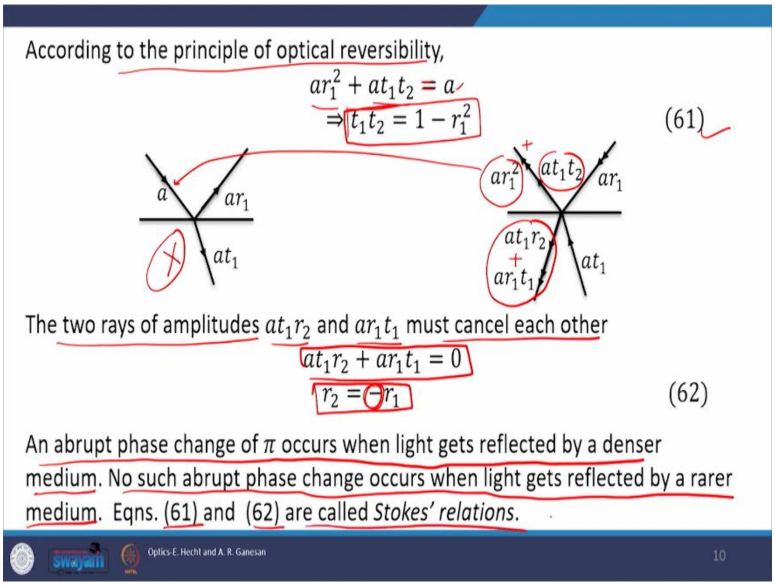

We will write these two conditions and see what is achieved here. Now, according to principle of optical reversibility, this magnitude would be added to this, and these two addition would give us a as per optical reversibility principle and this is what is done here $ar_1^2 + at_1t_2$ is added and equated with a, the input amplitude. And from here we get $t_1t_2 = 1 - r_1^2$ which is equation number 61.

Similarly, these two amplitudes would be added. And since there is no ray, this amplitude, the summation must be equated with 0. The two rays of amplitudes at_1r_2 and at_1r_1 must cancel each other and this is what is done here. And from here we get $r_2 = -r_1$, means amplitude reflection coefficients which you get if you go from one medium to another would be out of phase or would be 180 degree out of phase with respect to amplitude reflection coefficient if you get from second medium to the first one.

The same ray is met to incident from medium one to two and medium two to one. Then in these two reflection coefficients, apart from the usual difference of magnitude of the amplitude reflection coefficient, there would be a phase difference of π which is represented by minus sign here.

Now, we can bind these things in statement and the statement says an abrupt phase change of π occurs when light gets reflected by a denser medium and no such abrupt phase change occurs when light gets reflected by a rarer medium. And equation number 61 and 62 are called Stoke's relation. These two equation 61 and 62, these are called Stoke's relation. This is all about optical reversibility.

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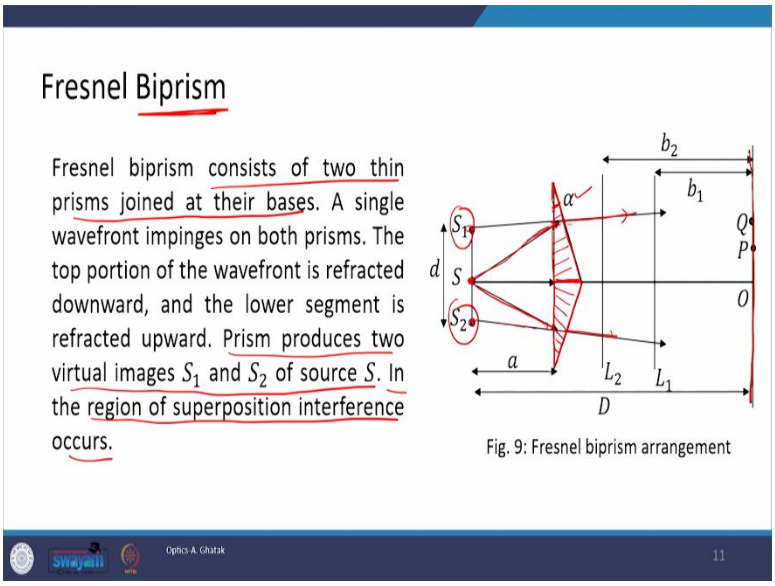

Now we will move to the last topic of today's lecture, which is Fresnel Biprism. This triangular say which you see here, this is biprism. Why do we call it a biprism? Because as you can see in this figure, it consists of two triangles, two prisms. This is the first prism and this is the second prison. These two prisms are glued together, since two prisms are glued to form a single prism therefore, the name this biprism. Fresnel biprism consists of two thin prisms joined at their bases. They are glued at their base and the prism angle is very small and this angle, prism angle is represented by angle α here.

Now, if you put a point source S here and you illuminates the prism, then what will happen is that the ray which is going in this direction, after refraction, it will bend towards the base of the prism and therefore it will bend in this direction. Similarly, the ray which is going here, it will bend in this direction. If you trace this back, then you will say that there are two virtual sources which would be formed in the source plane on the source side. And these virtual sources are called S_1 and S_2 which are separated by a distance d. Therefore, the prism produces two virtual images S_1 and S_2 of source S. And in the region of superposition, interference occur. Yeah, the two waves goes and whenever they superpose, interference occur and we see some fringes on the screen. The screen is here.

Okay, now, from the two sources, the two virtual sources are separated by distance d and the source plane is separated from the biprism by a distance a and the source plane is separated from observation plane by a distance D. Prism angle is α. These are known quantities.

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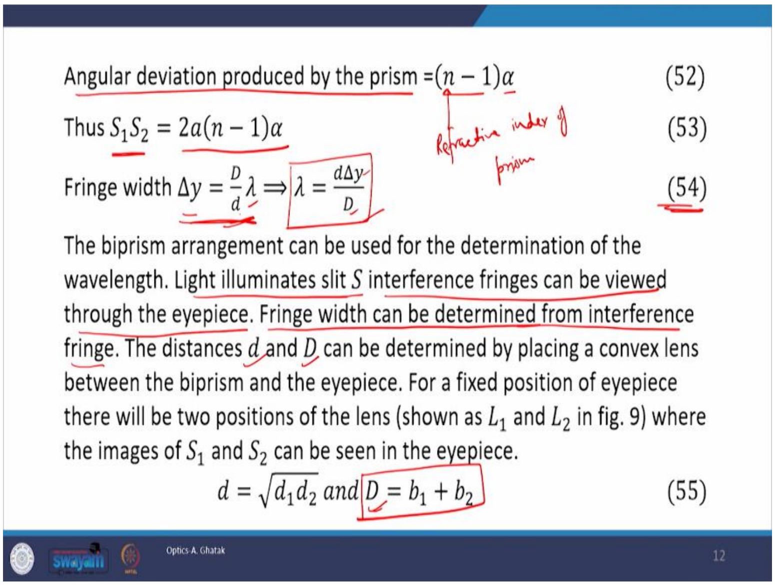

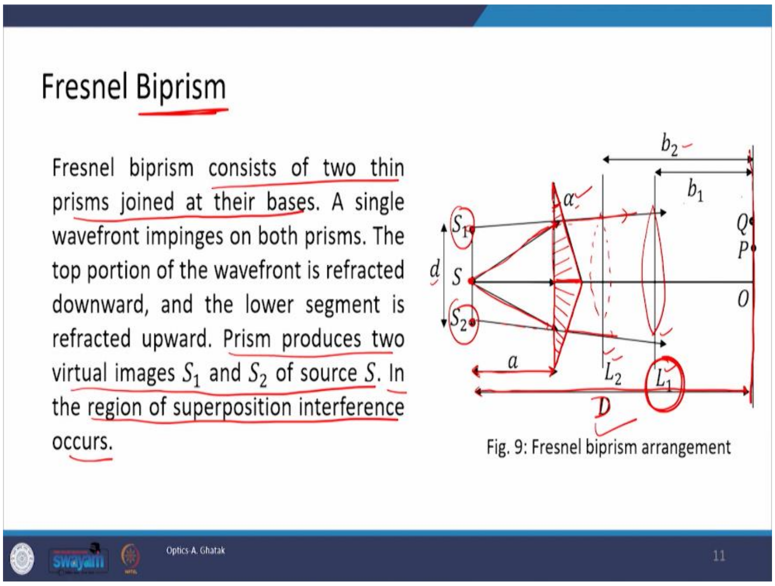

Now, the prism angle is very small. Therefore, the angular deviation produced by the prism would be equal to $(n-1)\alpha$, where n is the refractive index of the prism. Now once the expression for angular deviation is known, we can calculate S_1S_2 which is the separation between the two virtual sources which would be equal to $2a(n-1)\alpha$.

Now next we will calculate the fringe width, formula for the fringe width is known, which is Dλ/d. This is from Young's Double Slit experiment and from here we can calculate the wavelength of the light which is equal to $d\Delta y/D$. Δy is nothing but fringe width.

Now, from equation number 54, we can easily see that Fresnel Biprism can be used to determine the wavelength of the source. Now when the light illuminates slit S interference fringes can be viewed through the eyepiece. Instead of a screen, we put some eyepiece and then eyepiece collects the light, interference happens and we see those fringes.

Now, measuring the fringe width, we can determine the wavelength, this is we know. Now, how to measure the fringe width? In the eyepiece, we have a cross wire and the micrometer adjustment is associated with the cross wire. And we can move the cross wire in the horizontal plane and from there, we can measure the fringe width.

Now, once it is okay to measure the fringe width but how to measure D because S_1 and S_2 are the virtual sources. We do not know its position. Therefore, the quantities which are still unknown are small d and D. S_1, S_2 are the images of the source and small d is the separation between the two virtual sources and D is the separation between the observation plane and the source plane.

How to find out the correct values of d and D? This is done by placing a convex lens here and we can move the convex lens in the horizontal plane. Now when you move this convex lens in this setup then there would be two positions of the convex lens for which in the eye piece we will see distinct bright image of slits. Now we will note down the positions of the lens.

Suppose these positions are L_1 and L_2 and L_1 is at a distance b_1 from the observation plane and L_2 is at a distance b_2 from the observation plane. Then from there we cancalculate D. The $D = b_1 + b_2$ which is the distance between the source plane and the image plane or the screen plane.

Now when we move the lens in this longitudinal direction then what will happen is that for two positions of the lens we will see clear images of the two slits or two virtual images of the source which are S_1 and S_2. For each of these two positions, using eyepiece micrometer, we can record the separation between the two sources. I repeat, suppose we found the clear image of two sources for lens position L_1. Then at this lens position L_1, using the eyepiece micro, the cross wire in the eyepiece or using the micrometer scale, we can measure the separation between the two sharp sources.

And assume that this separation is d_1. Similar again, we will shift the position of the lens in the longitudinal direction and for another position of the lens we will again see a clear image of the two slits or two virtual sources. Once the image of two sources are distinctively and clearly seen in our eyepiece, than we again using the micrometer scale, we again measure the separation between the two sources or the image of two sources.

And say in this case the separation is d_2 then d is calculated by taking square root of multiplication of d_1 and d_2. $d = \sqrt{d_1 d_2}$. Using equation number 55 we can calculate D and d. And again using micrometer reading we can calculate the fringe width, once all the quantities are known we can calculate the wavelength. This is one of the application of this interferomatic setup. And this is all for biprism today. And with this I end my class. And thank you. See you all in the next class.