# Nuclear Astrophysics
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**Module – 05**
**Lecture – 22**
# Burning Stages of Stars and Hydrogen Burning

Welcome students to this lecture of different burning stages within a star. In the last lecture, I discussed the background of burning stages in a star starting with the hydrogen burning and some basic points regarding p + p reaction. What are the products emitted when two protons interact with each other? Which reaction is more feasible? And, which reaction is not possible? I also discussed neutron induced non-resonant reactions.

And, I have emphasized the importance of constant value of a reaction rate. Remember, it was valid only when s-wave neutrons and low energies of the neutrons are considered which is the normal case.

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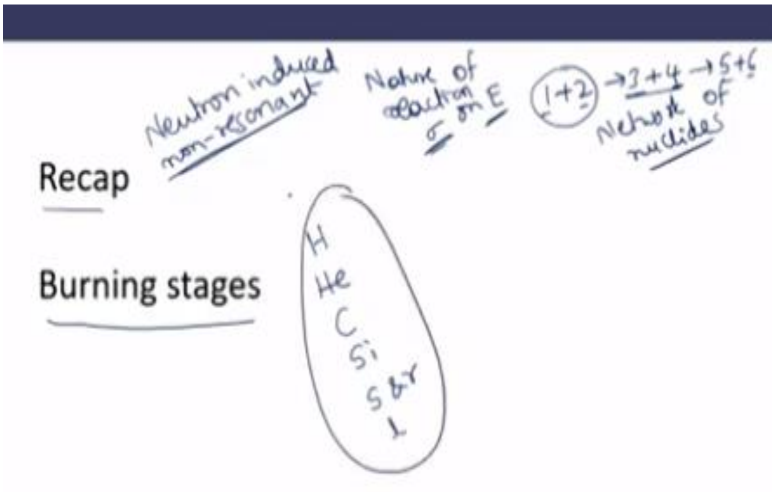

Recap

Burning stages

So, as a recap, I would like to highlight that neutron induced non-resonant reactions were discussed in the last class. Then, up to this, I have discussed basically the nature of the nuclear reaction. Is it smooth dependence of $\sigma$ on E or sudden increment or change of cross section of nuclear reaction on energy? So, it could be resonant or non-resonant.

So, after I discussed the individual nuclear reactions and their properties with some examples, we need to discuss the networking of nuclides which are participating in the evaluation of a star. So, because there are two main goals of this course, number 1, how to understand the energy produced from the stars? In which nuclear reactions play an important role? At the same time, what are the nuclear reactions which are responsible for the synthesis of elements that we see in periodic table?

These two different angles we are trying to understand. For that, till now, I have spent some time on the nature of nuclear reactions which are relevant for nuclear astrophysics. Now, because one nuclear reaction is not sufficient, so, when I discuss 1 + 2 gives rise to 3 + 4. So, when 1 and 2 nuclei are reacting to produce 3 and 4, so, this 1 and 2 got destroyed and this 3 and 4 got produced.

And again, when this 3 and 4 participate in some reaction you have 5 + 6. So, the nuclear reactions are taking place in large number inside the star. So, we need to understand the networking of these nuclides which are responsible for the evolution of a star. And, that is what makes different burning stages within a star: hydrogen burning, helium burning, carbon burning, silicon burning, s- and r-process and then l-process.

So, that is how we categorized the burning stages within a star. How? We can understand the categorization of burning stages. As I said, whenever nuclei with lowest Coulomb barrier is present at a certain time they will participate in the nuclear reaction. And, once they are consumed, then the remaining nuclei which are produced because of the original nuclei, they start undergoing gravitational contraction to a point when temperature is raised and sufficient to ignite another set of nuclei with next higher Coulomb barrier.

So, this is what decides the evolution of a star in terms of burning stages. So, we need to understand the 1st burning stage. That is hydrogen burning stage. And, in today’s lecture, I am going to inform and I am going to share some of the interesting points in the most fundamental nuclear reaction which is responsible for the evolution of universe. It is p + p reaction. I hope you will enjoy today’s lecture.

Let us see the salient features of this p + p reaction. Come, see, what I am going to list out here in terms of questions.

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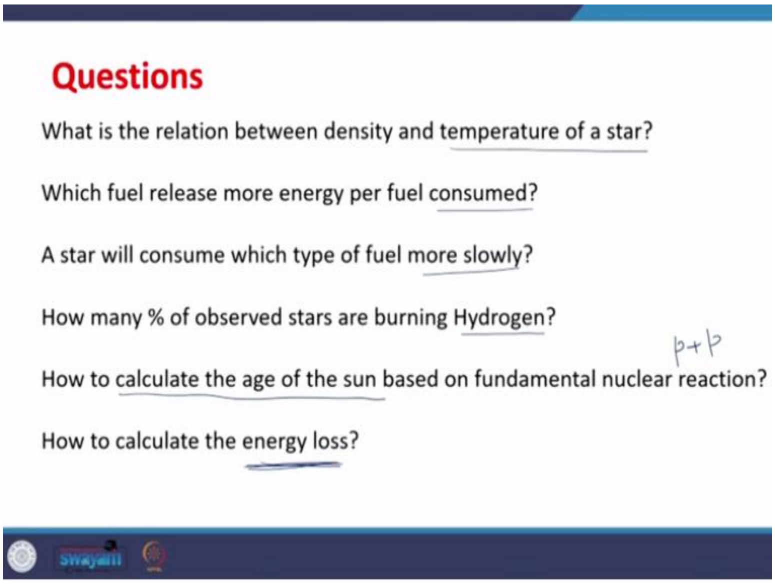

## Questions

What is the relation between density and temperature of a star?

Which fuel release more energy per fuel consumed?

A star will consume which type of fuel more slowly?

How many % of observed stars are burning Hydrogen?

p+p

How to calculate the age of the sun based on fundamental nuclear reaction?

How to calculate the energy loss?

What is the relation between density and temperature of a star? How to answer these questions? And, which fuel release more energy per fuel consumed? What is the relation with Coulomb barrier? And, when star consume different types of fuel, which type of fuel consume will be consumed more slowly and why? Can we try to understand with the help of some diagram?

And, because hydrogen is the most abundant element in the universe, out of absorbed stars, how many of them are burning hydrogen? And, what is the basis for that number though it is tentative? And, with this, is it possible to calculate the age of the Sun based on this fundamental nuclear reaction that is p + p reaction? And, what is the energy loss in every nuclear reaction which is not counted as part of energy produced from the star?

Though, this is nothing new for you. I have told earlier that it is because of escape of neutrinos and antineutrinos emitted during the different types of nuclear reactions because of their extremely weak ability to interact with others. They will escape from the star. So, that energy is not counted. That is fine, but that how much amount of energy is lost because of these neutrinos and antineutrinos.

We need to come up with some numbers. So, this set of questions I hope makes you interested to know the nature of p + p reaction. Let me bring back one diagram which I have shown earlier.

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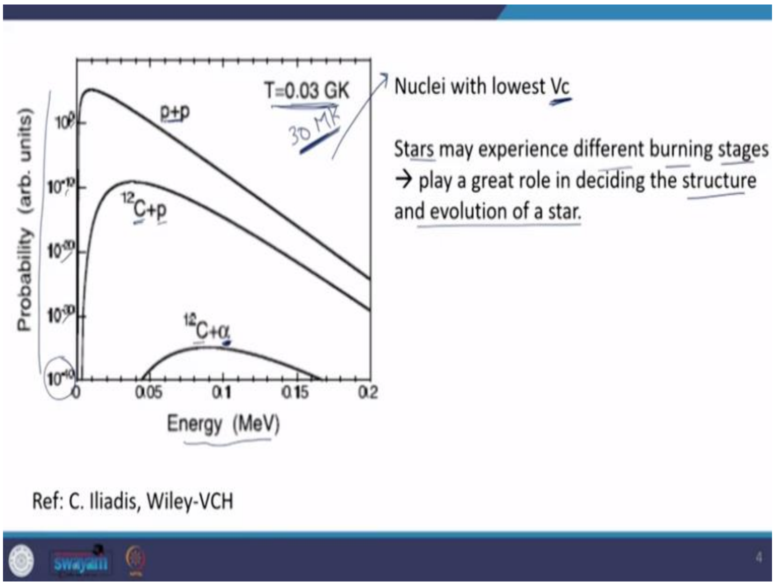

Nuclei with lowest Vc

Stars may experience different burning stages → play a great role in deciding the structure and evolution of a star.

Ref: C. Iliadis, Wiley-VCH

When we see the Gamow peak, see, this is the Gamow peak on the y axis in logarithmic scale (refer to the above slide). So, the probability versus energy, if we take at a specific temperature, say, $3 \times 10^7$ K that is 30 MK. How? The Gamow peas looks like for most fundamental nuclear reaction in the universe. That is p + p.

Then, this proton interacts with $^{12}$C and alpha interacts with $^{12}$C. So, here I have changed the projectile from proton to alpha. So, as I discussed earlier, the Gamow peaks centroid shift towards the higher side because of the strong dependence on Coulomb barrier. So, this figure establishes the fact that nuclei with lowest Coulomb barrier starts consuming within the star.

And as I said, the stars may experience different burning stages which can be categorized into 6 types. And, the existence of different burning stages itself is a testimony that the Coulomb barrier and then the different burning stages decides the structure and evolution of a star. So, this should provide enough information about the importance of different burning stages. The whole point I would like to highlight here is why it is important to study the different burning stages within a star.

Because it is the burning stage which is deciding the structure and evolution of a star and the 1st burning stage that is which is based on hydrogen we are trying to understand.

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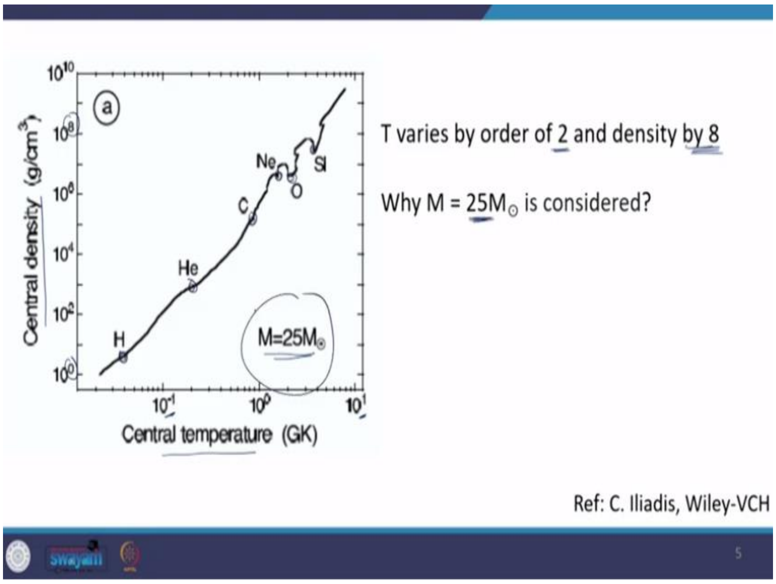

T varies by order of 2 and density by 8

Why $M = 25M_\odot$ is considered?

Ref: C. Iliadis, Wiley-VCH

Now, in a star, if I consider the central temperature on x axis and the density at the center of the star on y axis, you can see here the circles hydrogen, helium, carbon, neon, oxygen, silicon burning, so, these circles represent the different burning stages at different temperatures. Of course, here I have assumed the mass of the star as 25 times that of mass of the Sun. And, you see this interesting number. Temperature is changing by order of 2.

However, the density is varying from 0 to 8. So, the density is varying by the order of 8. So these numbers are quite important. When different burning stages are involved though I am not showing here s- and r-process because they are mainly induced by neutrons whereas these burning stages shown in this diagram, they are dependent on the charged particle induced non-resonant and resonant reactions. Why?

We have considered mass of the star as 25 times. Why not some other number? See, it has been established by the researchers that a star with the 25 times that of mass of the Sun is proved to be beneficial to understand the abundance of elements like solar system. So, that is the reason we have considered this mass of the star as 25 times that of mass of Sun.

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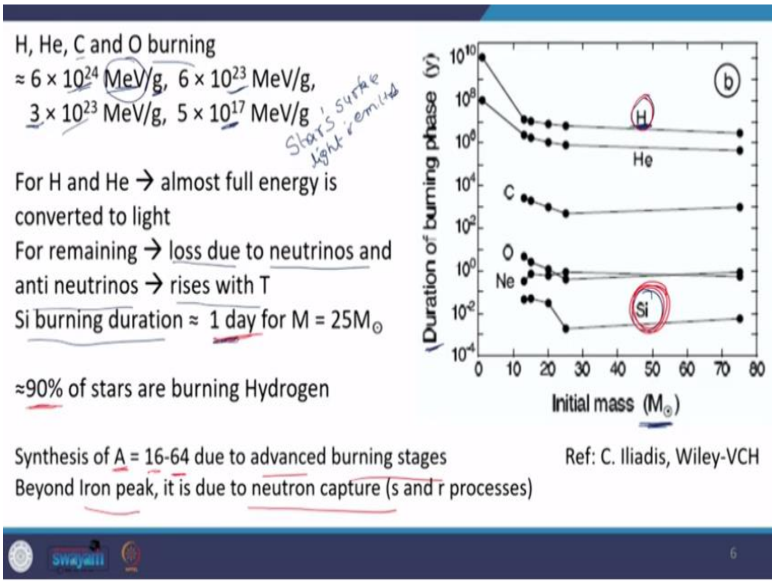

H, He, C and O burning
≈ $6 \times 10^{24}$ MeV/g, $6 \times 10^{23}$ MeV/g,
$3 \times 10^{23}$ MeV/g, $5 \times 10^{17}$ MeV/g

Star's surface light emitted

For H and He → almost full energy is converted to light
For remaining → loss due to neutrinos and anti neutrinos → rises with T
Si burning duration ≈ 1 day for M = $25M_\odot$

≈90% of stars are burning Hydrogen

Synthesis of A = 16-64 due to advanced burning stages
Beyond Iron peak, it is due to neutron capture (s and r processes)

Ref: C. Iliadis, Wiley-VCH

Now, let me start the actual part of the burning stages. And here, you can see that the hydrogen burning. How the duration changes with the mass of the star? Because it is the mass of the star which decides the composition and also the probability of different burning stages. Is not it? So, of course, it starts with the hydrogen burning. Initially, we can assume that because the fundamental nuclear reaction is p + p.

Depending on the original mass of the star, how the duration is changing? This diagrams provide interesting information, so, hydrogen, helium, carbon and oxygen burning. If we see the energy production rate per unit gram, here energy is in terms MeV, what is the energy produced per unit mass of the fuel? This gives some interesting numbers.

Like $10^{24}$ for hydrogen, $10^{23}$ for helium and order is same for carbon and much less for the oxygen. So, this is how one can feel the importance of energy production per the consumption of unit mass of the fuel per unit mass of the fuel. Now, if we see the energy can produced and the light emitted by the star from the star surface which we normally detect and whose light spectra gives information about the existence of different elements.

That is how I have shown that researchers have measured or discovered the technetium and some other elements as well. What is the relation between the energy produced and the light emitted? What might have happened in the star? Because whatever energy is emitted that we are not detecting. It is the light emitted by the star surface which we are able to detect. Now, in this burning stages, how to understand the relation between the energy produced in these burning stages and the light emitted by the stars surface from the outside.

Coming to the hydrogen, helium, almost all energy emitted is converted into the light. And for remaining things, because of the loss due to neutrinos and antineutrinos, the probability is more for remaining nuclear reactions. The loss is more. And, this loss increases with increase in the temperature. Now, if we take the duration of silicon burning, you see, it is just 1 day. Silicon burning, you see this one.

So, this silicon burning if you see the duration is just 1 day and whereas, hydrogen burning the duration you see, it is about you know $10^6$ to $10^7$ to $10^{10}$ years depending on the mass of the star. So, one can see the large difference in the duration of burning stages. Within 1 day silicon burning is over whereas initially the hydrogen burning is taking place for billions of years. Why so different?

Why there is so much difference? And, keeping this in mind, can we answer the question? How many percent of the observed stars are burning hydrogen? Because when things are happening so quickly like silicon burning, it is not possible to even identify them. It is very difficult. So, one can say that almost 90% of the stars are burning hydrogen and though remaining stars are not burning hydrogen.

In terms of energy production, they may not be important. But, in terms of synthesis of elements with mass number ranging from 16 to 64, the remaining nuclear reactions are important. Though they are not important for energy production before iron peak, they are important in terms of energy production that is in advanced burning stages.

Now, as I said beyond iron peak it is not the energy production which is very important. It is the synthesis of elements and which are happening because of the capture of the neutrons. And, in the last class, I have discussed, what are the reactions which are important for the production of neutrons? And, how the velocity distribution is takeing place? And, the reaction rate depends on the velocity of the neutron, 1/v law.

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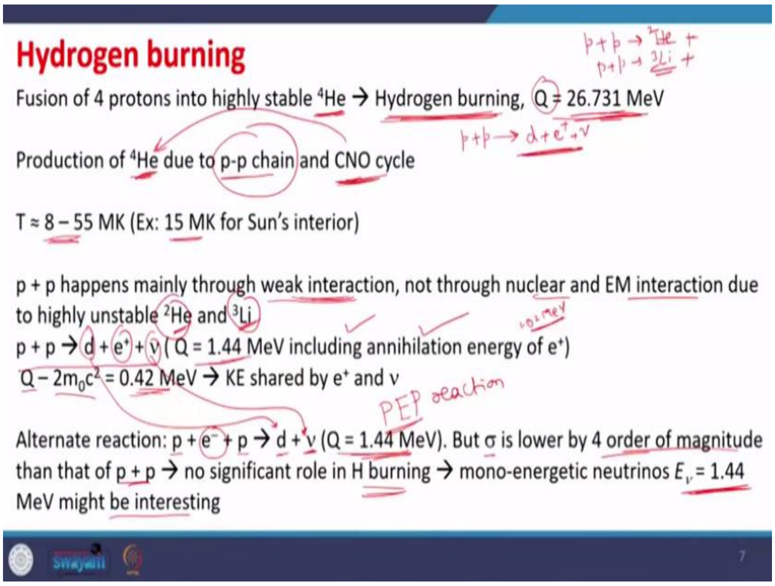

## Hydrogen burning

$p + p \rightarrow {}^{2}He +$

$p + p \rightarrow {}^{3}Li +$

Fusion of 4 protons into highly stable $^{4}He$ → Hydrogen burning, $Q = 26.731$ MeV

$p + p \rightarrow d + e^{+} + \nu$

Production of $^{4}He$ due to p-p chain and CNO cycle

$T \approx 8 - 55$ MK (Ex: 15 MK for Sun's interior)

p + p happens mainly through weak interaction, not through nuclear and EM interaction due to highly unstable $^{2}He$ and $^{3}Li$

0.42 MeV

$p + p \rightarrow d + e^{+} + \nu$ ( $Q = 1.44$ MeV including annihilation energy of $e^{+}$)

$Q - 2m_{0}c^{2} = 0.42$ MeV → KE shared by $e^{+}$ and $\nu$

PEP reaction

Alternate reaction: $p + e^{-} + p \rightarrow d + \nu$ ($Q = 1.44$ MeV). But $\sigma$ is lower by 4 order of magnitude than that of p + p → no significant role in H burning → mono-energetic neutrinos $E_{\nu} = 1.44$ MeV might be interesting

Now, let us see some more important features of the hydrogen burning. See, it was clear that fusion of 4 protons leads to the production of $^{4}$He. And, this process is called as hydrogen burning. And, the Q-value of this reaction is 26.731 MeV. So, you can take the mass excess of the values and find out the Q-value. And then, you can see that it is the Q-value of the p + p reaction.

I mean the fusion of 4 protons which is producing $^{4}$He is 26.3731. Fusion of 4 protons is giving rise to $^{4}$He because it is quite easy to assume that 4 protons have to be involved. Now, what about the simultaneous interaction of these 4 protons? Is it possible? Because simultaneous interaction of all these 4 protons leading to production of $^{4}$He initially seems like possible.

I mean $^{4}$He can be produced. But, the probability cannot be given as the reason for the observed abundance of the $^{4}$He. So, if it is not a single step, then what is other way? If it is not single step, it could be 2-step process. So, if it is 2-step process, what could be the sequence? Is it like this? p + p giving rise to $^{2}$He. Is not it?

What are the elements produced because of some other particles are also involved here. See, the reaction p + p giving rise to $^{2}$He or some other positrons, electrons or neutrinos, they are not sufficient to show that they are responsible for the production of $^{4}$He because highly unstable elements are involved. That is when the major thought process came up because of the thought that it is basically p + p.

Initially, it is giving rise to deuterium + positron + neutrino. Of course, we will see more features of this p + p reaction. And, the production of $^4$He which is the 2nd abundant highest abundant element in the universe, it happens due to p + p chain where 4 protons are involved to produce $^4$He. At the same time, 4 protons are also used to produce CNO cycle. And, they are in CNO cycle for the production of same $^4$He.

So, production of $^4$He is because of both pp chain. I mean what are the reactions involved in this pp chain? I will discuss. And, it can also be due to CNO cycle, carbon nitrogen oxygen cycle. That also will lead to the production of $^4$He. And, this pp chain happens in the stars when temperatures are in the range of 8 to about 55 MK. For example, in the Sun's interior, we have 50 MK.

And, p + p happens mainly through the weak interaction only not through the nuclear or electromagnetic interaction, because as I have written here, $^2$He or whatever they are produced, they are highly unstable. So, it is not possible to assume that $^4$He could have been produced because of production of $^2$He. So, it is mainly through weak interaction. And, this is the fundamental reaction.

p + p is producing deuterium, positron and neutrino. The Q-value of this p + p reaction is 1.45 MeV. Please note that it includes annihilation energy of the positron. That is 1.02 MeV, 511 keV + 511 keV. And, what is the kinetic energy shared by positron and neutrino? It is simply ($Q - 2m_0c^2$). That is 0.42 MeV and remaining 1.02 MeV goes into the annihilation energy. Now, one more reaction could be possible.

When two protons are undergoing fusion, it can take up 1 electron like capture and give same deuterium and neutrino. Here also, it is the same deuterium and same neutrino. And, the Q value is same, 1.44 MeV. But, remember, the cross section is lower by 4 orders of magnitude when compared to p + p reaction. So, this has no significant role in the hydrogen burning though the interesting point in this reaction which is called as proton electron proton reaction PEP reaction.

This emits mono energetic neutrinos. That is 1.44 MeV. This might be interesting for the detection of neutrinos from the sun, because already the detection of neutrinos is quite challenging. And, the energy is different, it is much challenging task. If we have a mono energetic neutrinos, things can be planned maybe in a better way when compared to the detection of poly energetic neutrinos.

But, for that you need to know the source of this mono energetic neutrinos. And, that is PEP reaction, of course, which has no real significant role in the production of $^4$He as part of hydrogen burning.

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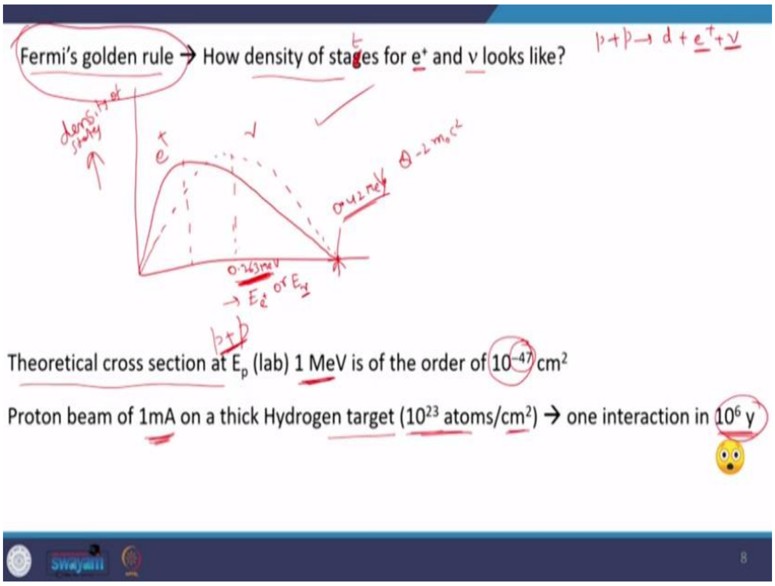

Fermi's golden rule → How density of stages for $e^+$ and $\nu$ looks like?

Theoretical cross section at $E_p$ (lab) 1 MeV is of the order of $10^{-47}$ $cm^2$

Proton beam of 1mA on a thick Hydrogen target ($10^{23}$ atoms/$cm^2$) → one interaction in $10^6$ y

Now, this mathematics of a Fermi golden rule I will not discuss, but I want to say that the density of states for positron and neutrino when $p + p \rightarrow d + e^+ + \nu$, when they are emitted, what is the energy distribution of positron and neutrino? So, this looks like this (slide 8). The density of states if you go for the energy and the density of states the details I am not sharing here.

But, one interesting thing I want to share you here is that the density of states for positron and for neutrino. The maximum energy is 0.42 MeV. And, the calculation I have shown this is nothing but ($Q - 2m_0c^2$). And, the maximum energy, this is for the neutrinos and this is for the positron. So, this would be either for the positron or this could be for energy of the neutrinos.

So, the average energy of the neutrino if you see, it is about 0.263 MeV. And, the average energy of the positron is less than this one. And, this value will be useful in calculation that we will discuss very soon. So, this diagram basically shows that the distribution of energy among positron and neutrino takes place which can be understood from the density of states available for the positron and neutrino.

And, mathematically, one can know more with the help of Fermi's golden rule which is not the topic of this course. For the sake of information, I would like to give this diagram. But, please remember this 0.263 MeV, the average energy of the neutrino. Why? Because we need to understand the loss of energy from the stars and the loss of energy is taking place because of the escape of the neutrinos and antineutrinos.

So, when you do the calculation of energy produced from the stars, and if you want to know the energy laws, you need to know the number regarding the energy of neutrinos. So, that is where this diagram is useful. I hope it is clear to you. Anyway, if we find out the cross section of this p + p reaction theoretically at 1 MeV say, then the cross section is of the order of $10^{-47}$. You see such a big number.

Earlier also, I have tried to emphasize the importance of this reaction and why it is not possible to measure experimentally. The cross section is of the order of $10^{-47}$. Even if you use a maximum current available for the protons and the energy of the proton is say 1 MeV. Now, if I take the 1 mAe kind of current for the protons, and if I take thick hydrogen target where per $cm^2$ you have $10^{23}$ atoms, then you see, one interaction will happen in $10^6$ years.

If one interaction is happening in $10^6$ years, how the present generation when we do the experiment will observe this and even in the $10^6$ years if you get one interaction, when you have 1000s of events so that you can measure the cross section. So, that is the reason. It is almost impossible to measure the cross section of p + p reaction with the available facilities.

Maybe in future if the technology is developed with a high current and better targets and better data equipped system and extremely low background, things may change in future. But, for now, there is no hope.

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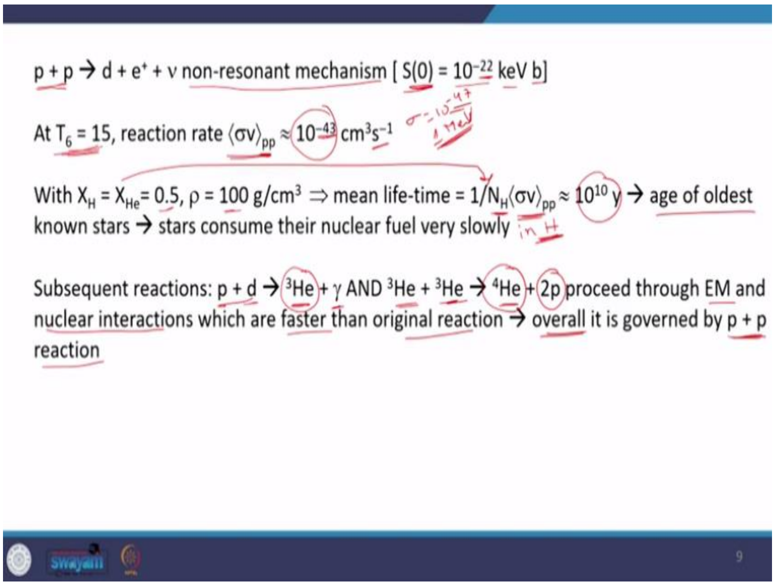

$p + p \rightarrow d + e^{+} + \nu$ non-resonant mechanism [ $S(0) = 10^{-22}$ keV b]

At $T_6 = 15$, reaction rate $\langle \sigma v \rangle_{pp} \approx 10^{-43}$ cm$^3$s$^{-1}$ $\sigma \sim 10^{-47}$ 1 MeV

With $X_H = X_{He} = 0.5$, $\rho = 100$ g/cm$^3$ $\Rightarrow$ mean life-time = $1/N_H\langle \sigma v \rangle_{pp} \approx 10^{10}$ y → age of oldest known stars → stars consume their nuclear fuel very slowly in H

Subsequent reactions: $p + d \rightarrow {}^3He + \gamma$ AND ${}^3He + {}^3He \rightarrow {}^4He + 2p$ proceed through EM and nuclear interactions which are faster than original reaction → overall it is governed by p + p reaction

And earlier, we have seen that this p + p both are charged particles, so, they follow non-resonant mechanism. And, if you see the extrapolated extracted diagram at zero energy, it is about $10^{-22}$ keV b. And, based on this, if you try to understand the reaction rate $\langle \sigma v \rangle_{pp} \approx 10^{-43}$ cm$^3$s$^{-1}$ at $T_6 = 15$, that is interior of the Sun.

Earlier, I have shown you σ is of the order of $10^{-47}$ if 1 MeV proton is involved. Now, see some more values regarding the lifetime. If the mass fraction of hydrogen and helium is given us $X_H = X_{He} = 0.5$ and say density is $\rho = 100$ g/cm$^3$, the mean lifetime can be calculated by using $1/N_H\langle \sigma v \rangle_{pp}$ where $N_H$ is the number density of proton which can be found out from the mass fraction. It appears like $10^{10}$ years. Mean lifetime of the p + p reaction is $10^{10}$ years. That means age of the oldest known stars. This is the reason stars consumed their nuclear fuel very slowly in terms of hydrogen as fuel. So, I hope you got the answer for one of the questions with which I have started this lecture.

Now, once deuterium is produced, next reaction is p + d gives rise to $^3$He and then gamma. And again, when this $^3$He is produced, it reacts with $^3$He produced in another reaction to give rise to $^4$He. So, this is called as pp I chain where overall 4 protons are involved. Please consider these 2 protons emitted in the reaction. And, then p + d and $^3$He + $^3$He, they do not proceed through weak interaction.

But, they proceed through electromagnetic and nuclear interactions, of course, which are larger and which are faster than the original reaction. So, because they are faster compared to the p +

p reaction, overall the reaction rate is governed by p + p reaction only. So, now, let me do the calculation of age of Sun based on these calculations.

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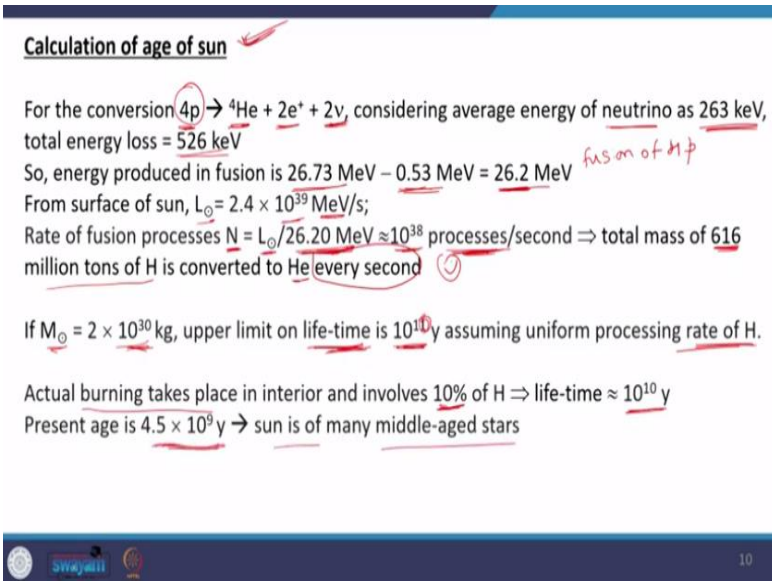

**Calculation of age of sun**

For the conversion $4p \rightarrow {}^4He + 2e^+ + 2\nu$, considering average energy of neutrino as 263 keV, total energy loss = 526 keV

So, energy produced in fusion is 26.73 MeV – 0.53 MeV = 26.2 MeV

fusion of 4p

From surface of sun, $L_\odot = 2.4 \times 10^{39}$ MeV/s;

Rate of fusion processes $N = L_\odot/26.20$ MeV $\approx 10^{38}$ processes/second ⇒ total mass of 616 million tons of H is converted to He every second

If $M_\odot = 2 \times 10^{30}$ kg, upper limit on life-time is $10^{11}$ y assuming uniform processing rate of H.

Actual burning takes place in interior and involves 10% of H ⇒ life-time ≈ $10^{10}$ y

Present age is $4.5 \times 10^9$ y → sun is of many middle-aged stars

Now, you see the conversion of 4 protons into $^4$He overall involves emission of 2 positrons and 2 neutrons, 2 neutrinos. Please recollect what I have said regarding the average energy of the neutrino. It is about 0.263 MeV. So, because 2 neutrinos are emitted in overall 4 protons conversion into $^4$He, the total energy loss because of the 2 neutrinos is 526 keV. So, the energy produced in fusion of 4 protons whose Q-value is 26.73 is basically 26.2 MeV which is the effective energy produced in the fusion of 4 protons. Now, we know that the surface luminosity of the Sun is about $10^{39}$ MeV/s. So, based on this, we can calculate the number of processes taking place per second, because we know the luminosity, that is energy emitted per unit time.

And, what is the energy emitted in one set of fusion of 4 protons? That comes out to be around $10^{38}$ fusion processes per second. In each process, 4 protons are involved. And, we know the mass of each proton. If you calculate, we can see that 616 million tonnes of protons is converted to helium every second.

How much? What is the mass of proton converted to helium 4 at the center of the Sun in every second? It is about 616 million tonnes. Now, if we consider the mass of the Sun, $10^{30}$ kg, what is the upper limit on the lifetime? Because, now, our aim is to find out the age of the sun, so, considering the mass of the Sun, what is the upper limit on lifetime?

So, this can give us the value that $10^{11}$ years assuming the uniform processing rate of hydrogen. In the previous slide, I have shown you that duration of the p + p reaction. However, the actual burning take place in the interior. And, it does not involve 100% of hydrogen, but only 10%. So, take out the 10% of from $10^{11}$. So, the lifetime is $10^{10}$ years.

And, the present age of the Sun is observed to be $4.5 \times 10^9$ y. So, we can say that Sun is of middle many middle aged stars. So, to summarize, in today's lecture, I have tried to provide some information on the different burning stages, durations and salient features of p + p reaction. What could be the possibility for the next reaction?

And, to produce $^4$He from hydrogen, what are the reactions involved? And, some numbers regarding the energy produced which can give us an idea about the age of the Sun. More points about the p + p reaction, how temperature decides the reaction rate and some more features in the next lecture. See you very soon, thank you very much.