Applied Optics
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Lecture – 17
Interference of Light Waves, Interference of Polarized Light - II

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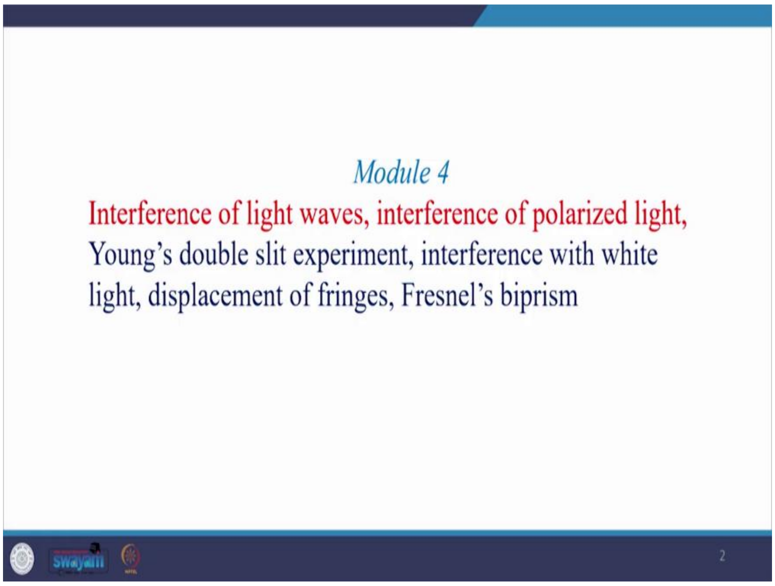

Hello everyone, welcome back to my class and we are in module 4 and this module we started in last lecture and we are learning interference. Now in the last lecture we talked about interference of light and we also saw the effect of polarization on interference, on the visibility of the interference.

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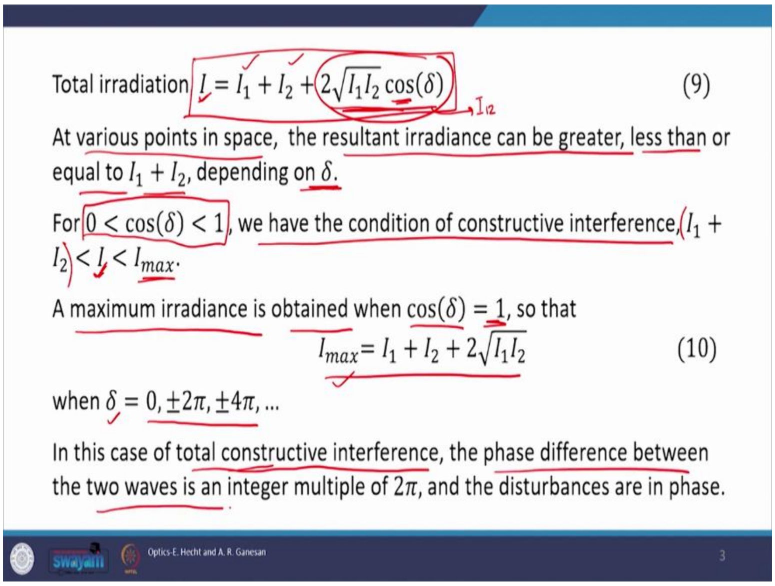

Now, today we will start from where we ended in the last lecture. We calculated the expression of total irradiance, this equation number 9, we already derived in the last class where I_1 and I_2 are irradiances due to source 1 and source 2 which have electric field contribution $\vec{E}_1$ and $\vec{E}_2$ at the point of observation P. And this is the interference term, this term we named it as interference term and it is expressed by I_{12}.

Now to remind you, we started with two point sources and from these two point sources spherical waves starts and then they overlap in a region of space. Now a point of observation P is very far from these two-point sources S_1 and S_2. Since the point of observation is very far, the waves which the observer at point P will observe would be plane wave because the curvature of the spherical wave front, it would become almost infinity.

The radius of curvature would become almost infinity and therefore, the spherical wave front would be treated as plane wave at point of observation P. With that assumptions, we started our calculations and we calculated the total disturbance at point of observation P due to presence of source S_1 and source S_2 and from that disturbances we calculated irradiance which is nothing but time average of square of total electric field at point of observation P.

Now at various points in space, the resultant irradiance can be greater, less than or equal to $I_1 + I_2$ and this depends on the phase δ. You see that, in this interference term, we have a cosine term and due to this cosine term, the irradiance $I_1 + I_2$ will be modulated and therefore the total irradiance at few points in space would be equal to $I_1 + I_2$ and at few place it would be larger than $I_1 + I_2$ and in few places it would be smaller than $I_1 + I_2$.

Now the places, the total irradiance is larger than $I_1 + I_2$, it will see more intense light, our total irradiance would be larger and therefore we will say that these are the reasons of constructive interference. Similarly, the places where total irradiance is smaller than $I_1 + I_2$, these are called the places of destructive interference.

Now, how to define these points in space or these places in space. Now, if $cos\delta$ is in between 0 and 1, then we will have the condition of constructive interference and in this condition the resultant irradiance I would be in between $I_1 + I_2$ and I_{max} and I_{max} is maximum irradiance and what is the maximum irradiance? Maximum irradiance is equal to $I_1 + I_2 + 2\sqrt{I_1 I_2}$.

We get I_{max} when $\delta = 0$ and this is what it is written here. Maximum irradiance is obtained when $cos\delta = 1$ and in this case $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$. When will we get this maximum irradiance, we will get maximum irradiance when δ is equal to integral multiple of 2π which is written here when $\delta = 0, \pm 2\pi, \pm 4\pi$ and so on. For these values of δ, we will get maximum irradiance.

And in the case of total constructive interference, the irradiance becomes equal to I_{max}, the total irradiance in case of total constructive interference becomes equal to I_{max} and when the interference is constructive but not totally constructive then the value of a irradiance will vary between $I_1 + I_2$ and I_{max}. Now, for total constructive interference, the phase difference between the 2 waves is an integer multiple of 2π and the disturbances are in phase.

And this is very much clear because if the phase difference is integral multiple of 2π, the two fields would always be in phase because the phase difference between them is one full complete cycle or integral multiple of complete cycle and therefore they would be called to be in phase.

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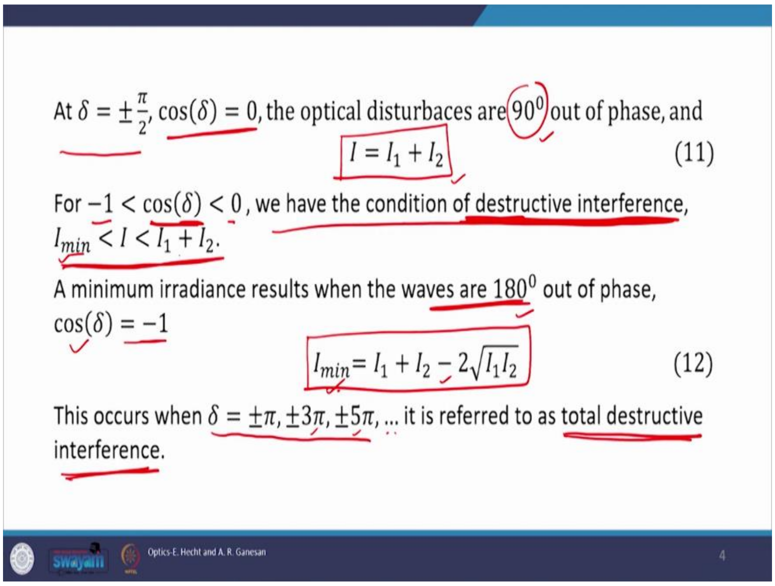

Now at places where $\delta = \pm\pi/2$, where phase difference is $\pm\pi/2$. In this particular case the cosine term would be equal to 0 and therefore the resultant optical disturbance would be $I_1 + I_2$ and when will this happen? This happens when the two interfering waves are out of phase by 90 degree. If the two interfering waves are out of phase by 90 degree in that particular case the resultant irradiance at the point of observation P is equal to $I_1 + I_2$.

Now, if the value of $cos\delta$ is between -1 and 0, then we have condition of destructive interference and the intensity or the irradiance at the point of observation P would be between I_{min} and $I_1 + I_2$. This is very much clear, if you substitute back the $cos\delta$ in this expression, in this equation number 9 you will see the variation in the irradiance.

Now what is I_{min}, now to calculate the minimum irradiance the two waves must be out of phase by 180 degree and if they are out of phase by 180 degree then $cos\delta$ would be equal to -1 and then the expression of $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$.

Here we have now minus sign then intensity is even smaller than $I_1 + I_2$ and this occurs when $\delta = \pm\pi, \pm3\pi, \pm5\pi$ and so on and these particular phenomena where we get irradiance equal to I_{min} is called total destructive interference. Here observes the wording, here I am using total destructive interference and here destructive interference only.

Total destructive means the irradiance is minimum, the intensity at the dark fringe would be least while in usual destructive interference there would be variation in the intensity, there would be some modulation in the intensity but the minima would not be as dark as it is in the case of total destructive interference and similarly with the total constructive and usual constructive interference.

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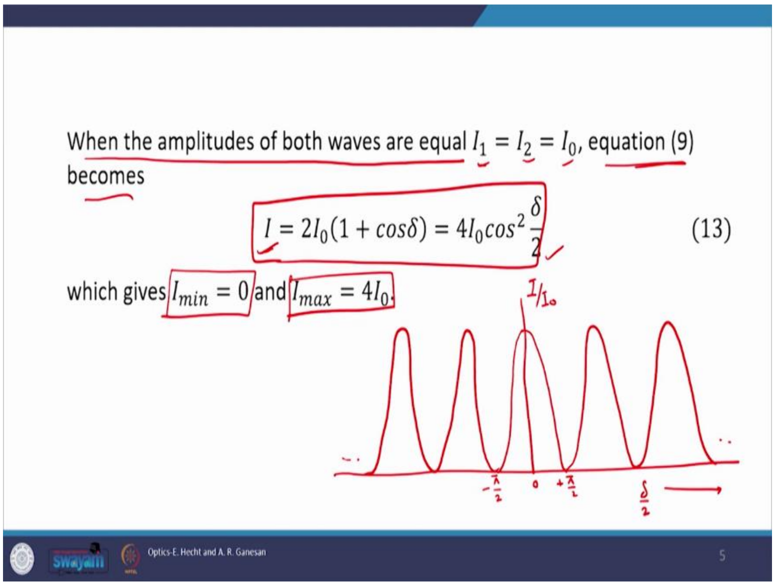

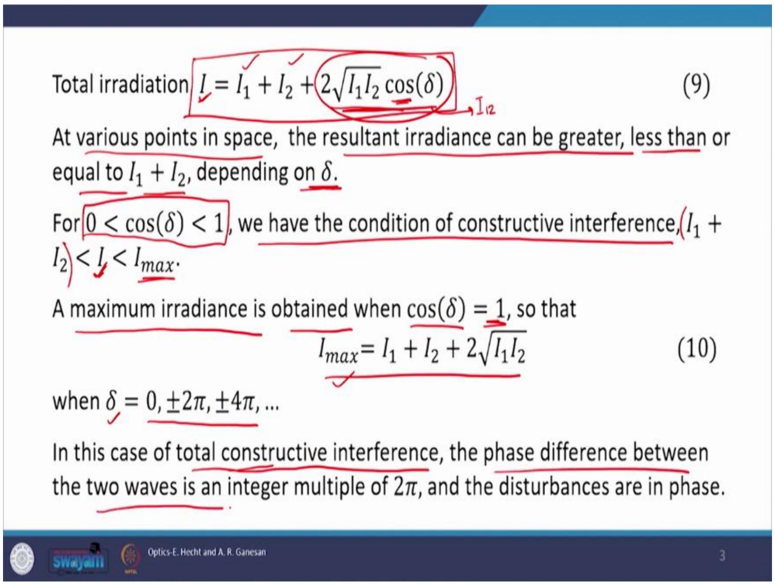

Now, consider a special case when the amplitudes of both the waves are equal that is irradiance $I_1 = I_2$ and suppose they are equal to I_0, in this particular case equation 9 which we derived in the last lecture, which is given here this is our equation 9, in this equation if we put $I_1 = I_2 = I_0$, then the resultant irradiance I can be expressed by equation number 13 and which says that $I = 4I_0 cos^2\delta/2$.

And let us calculate the I_{max} and I_{min} for this particular case, the irradiance would be maximum when $\delta/2 = 0$, or $\delta = 0$ and when $\delta = 0$, $I_{max} = 4I_0$. Similarly, the minimum irradiance will be equal to 0 and other values which we already discussed.

Now let us plot equation number 13 and see the variation of irradiance at the screen. Now if you plot $\delta/2$ here on the horizontal axis and I/I_0 on the vertical axis, then you see that the intensity maximizes when $\delta = 0$ and then it goes to reduces down and then it goes to 0 when $\delta/2 = -\pi/2$ here and $\pi/2$ and positive and then it again goes back to its maximum value and then again it goes down and then again it goes up and this again goes down and this repeats on the other side also. This type of periodic variation would be observed, if we plot equation number 13 and these are nothing but our interference fringes.

Now through these derivations or through this analysis we found that if we have two-point sources and a point of observation P, which is very far situated from the point sources S_1 and S_2, then we observe interference pattern and where we will see maxima and minima.

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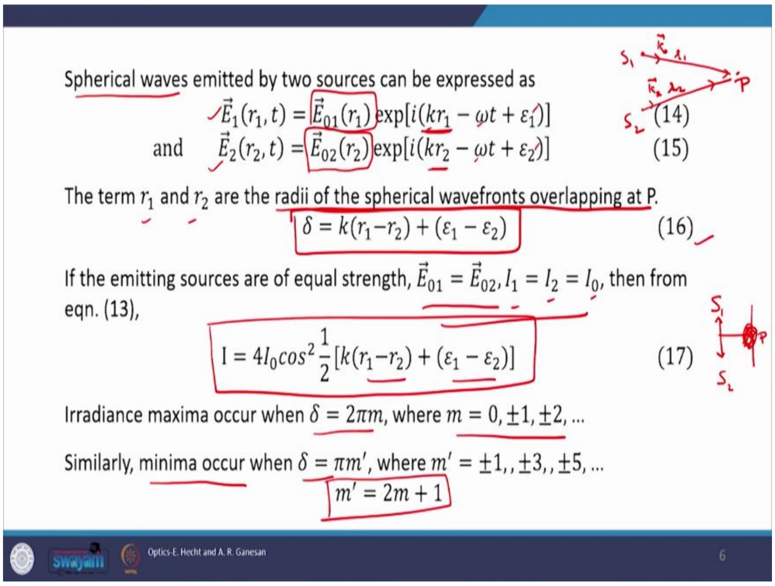

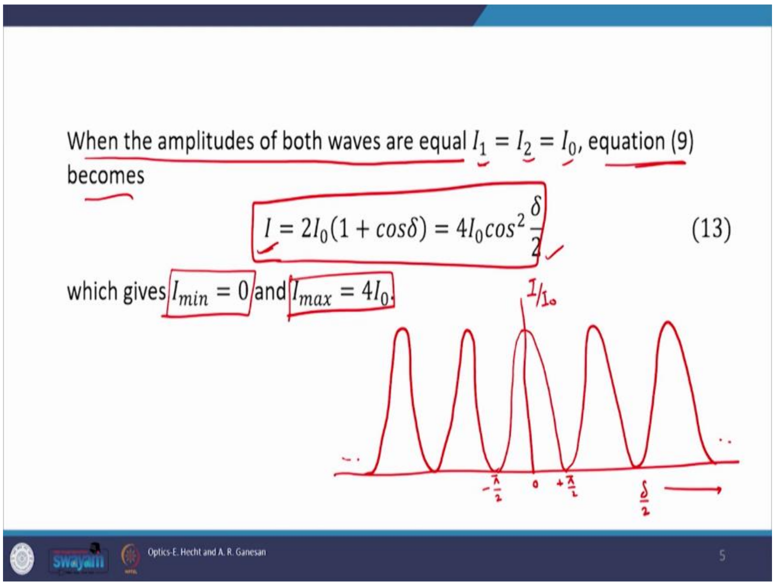

Till now, we were considering that the waves which are arriving at the point of observation P are plane but what will happen if we put the point of observation P close to the sources or alternatively what will happen if the waves reaching at point P, the observation point are spherical not plane.

Now in case of a spherical wave, we will have to rewrite the expression for our field. The field which reaches at the point of observation P. Now here for spherical waves emitted by two sources we have rewritten the expression for $\vec{E}_1$ and $\vec{E}_2$, what are vector $\vec{E}_1$ and vector $\vec{E}_2$? These are the contributions from sources S_1 and S_2 respectively to the disturbance at point of observation P.

Now since the wave is spherical, the wavefront is a spherical, the amplitude here is now a function of r_1. Similarly, the amplitude here in the second field expression is also a function of r_2. They are now function of distance, function of radial coordinate.

And observe here too in this exponential part earlier it was $\vec{k}.\vec{r}$ We were having two sources and we were having point of observation P here, this is S_1, this is S_2 and the waves starting from S_1 and S_2 are reaching to point P. Now this is $\vec{r}_1$ and this is $\vec{r}_2$ and the $\vec{k}$ is also in the same direction because in the spherical wave the radial vector and the wave vector would be in the same direction.

Now since the wavelength of the two waves are the same. Therefore, we will omit this subscript 1 and 2 from the expression of $\vec{k}$. Now if we take $\vec{k}.\vec{r}$, now since both $\vec{k}$ and $\vec{r}$ are pointing in the same direction, we will get scalar kr_1 because they are pointing in the same direction therefore $\vec{k}.\vec{r} = krcos\theta$, θ is the angle between the two and θ in this case is equal to 0 therefore, instead of writing things in form of dot product usually scalar multiplication is written here, the dot products are now replaced here with usual multiplication.

As usual ω (omega) is the frequency of waves starting from S_1 and S_2 and ϵ_1 and ϵ_2 are the initial phases. Now r_1 and r_2 are the radii of the spherical wave fronts overlapping at P quite clear, then therefore the resultant phase difference would now be written in this form here, $\delta = k(r_1 - r_2) + \epsilon_1 - \epsilon_2$. Equation 16 represents the effective phase difference between the waves super imposing at point P. Now, let us consider that the sources are of equal strength and they are very closely spaced.

Now what we are doing is that we pick two sources and the separation between the two are reduced. They are reduced in such a way that the separation between the two sources is much-much smaller than the distance of point of observation P from either of the sources and if this is the case and if sources are of same strength, then we can safely put $\vec{E}_{01} = \vec{E}_{02}$ and $I_1 = I_2 = I_0$.

I repeat, here we are assuming that sources are very closely spaced and point of observation P is situated at a distance which is much-much larger than the separation between the sources and interference pattern is observed around the centre of the screen. The observation space of the interference pattern is quite confined, if these are the source and this is the screen then we are confining ourselves in this shaded region around P.

If we consider this, then we will again go back to equation number 13, we replace the expression of δ and this is what we get. The expression for resultant irradiance at the point of observation P. Now here we see that the irradiance is now function of $(r_1 - r_2)$ and of course it is a function of the difference between the initial phase.

Now moving ahead, we can easily calculate the conditions on maxima and minima, maxima occur when $\delta = 2\pi m$, where $m = 0, \pm 1, \pm 2$, basically integer, and similarly minima occur when $\delta = \pi m'$, where $m' = \pm 1, \pm 3, \pm 5$ or so on. We can relate m and m' with this relation here. In the second case δ is odd integral multiple of π while in the first case where we are seeking maxima δ is integral multiple of 2π.

Now with this like let us substitute δ back into equation number 16, these expression of δ are substituted back in 16 and once we substitute them back.

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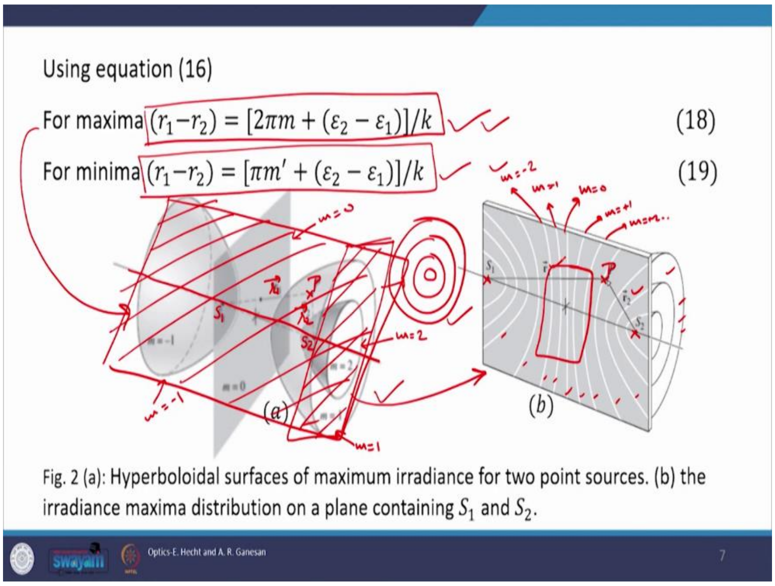

Fig. 2 (a): Hyperboloidal surfaces of maximum irradiance for two point sources. (b) the irradiance maxima distribution on a plane containing S_1 and S_2.

Then we get these two expressions for maxima and minima respectively. For maxima we get this expression and for minima we get this expression and you can see that on the right-hand side, it is a constant. On right hand side of equation 18 and 19 we have a constant quantity.

Now if we plot equation number 18 and 19 then we get Hyperbolide of revolution, which are shown here in figure 2(a). And for different values of m, we get different Hyperbolide of revolution.

Now in this figure, S_1 is source 1 and S_2 is source 2. S_1 and S_2 are the two sources which work as foci of the Hyperbolide of revolution, the right-hand side in equation 18 and 19, they represent the difference between the vortices in the Hyperbolide of revolution. Now if you vary m like this hyperboloid of revolution is for m=1, this hyperbolide of revolution is for m=2, this is for m=0.

Similarly, this is for m=-1, keep varying m and you will get different Hyperbolide of revolution. But with common foci. The point sources S_1 and S_2, here in this particular case works as foci and any point of observation P, it is at a distance r_1 from S_1 and r_2 from S_2. This is how they are defined.

Now you see that, when m is equal to 0, we get a plane which is passing through a point which is at an equal distance from vertices of the two Hyperbolide and as we increase the value of m, then we get different types of Hyperbolide, it is a three-dimensional Hyperbola. Now these Hyperbolide, Hyperbolide of revolution, these are fringes in 3-D, these are maxima, this Hyperbolide of revolution here they are plotted for equation number 18. Similar Hyperbolide revolution you will get for equation number 19.

The 18 represents the maxima, these Hyperbolide of revolution are locus of points where intensity or irradiance maximizes and these are the 3-D fringes. The interference is happening in space in 3-D and the dark and bright fringes are forming in this three-dimensional space and if we trace the bright fringes then this Hyperbolide of revolutions are formed.

Now if you cut this Hyperbolide of revolution through a plane which is perpendicular to the axis of this Hyperbolide, then you will get concentric circles. If you cut it through this plane here then you will get concentric circles. Here you get circular fringes, what do I mean by saying cutting this hyperbolide of revolution, cutting means, if you place a screen which is perpendicular to the line joining the two sources S_1 and S_2, this is the line joining the two sources S_1 and S_2 and this screen is placed perpendicular to this line S_1S_2.

If we place this screen in this way then at the screen, we will get fringes which would look like this, the fringes would be concentric circles. Alternatively, if you cut this structure through a plane which contains the line S_1 and S_2, what do I mean here is that if you cut this structure with this plane which contains the line S_1 and S_2, then you will get this.

Now here you can clearly see that source S_1 is sitting here, source S_2 is sitting here, the point of observation P is here and this is your vector r_1 and vector r_2. These white lines are the bright fringes which you observe when you cut the Hyperbolide of revolution through a plane which contains this S_1S_2 line and this circles which you see here on sides, these circles are observed when you cut it through a plane which is perpendicular to the line S_1 and S_2. These are two different types of fringes.

Now this fringe the central fringe which is a straight line, this is for m=0, the next one is for m=1. The next one is for m=2 and so on. Similarly, here on the other side, this one is for m=-1, this one is for m=-2 and so on.

Now, you see that close to m=0 line, there are so many fringes, there are so many dark fringes which are very closely spaced and they are very close to m=0 fringe. Why is it so? Because our wavelength is very small and therefore you see almost parallel fringes here in this region. And this is what exactly we observe in Young's Double Slit experiment.

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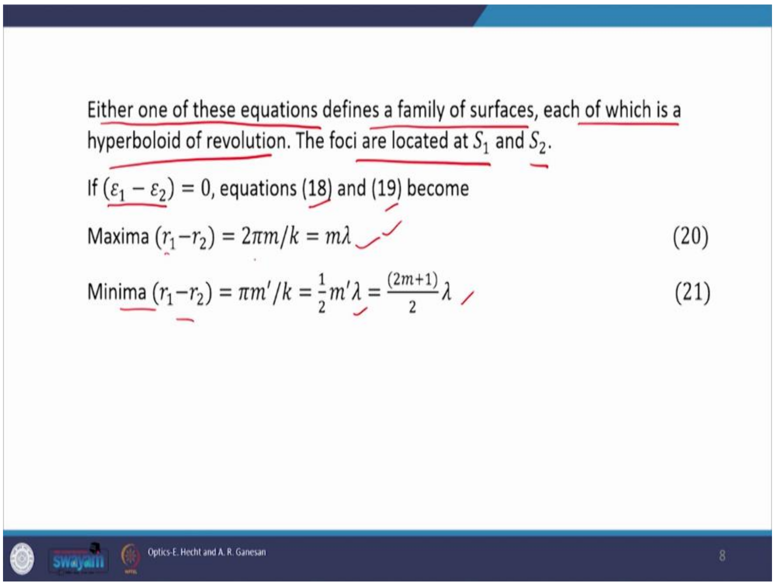

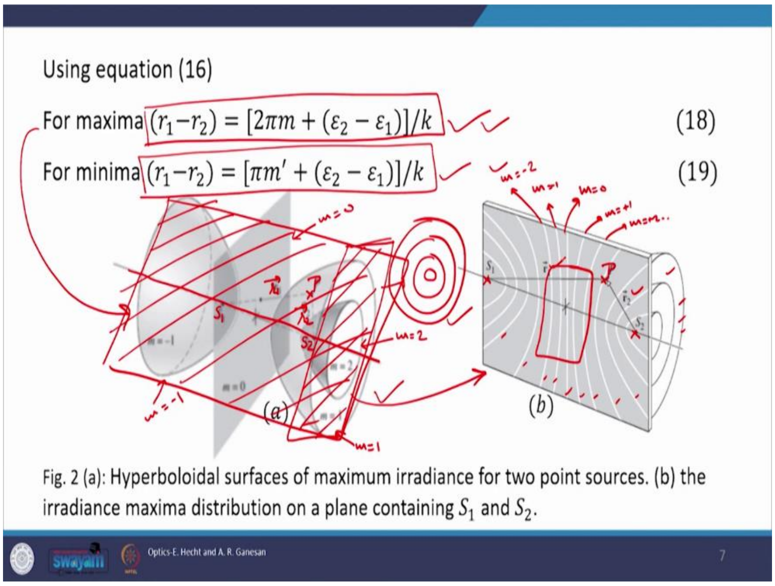

Now, as I discussed earlier either one of these equations, which equations? These equations equation number 18 and equation number 19, they defines a family of surface, each of which is a Hyperbolide of revolution and the foci are located at sources S_1 and S_2.

Now let us consider a particular case when the two sources, the waves which are emanating from the two sources are in same phase. If they are in the same phase then $\epsilon_1 - \epsilon_2$ which is the phase difference due to the initial phase, it is 0. Then the equation 18 and equation 19 will become this $r_1 - r_2 = m\lambda$ and for minima $r_1 - r_2 = m'\lambda/2$. They become quite simple and if you plot it, you will again get Hyperbolide of evolution.

Now here we saw that irrespective whether we are having a plane wave front or a spherical wavefront, we are getting fringes in interference with a spherical wave front. We get Hyperbolide of revolution, now depending upon the orientation of our screen where we want to observe the fringes, the shapes of the fringes vary. Sometimes we see straight line fringes, sometimes Hyperbolic fringes, sometimes concentric circular fringes.

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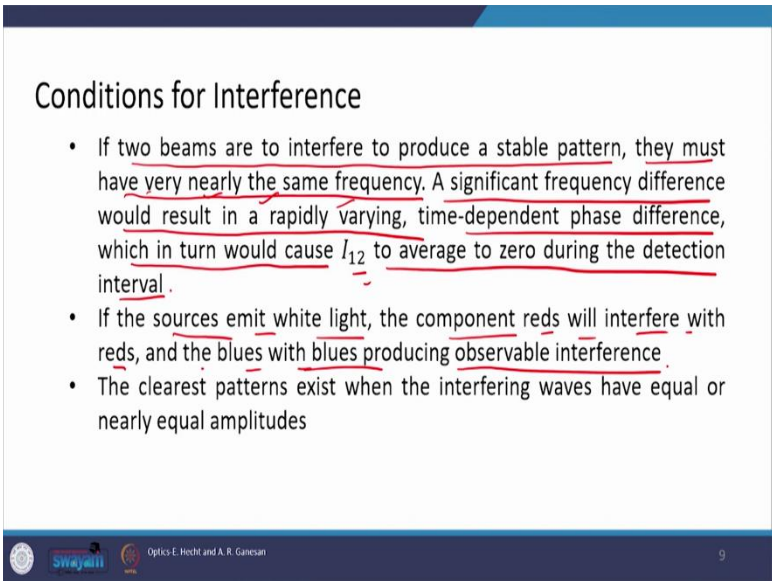

Now few take aways from this analysis are, if two beams are to interfere to produce a stable pattern, they must have a very nearly same frequency. These are the prerequisite to observe beautiful fringes, please observe on the wording beautiful fringes. Interference will happen anyway irrespective whether the sources are coherent or not, irrespective whether their frequencies are very close to each other or not, irrespective of their amplitudes.

The phenomena of interference will always happen, it would always be there because the waves are anyway overlapping but to observe beautiful fringes which does not vary with time. We must have a very nearly the same frequency, the 2 sources must have almost same frequency.

A significant frequency difference would result in a rapidly varying time dependent phase difference and how would it affect our fringe pattern, now if the phase difference is varying very rapidly then the positions of maxima and minima will vary and if these positions varies then if you take time average, then you will see a uniform illumination on the screen.

We would not be able to detect distinct fringes and this is what it is written here, which in turn would cause I_{12} to average to 0 during the detection interval. And therefore, a uniform illumination would be visible, if the frequency difference between the two sources is large. Second take away, if the sources emit white light the component reds will interfere with reds and the blues with blue producing observable interference.

Whenever we talk about interference, we always tell that the source must be a coherent source and the second criteria is that the phase difference between the interfering waves must be constant throughout. But here what we are claiming is that, even with the white light we will be able to observe interference fringes and we will talk more about white light interference in forthcoming lectures.

The next point is, the clearest patterns exist when the interfering waves have equal or nearly equal amplitudes, why, because when it is constructive interference the waves amplitude will sum up and a very nice bright fringe will appear and when it is destructive interference, all the energy will go away and we have very nice dark fringe and due to this nice contrast, the very clear pattern would exist and therefore if we want to observe a very nice beautiful well contrasted interference pattern, the waves must have almost equal amplitudes.

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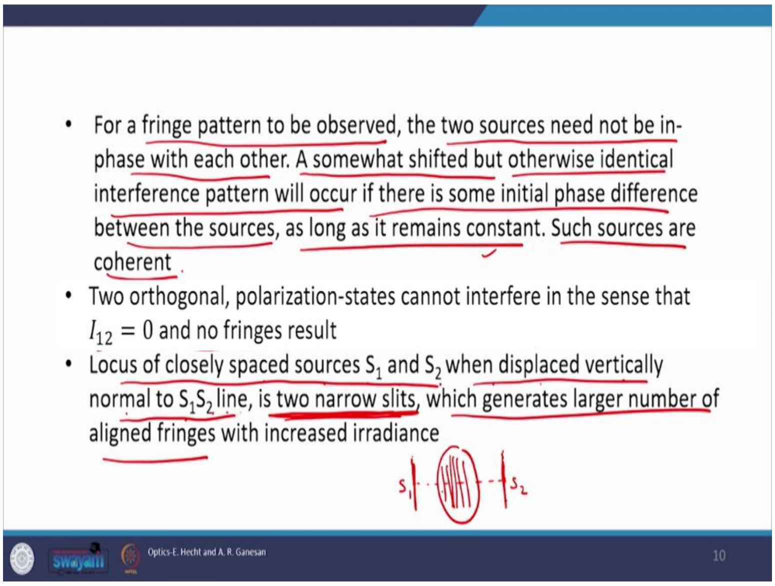

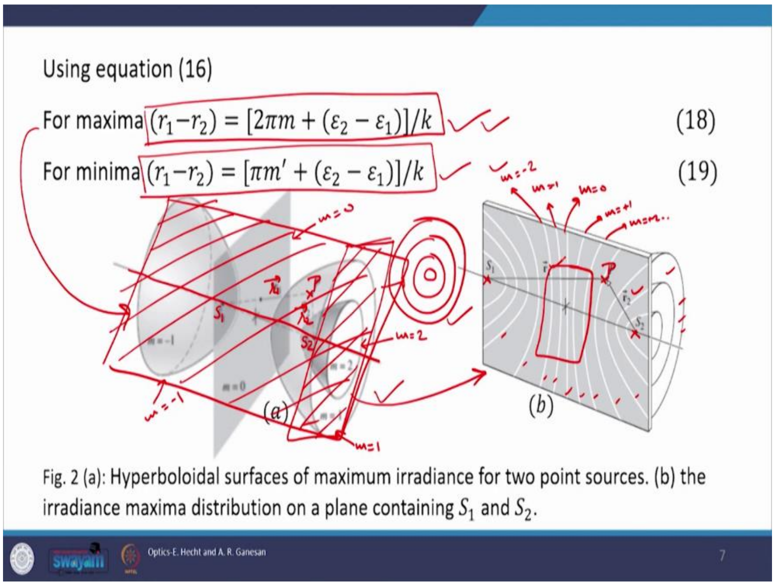

Next point, for a fringe pattern to be observed the two sources need not be in phase with each other. The important point is that the waves emanating from the two sources must maintain the constant phase difference.

Irrespective of the value of this constant. If the phase difference is 0 initially, you will observe well beautiful interference fringe and its symmetric but what if the phase difference is not 0 but constant. If it is not 0 but constant then somewhat shifted but otherwise identical interference pattern will occur and if there is some initial phase difference between the sources then there would be a shift in the interference pattern.

But if it is a linear vertical lines then this shift would be indistinguishable and as long as this initial phase difference is constant, you will see a nice bright and beautiful interference pattern and this shift would not matter, whatever applications of the interferometer have, it would still be able to satisfy those applications but the fringe pattern would be a bit shifted due to non-zero value of this constant phase difference.

Now if we have two sources and the phase difference between the two is constant then such sources are called coherent. If the two sources maintain constant phase difference over a long period of time, then these sources are called coherent sources. We will discuss more about coherence in our forthcoming lecture. The next point which we observed is that, two orthogonal polarization, the state cannot interfere in the sense that I_{12} is equal to 0 and no fringes result.

During our analysis, we saw that if we let interfere to cross polarized light or to cross polarized wave or the waves which are orthogonally polarized then the interference term is equal to 0 and when interference pattern is 0, then you will see a uniform illumination on the screen, on the observation plane. We will not be able to see fringe pattern, all the interference is happening there too but due to this cross polarized nature of the interfering wave the fringes will not be there.

Now the locus of closely spaced sources S_1 and S_2 when displaced vertically normal to the S_1 and S_2 line is two narrow slits. We started with 2 slits S_1 and S_2, these are the 2 slits and this is the line joining the 2 slits. Now if we shift the position of these two-point sources $S_1$and S_2 along a line which is perpendicular to the line joining S_1 and S_2 then this point will either go up in this direction or in this direction.

But if we shift the position of these two sources vertically then what will happen, let us go back, now see figure number 2b, now if you shift S_1 and S_2 vertically up or down then what will happen is that the whole fringe pattern will either go up or down. If S_1 and S_2 are shifted vertically up, the whole fringe pattern either will go up and if the S_1 and S_2 are shifted down then the whole fringe pattern will go down. In this exercise, the position of almost vertical fringes which are here sitting at the centre, they will remain unaffected, they will remain intact. The distribution at the centre would almost remain intact.

There would not be any change and therefore what we can do is that we can create a line source just by shifting these two-point sources S_1 and S_2. Let us consider several S_1 and S_2 and put them along a line, then S_2 will create a line source and similarly S_1 will create a line source.

And then if you generate interference pattern out of these line sources then we will get these vertical lines at the centre but since the number of sources has got increased the irradiance of these bright lines would be larger because the number of point sources which are contributing in the formation of this pattern has increased and this will also therefore increase the irradiance or the brightness of the bright fringes, the bright vertical lines.

And this is what exactly happens in Young's Double Slit experiment and this is what is also written here. Locus of closely spaced sources S_1 and S_2 when displaced vertically normal to S_1 and S_2 line is two narrow slits.

This will create two narrow slits which generates larger number of aligned fringes here, the central fringes will be generated and they would be large in number and with increased irradiance. Their radiance will of course be increased because the contributing source has increased now.

Now this is all for today, today we discussed in detail the basics of interference and in the next class, we will talk about Young's Double Slit experiment, the famous experiments on interference. Thank you all for listening me.