Nuclear Astrophysics
Prof. Anil K. Gourishetty
Department of Physics
Indian Institute of Technology – Roorkee
Module – 05
Lecture – 21
Neutron Induced Non-resonant Reactions

Welcome to today's lecture, in which I am going to discuss neutron induced non-resonant reactions. In the last couple of lectures, I could not cover the topic. So, in today's lecture, I am going to cover this topic initially.

Then, I will discuss one new part that is in the syllabus. If you see, it is the 4th part out of 8 parts which covers the different burning stages within a star. So, what do we discuss in the last lecture? Let us quickly take a summary and then let us start today's lecture.

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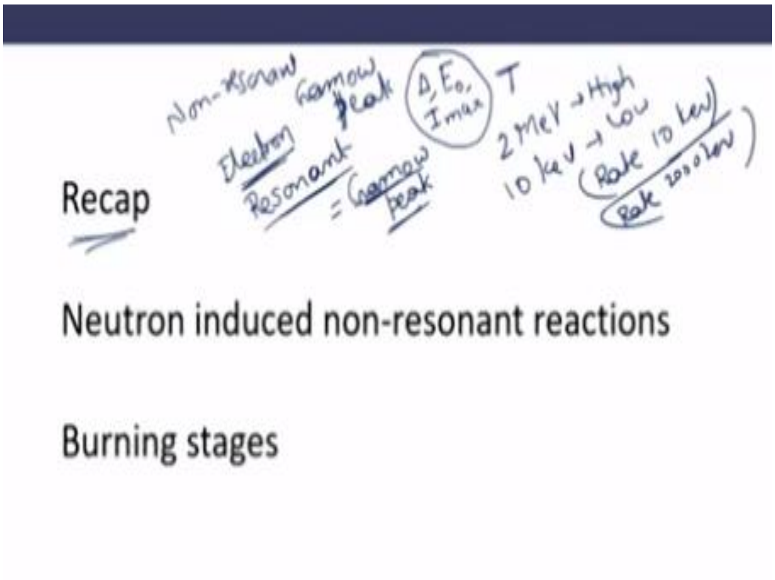

Recap

Non-Resonan Gamow peak

(Δ, E_0, I_{max}) T

Electron

Resonant = Gamow peak

2 MeV → High

10 keV → Low

$\left(\frac{\text{Rate } 10\text{ keV}}{\text{Rate } 2000\text{ keV}}\right)$

Neutron induced non-resonant reactions

Burning stages

So, in the recap, I would like to emphasize that in the non-resonant type reactions, we have seen, how Gamow peak plays important role to understand the centroid and width of the peak and the I_{max}, that is intensity of the Gamow peak. And, how these parameters depends on the Coulomb barrier and also the temperature. Then, I have discussed electron screening effect which matters when high stellar density is available.

And then, resonant type reactions where the resonance region coincides with the Gamow peak. Is not it? And, some of the important features of this charged particle induced resonant reactions and non-resonant reactions I have discussed. And, in resonant reactions in particular, you must understand the importance of low energy resonances. I have discussed one example in brief where the resonance set at 2 MeV which we are calling as relatively high energy resonance.

And, set 10 keV. That is low energy resonance. And we have seen how the reaction rate at a 10 keV and reaction rate at 2,000 keV. What is the ratio of this reaction rates? And, that has given a beautiful insight into the importance of low energy resonances. So, one has to understand how dramatically the stars evolution changes because of the change in the resonance energy. And, we also have seen the importance of Breit-Wigner formula.

So, after this, in today's lecture, let me shift from charged particle to neutron induced reactions. Overall, we can in general divide the type of nuclear reactions induced in terms of particle induced as charged particles like protons and then alpha particles or very less times you can say some kind of electrons rarely and then we can go for heavy ions which are really discussed.

But, mainly, it is proton and alpha induced reactions, so, which comes under charged particle category. And, they are responsible for the synthesis of elements below iron peak. That is what I have discussed when elemental abundance curve, I have discussed in one of the previous lectures. And, after iron peak, almost all the elements have been synthesized. And, the synthesis is still going on because of not charged particles.

But, almost all the nucleus particles behind the iron peak they are synthesized by neutrons. Neutrons are initiating the reactions. Now, when we say neutrons are initiating the reactions, are they basically of non-resonant type or resonant type? And, if it is non-resonant type, can we come across something called as Gamow peak? sounds bit crazy, because neutrons do not face the problem of Coulomb barrier penetration.

So, keeping it in mind, existence of Gamow peak no more arises in that case of neutron induced reactions. But, there are some interesting features of neutron induced non-resonant reactions. What are the switches? I will discuss in today's lecture. Topic number 2 of today's lecture, as I said, it is a 4th part out of 8 parts in this course, nuclear burning stages and processes in stars.

So, I will try to provide some kind of background of this nuclear burning stages within the stars. And, I will initiate the hydrostatic hydrogen burning which is nothing new. In one of the previous lectures, I have categorized the burning of different burning stages in the stars. That is hydrogen burning, helium burning and then oxygen burning, neon burning, silicon burning, and then comes into picture s-process, r-process and finally, l-process.

So, I will be starting the first one. I will be starting this different burning stages topic with hydrogen burning. So, to conclude the introduction of today’s lecture, I am going to discuss salient features of neutron induced non-resonant reactions and introductory part of nuclear burning stages within the stars. Let us start.

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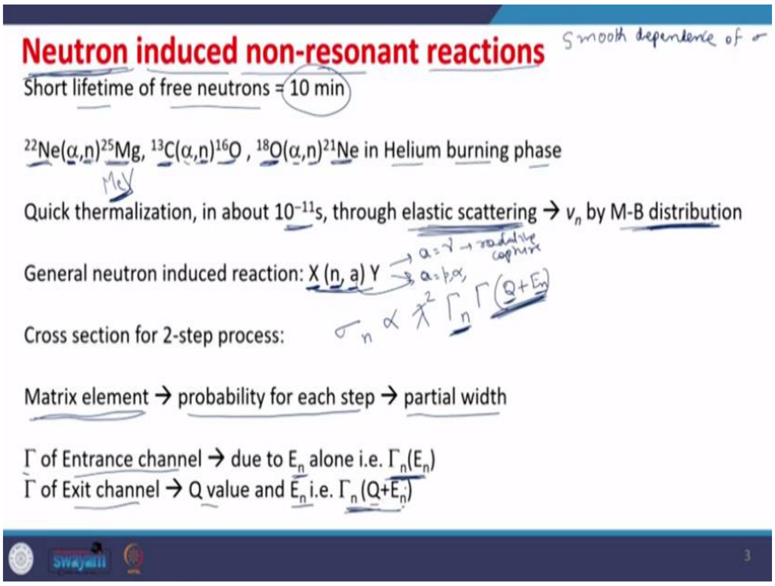

Neutron induced non-resonant reactions

Smooth dependence of σ

Short lifetime of free neutrons = 10 min

$^{22}Ne(\alpha,n)^{25}Mg$, $^{13}C(\alpha,n)^{16}O$, $^{18}O(\alpha,n)^{21}Ne$ in Helium burning phase

MeV

Quick thermalization, in about 10^{-11}s, through elastic scattering → v_n by M-B distribution

General neutron induced reaction: X (n, a) Y

$\rightarrow a = \gamma \rightarrow$ radiative capture

$\rightarrow a = p, \alpha,$

Cross section for 2-step process: $\sigma_n \propto \lambda\!\!\!^{-2}\, \Gamma_n\, \Gamma(Q+E_n)$

Matrix element → probability for each step → partial width

Γ of Entrance channel → due to E_n alone i.e. $\Gamma_n(E_n)$

Γ of Exit channel → Q value and E_n i.e. $\Gamma_n(Q+E_n)$

Non-resonant reactions, I have already discussed the smooth dependence of cross section on the projectiles energy. We categorize them as non-resonant reactions, means smooth dependence of cross section on the energy. So, when I say neutron induced non-resonant reactions, what kind of neutrons I am talking about, bound neutrons or free neutrons. Of course, free neutrons.

Remember, I have given a few examples where neutrons are emitted in the nuclear reactions. And, these neutrons will be reacting with some seed nuclei like iron to produce the elements beyond iron. So, when I say that free neutrons are reacting with the nuclei which are produced up to iron peak, you must remember that free neutrons are not stable unlike free protons which are expected to have around 1 billion years of life.

So, it is well known to you that the lifetime of free neutrons is about 10 minutes. It is quite short when compared to free protons lifetime. What are the reactions responsible for the production of neutrons? Mainly, alpha which is a charged particle when it reacts with ^{22}Ne it produces ^{25}Mg and a neutron, which can be written as ^{22}Ne(α,n)^{25}Mg. Mainly, ^{22}Ne(α,n)^{25}Mg, ^{13}C(α,n)^{16}O, ^{18}O(α,n)^{21}Ne these are the reactions, which take place in the helium burning phase, responsible for the production of neutrons, which are inducing the reactions which are responsible for the production of elements beyond iron.

Now, this helium burning phase I will discuss in detail in coming lectures. But, because I have to use the names of those reactions which are responsible for the production of neutrons, because our idea is to discuss neutron induced non-resonant reactions, how the reaction rate can be expressed. So, I had to choose this word helium burning phase though it was not covered until now.

But, quickly, let me tell you what exactly is this helium burning phase. So, you know initially when hydrogen burning starts once the hydrogen fuel is over, then, whatever remnants are there that is helium is there, they try to contract in the presence of gravitational attraction. This gravitational contraction takes place. It continues until it converts into thermal pressure.

And, it forces the helium nuclei to undergo nuclear fusion. And, this nuclear fusion between helium nuclei initiates nuclear reactions. And, the energy produced in these reactions prevents further gravitational contraction. This is how the stabilization of star happens. This I have been saying many times and of course, I will repeat this whenever the time demands during my course.

When hydrogen consumption is almost over at the core of the star, so, here, remaining part is helium and how hydrogen is converted into helium those reactions and steps I will discuss in detail. Wait for some time to know the steps involved in the conversion of hydrogen into helium. So, once hydrogen is finished, whatever helium is present at the core of the star they start burning.

And, outside this helium burning core, there will be a thick shell in which hydrogen burning continues whatever remaining hydrogen is there. So, at the core, when helium burning is taking place, these are the reactions which are producing the neutrons. And, these are the neutrons which are responsible for the production of elements beyond iron.

So, I hope, now, at this phase, you are able to experience or feel the role of nuclear physics in stars. So, let us see some more features of this neutron induced non-resonant reactions. Now, whatever neutrons are emitted, the energies of these neutrons are of the order of MeV. And, they undergo quick thermalization in the order of about pico seconds.

And, the phenomenon is elastic scattering. This is nothing new. Already you have been taught the concept of elastic and inelastic collisions in mechanics course. So, through elastic collisions, these high energy thermal neutrons will be reduced to thermal region. And, the velocity of these thermalized neutrons will be expressed by Maxwell Boltzmann distribution.

Now, you can always ask, those small deviation from the ongoing topic. This thermalization is happening because a lot of nuclei are present in the stars. What about same thermalization we have to do in the laboratory? When I carry out the nuclear reactions using alpha, neutrons are produced with high energies. Use moderators which quickly thermalizes this high energy neutrons.

Hope, when I am using the word moderator you might be recollecting the basics you have been taught regarding the working principle of nuclear reactor. Do you remember the nuclear reactor? What are the materials used to bring down the energies of the high energy neutrons? Water, heavy water, so, in the laboratory, one can keep the light materials like hydrogen I mean the material which has high content of hydrogen and some more light elements.

By colliding with these light elements like hydrogen mainly and deuterium in case of heavy water, you take some kind of candles and melt them, you will get some kind of wax and cool it and that will be one of the excellent moderators. So, the neutrons when passes through this kind of materials, they are quickly thermalized. And, use those thermalized neutrons to react with the targets of interest to study the synthesis of elements beyond iron.

Anyway, coming back to the topic, the neutrons produced in alpha induced reactions they will be quickly thermalized within like 10^{-11} seconds. And, the velocities of these thermalized neutrons they can be described by Maxwell Boltzmann distribution. Some more points, one can see the general representation of a neutron induced reaction as X (n, a) Y.

If a is equal to gamma ray, then we call it as radiative capture reaction. And, depending on the nature of the reaction, this particle a could be proton. Sometimes, it could be alpha. So, it could be some kind of heavy ion as well depending on the nature of the reaction. So, this is a general representation of a neutron induced reaction.

Now, to understand the cross section of this neutron induced reaction, we can consider it as a 2 step process. And, as I have explained in one of the previous lectures, there matrix element can be understood as probability for each step and which is further represented by partial width. For example, σ_n cross section of the neutron induced reaction is proportional to ƛ. This is the maximum geometrical cross section one can imagine.

Then, the cross section which is represented by the partial width for the neutron in the entrance channel this is the probability. And, in the exit channel, this is represented as mainly the exit channel is represented as Q plus E n. So, it is a 2 step process. In the first step, neutron is interacting with the target nucleus. So, the probability depends on the energy of the neutron only.

And, in the exit channel, the probability for a particular exit channel to happen depends on the Q-value of the nuclear reaction and the energy of the neutron. And, you know how to write down the probability. That is cross section for these 2 steps in terms of matrix elements. Already I have discussed enough. Now, the partial width of the entrance channel is basically controlled by the energy of the neutron alone.

So, that is the partial width that is depending on the energy of the neutron. And, for the exit channel, it depends on Q-value and also on E_n. That is why I have written the partial width dependence on the Q-value and the energy of the neutron.

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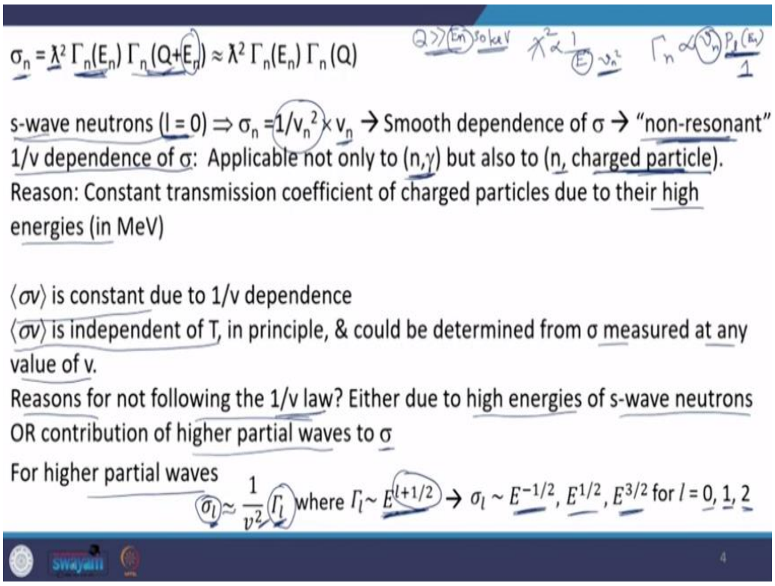

$\sigma_n = \lambdabar^2 \Gamma_n(E_n)\,\Gamma_n(Q+E_n) \approx \lambdabar^2 \Gamma_n(E_n)\,\Gamma_n(Q)$ $Q \gg E_n$ 30 keV $\lambdabar^2 \propto \frac{1}{E}$ v_n^2 $\Gamma_n \propto v_n \frac{P_l(E_n)}{1}$

s-wave neutrons ($l = 0$) $\Rightarrow \sigma_n = 1/v_n^2 \times v_n$ → Smooth dependence of σ → "non-resonant"
1/v dependence of σ: Applicable not only to (n,γ) but also to (n, charged particle).
Reason: Constant transmission coefficient of charged particles due to their high energies (in MeV)

$\langle \sigma v \rangle$ is constant due to 1/v dependence
$\langle \sigma v \rangle$ is independent of T, in principle, & could be determined from σ measured at any value of v.
Reasons for not following the 1/v law? Either due to high energies of s-wave neutrons OR contribution of higher partial waves to σ
For higher partial waves $\sigma_l \approx \frac{1}{v^2}\Gamma_l$ where $\Gamma_l \sim E^{(l+1/2)} \rightarrow \sigma_l \sim E^{-1/2}, E^{1/2}, E^{3/2}$ for $l = 0, 1, 2$

Now, let us see. The cross section of the neutron induced reaction is $\sigma_n = \lambdabar^2 \Gamma_n(E_n)\,\Gamma_n(Q+E_n)$. It is 2-step process. Now, in general, the value of E_n is much less than Q. That means Q-value is much higher than the energy of the neutron. Why? Because, already, it is thermalized and the normal energy that is considered is about 30 keV whereas Q is in MeV, so, we can say the cross section for the exit channel which is expressed as partial width of the exit channel depends on Q value only. $\sigma_n = \lambdabar^2 \Gamma_n(E_n)\,\Gamma_n(Q+E_n) \approx \lambdabar^2 \Gamma_n(E_n)\,\Gamma_n(Q)$.

But, remember Q-value is a constant whereas energy of the neutron maybe slightly varying. But, Q-value for a given combination of projector and target it is a constant. So, keep this in mind. The partial width of the exit channel is a constant. It depends on the constant value whereas the entrance channel partial width is depending on the energy of the neutron which is not exactly constant slightly varying. Keep this in mind.

And, try to see how cross section can be understood in a better way. And, also, reaction rate can be expressed. See, and in general, if I consider the s-wave neutrons, the orbital angular momentum is considered, l is equal to 0. And, in general, at the low energies s-wave neutrons are dominated. For l = 0, we can write down $\lambdabar^2$ as $1/v_n^2$. Is not it?

We know $1/\lambdabar^2$ is proportional to 1/E. And, E is proportional to v_n^2. And, this partial width depending on the energy of the neutron for s-wave is expressed as that means, partial width of entrance channel is normally is proportional to velocity of the neutron. And, some kind of you know Legendre polynomial which I will not discuss here.

So, we can ignore this. For s-wave neutrons, it is 1 basically. And, this v_n is coming here. And, one v_n is getting cancelled. $\sigma_n = 1/v_n^2 \times v_n$. So, the cross section is proportional to $1/v_n$. That means σ_n is inversely proportional to the velocity of the neutrons. And, the smooth dependence is termed as non-resonant. So, this is not only applicable to only radiative captures like (n,γ), but also for n as a projectile and charged particle as an ejectile. Why?

See, the interesting part here is the charged particle which are normally of high energies, the transmission coefficient is constant because of their high energies. So, because transmission coefficient is constant, one can assume that this 1/v dependence can be applied to not only gamma as an ejectile when neutrons are projectiles, but charged particles as ejectiles. So, this leads to the constant value of $<\sigma v>$. That is reaction rate because of the 1/v dependence.

Remember, this clearly says that it is independent of temperature in principle. So, it could be determined from σ measured at any value of the velocity. Are there any reasons for not following the 1/v law? Yes, there are reasons. There is no need to say all the time that 1/v dependency is there. Though it is a general presentation, the reason could be s-wave neutrons of not low energies, if high energies are involved or if higher partial waves are contributed to the cross section.

Under these 2 circumstances, 1/v law is not applicable. And, if I consider higher partial waves, then that means $l \neq 0$, but $l = 1, 2, 3$ like that. Then, σ_l is equivalent to $\frac{1}{v^2}$, because $\lambda\!\!\!^{-2}$ is $1/v_n^2$. Then, it is partial width corresponding to a particular value of l. And, the partial width corresponding to a particular value of l is expressed in terms of energy as, $\Gamma_l \sim E^{l+1/2} l$.

So, this is an important relation. We need to understand the dependence of partial width on the energy. And, if you know the energy dependence, you can always go for the velocity dependence. So, this is the information which I would like to share with you how higher partial waves can be handled while discussing the neutron induced non-resonant reactions. For example, if I go for different values of l.

For $l = 0, 1, 2$ respectively $\sigma_l \sim E^{-1/2}, E^{1/2}, E^{3/2}$. Like this, l dependence can be included in the cross section. And, this is the case when 1/v dependence is not followed. I hope you have understood the features of this neutron induced non-resonant reactions. Some more features are there with that I will complete this topic.

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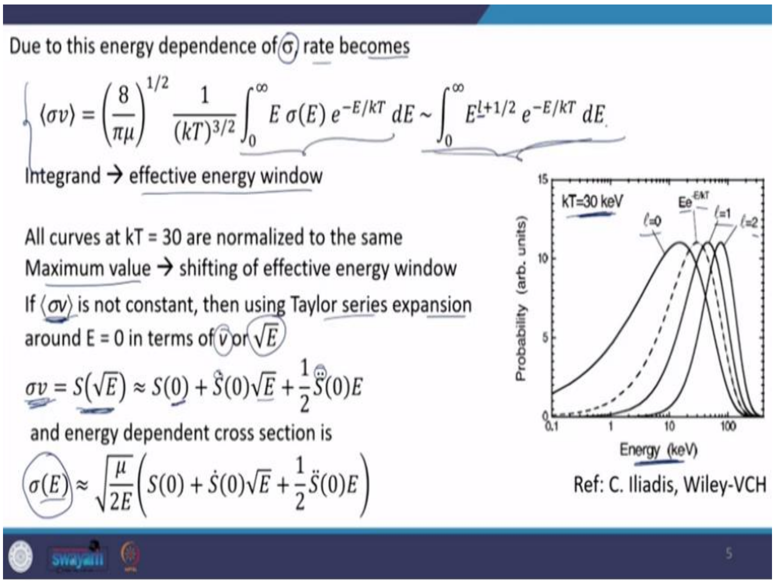

Due to this energy dependence of σ rate becomes

$$\langle\sigma v\rangle = \left(\frac{8}{\pi\mu}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE \sim \int_0^\infty E^{l+1/2}\,e^{-E/kT}\,dE$$

Integrand → effective energy window

All curves at kT = 30 are normalized to the same Maximum value → shifting of effective energy window

If $\langle\sigma v\rangle$ is not constant, then using Taylor series expansion around E = 0 in terms of v or $\sqrt{E}$

$$\sigma v = S(\sqrt{E}) \approx S(0) + \dot{S}(0)\sqrt{E} + \frac{1}{2}\ddot{S}(0)E$$

and energy dependent cross section is

$$\sigma(E) \approx \sqrt{\frac{\mu}{2E}}\left(S(0) + \dot{S}(0)\sqrt{E} + \frac{1}{2}\ddot{S}(0)E\right)$$

Ref: C. Iliadis, Wiley-VCH

Now, because of the energy dependence of this σ, the reaction rate becomes $\langle\sigma v\rangle = \left(\frac{8}{\pi\mu}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE \sim \int_0^\infty E^{l+1/2}\,e^{-E/kT}\,dE$ when l is coming into picture like this. And, the integrand is nothing but effective energy window. And here, I have plotted the probability versus energy for different values of l. And, you can see the centroid of this probability curve is shifting towards the high energy (refer to the above slide).

And, this is the most convincing value of the kT, 30 keV. Thermalized neutron energy range one can go for 30 keV kind of thing. And here, all curves are normalized at 30 keV only. And, as I said, the maximum value shifting towards the effective energy window. And, if at all the reaction it is not constant, then one can use the Taylor series expansion around E = 0 in terms of v or $\sqrt{E}$.

Now, how to do this Taylor series expansion? You know how $\sqrt{E}$ can be used in different alternate ways and the single differentiation and double differentiation is used here. And, from this, one can find out the energy dependent cross section as like $\sigma v = S(\sqrt{E}) \approx S(0) + \dot{S}(0)\sqrt{E} + \frac{1}{2}\ddot{S}(0)E$. There is no need to remember this formula seriously.

But, I would like to emphasize here how the reaction rate and the cross section can be expressed if the reaction rate is not constant. So, this I have basically equated to this term. $\sigma(E) \approx \sqrt{\frac{\mu}{2E}}\left(S(0) + \dot{S}(0)\sqrt{E} + \frac{1}{2}\ddot{S}(0)E\right)$.

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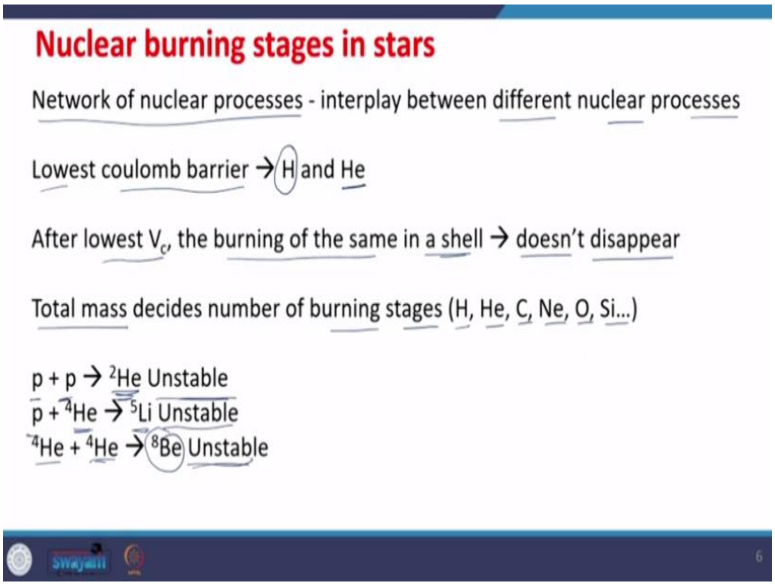

Nuclear burning stages in stars

Network of nuclear processes - interplay between different nuclear processes

Lowest coulomb barrier → H and He

After lowest V_c, the burning of the same in a shell → doesn't disappear

Total mass decides number of burning stages (H, He, C, Ne, O, Si...)

$p + p \rightarrow {}^{2}He$ Unstable
$p + {}^{4}He \rightarrow {}^{5}Li$ Unstable
${}^{4}He + {}^{4}He \rightarrow {}^{8}Be$ Unstable

So, with this, I have completed the items in 3rd topic of the course. That is reactions induced by charged particles both non-resonant and resonant type and reactions induced by neutrons. Coming to the 4th part of the course which has 8 parts, remember until now, we have seen the nuclear reaction in an isolated manner. Two nuclides are involved in the nuclear reaction and two different types of nuclides are produced, sometimes particles are produced.

Sometimes the gamma is produced. Though I have not discussed a photon induced nuclear reactions, so, this I am skipping. It is not very much important. Though it plays some important role in the energy production or nucleosynthesis, the major part goes to charged particle induced and neutron induced reactions. And mainly, neutron induced reactions plays role for the synthesis of elements not the energy production in majority cases.

Now, it is important to understand the networking of the nuclear reactions not in an isolated manner, because, if you want to understand the evolution of the star, it is important to know the reaction network. So, when it comes to the reaction network, you should understand, what are the phases of this nuclear reactions? Based on which we can categorize that we can categorize the evolution of a star.

So, let me try to provide some background in this lecture. More details, I will discuss in the next lecture. So, in the nuclear burning stages in stars, as I said, network of nuclear processes, we can say that the interplay of different nuclear processes, not one nuclear process. As I said many times, whenever you have some fuel, the nuclides having lowest Coulomb barrier will start burning initially because it is more probable.

Anyway, it is a quantum tunneling process. So, naturally the nuclides which are having lowest Coulomb barrier will try to participate in the nuclear burning stage compared to 2nd or 3rd lowest Coulomb barrier. So, considering that it is very clear that hydrogen burning is the 1st stage. Once hydrogen is over, then helium comes into picture. So, once the lowest Coulomb barrier corresponding nuclides have been consumed, as I said earlier, the burning of the same happens in a shell outside the core.

So, if hydrogen starts consuming, once hydrogen is completed, it does not mean that hydrogen is completely finished. Whatever amount is left, the burning in that small amount of hydrogen will happen not at the core, but in the shell around the core in which second lowest Coulomb barrier corresponding nuclides will start burning. So, the nuclides with the lowest Coulomb barrier will not disappear completely.

And, it is easily understood that it is the total mass which decides the burning stages whether hydrogen burning, helium burning, carbon burning, neon burning, oxygen burning and silicon burning has to take place or not. It depends on the mass of the stars. So, it is not very difficult to imagine the first nuclear reaction, that is p + p which gives rise to ^{2}He which is highly unstable.

So, I am introducing the steps involved in the hydrogen burning which is the 1st burning stage in a star. So, when protons are present, what are the probable processes my dear? Of course, you can imagine a few helium also involved in that. So, if $p + p \rightarrow {}^2\text{He}$ happens, immediate reaction which comes to our mind, but, it is highly unstable.

Another process which is highly probable is $p + {}^4\text{He} \rightarrow {}^5\text{Li}$. But, it is also highly unstable. Then, ${}^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be}$ which is also highly unstable.

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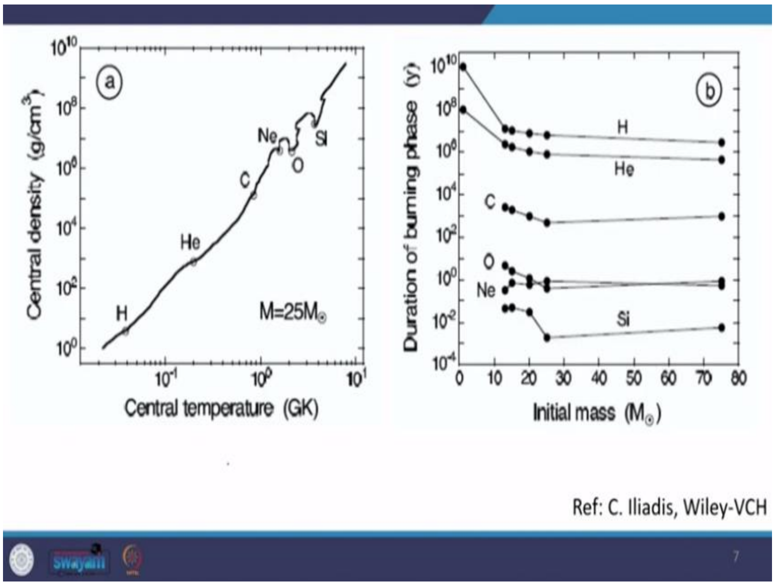

Ref: C. Iliadis, Wiley-VCH

So, when these expected nuclear reactions are producing highly unstable nuclei, what kind of other reactions might be taking place which are trying to stabilize the star? So, that gives us a beautiful insight into the hydrogen burning stages. That I will discuss in the next lecture. Thank you so much for your attention, bye.