Nuclear Astrophysics
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Module –04
Lecture – 18
Gamow Peak and Electron Screening Effect

Welcome students, we are in process of understanding the relation between various parameters as part of discussion on non-resonant reactions, that means the smooth dependence of cross section on energy, that too reactions are induced by charged particles, so please ignore the involvement of neutrons and photons which means gamma rays as projectiles. So, I am confining the discussion only on the charged particle induced reactions. What did we learn in the previous lecture? So, let me start with the slide with which I have ended the previous lecture.

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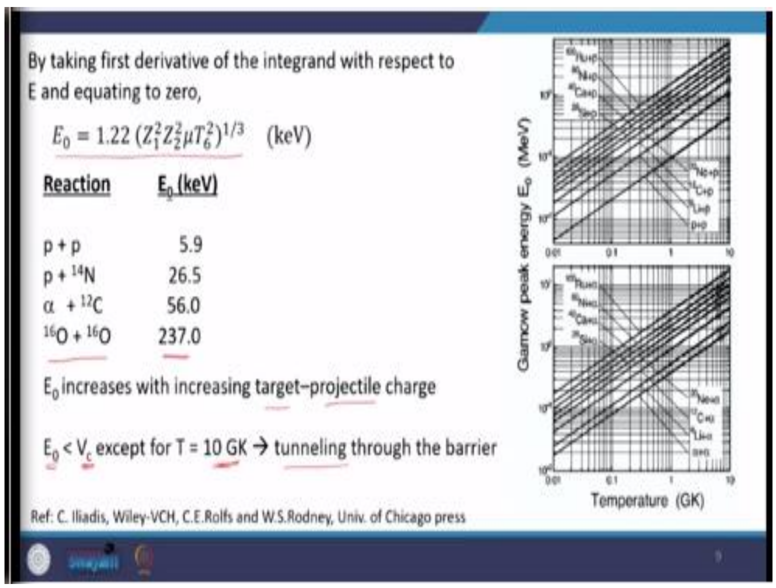

So, here we have seen the dependence of centroid of the Gamow peak on the charge of target and projectile. And except at very high temperature like 10 GK energy is always less than the Coulomb barrier and the reaction is not classically happening but it is happening quantum mechanically via tunneling process. $E_0 = 1.22\,(Z_1^2 Z_2^2 \mu T_6^2)^{1/3}$ is the formula to calculate the most effective energy at which nuclear reactions are taking place in the stars. So, now we need to answer some more questions.

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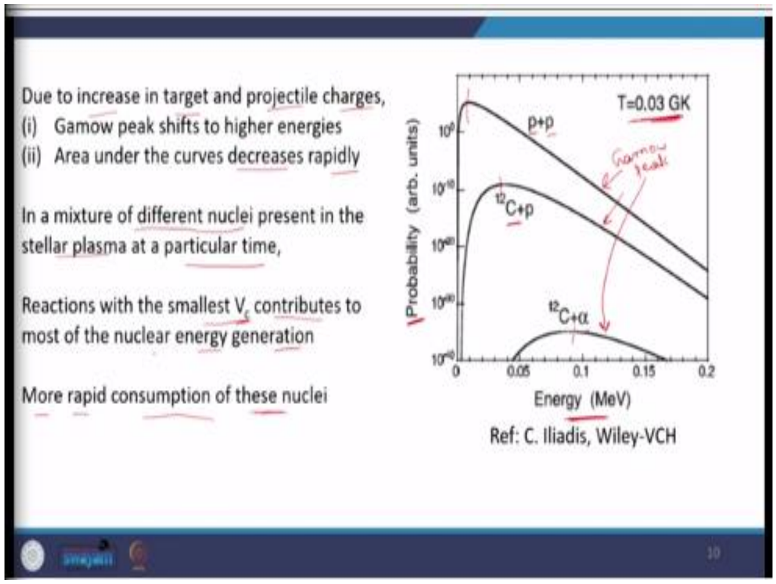

For that let me show you another diagram (refer to the above slide) at a specific temperature less than the Coulomb barrier corresponding temperature0.03 GK, that means 30 MK, how the probability is changing with respect to the energy? You can see all these are Gamow peaks in logarithmic scales. This shows that because of the increase in target and projectile charges, you see here projectile charge is increasing from proton to alpha and target charge is also increasing from a proton to ^{12}C.

So, when there is increase in the charges of target and projectile the Gamow peak is shifting towards higher energies, you see the Gamow peak for p + p is 5.9 keV for p + ^{12}C, it is about less than 50 keV and for ^{12}C + α it is around0.1 MeV. So, the Gamow peak centroid is shifting to higher energies because of the increase in the target and projectile charges. Number 2, the area under the curves decreases rapidly. So, do you remember one of the previous questions which I have listed in the previous lecture?

How the area of the Gamow peak changes with the charges of the target and projectile which of course decides further the Coulomb barrier. So, please do not get confused to relate all these parameters. So, in this slide I am showing you the relation between the Gamow peak and the charges of target and projectile. Also the impact on the area of the Gamow peak which is decreasing rapidly as the Coulomb barrier is increasing because of the increase in charges of turret and projectile.

So, the area is decreasing, means what? The probability for the reactions to happen is also coming down, the product of probability and the energy. Now for example you consider a scenario where different nuclei are present in a mixture in the stellar plasma at a particular time, now how we can understand the burning process? What kind of nuclear reactions will take place? Whatever reactions with the smallest Coulomb barrier are there they contribute to the most of the nuclear energy production, why it is so because for nuclear reactions with least Coulomb barrier centroid of the Gamow peak is also less.

So, as a first step those nuclei having less charges and the combination of nuclei participating in the nuclear reaction with the smallest Coulomb barrier are responsible for the major production in the energy production of stars. This is a very significant statement you need to understand from this diagram. Not only most of the nuclear energy production but also rapid consumption of the nuclei also takes place at this stage where nuclear reactions with the smallest Coulomb barrier are happening within the star.

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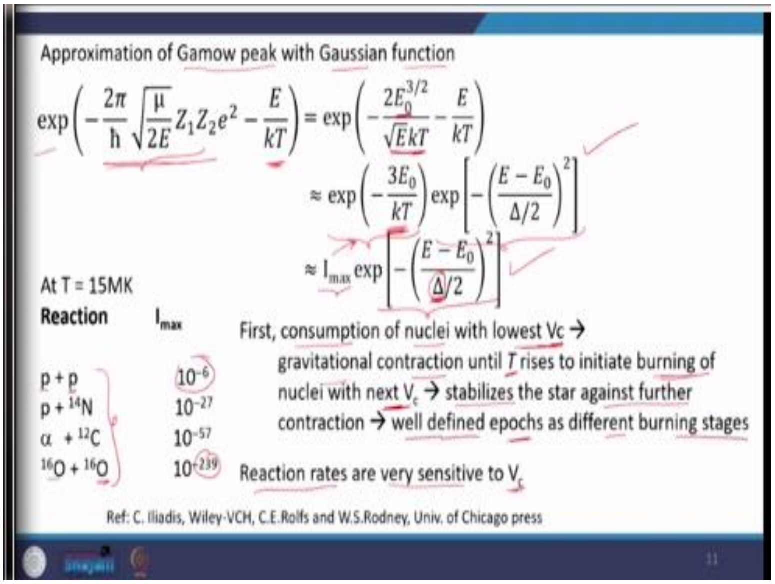

Now as I have told you earlier this Gamow peak is almost close to the Gaussian function. Now let me approximate the Gamow peak with a Gaussian function, then how it looks like mathematically? So, $e^{-2\pi\eta-E/kT}$, this $-\frac{2\pi}{\hbar}\sqrt{\frac{\mu}{2E}}Z_1Z_2e^2$ is nothing but $-2\pi\eta$ and $-E/kT$, these 2 terms were always present in the integrand of the reaction rate formula which I have represented which are presented in the previous lecture.

Here the moment I represent the Gamow peak with Gaussian function I can always write down around E_0 I am approximating this. So, in terms of kT and E_0 I can approximate this expression as $\approx \exp\left(-\frac{2E_0^{3/2}}{\sqrt{E}kT} - \frac{E}{kT}\right)$. So, there is a slight difference in the curvature but we are assuming at the same centroid but at the same curvature we are approximating the Gamow peak with the Gaussian function. What is the advantage of this?

It will be clear to you very soon. I am separating the terms as $\approx \exp\left(-\frac{3E_0}{kT}\right) \exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)^2\right]$. I am separating 2 terms the expression of integrand in the reaction rate equation. Now what is this I_{max} my dear? In the Gamow peak what is the value corresponding to the centroid of the Gamow peak, the corresponding value on the y axis is the maximum intensity of the peak.

That maximum intensity of the peak is represented mathematically by $\exp\left(-\frac{3E_0}{kT}\right)$ and remaining term you have here. Here in the next slide you can see the expression for the width; $\Delta = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.749(Z_1^2 Z_2^2 \mu T_6^5)^{1/6}$ (keV), this is the width of the Gaussian peak basically. Now how I can tabulate the I_{max} values for different types of nuclear reactions starting from low charges of target and projectile and high charges of target and projectile?

You can see the I_{max} is decreasing from 10^{-6} to 10^{-239}, so the maximum intensity of the Gamow peak is very much sensitive to E_0, also which further depends on the Coulomb barrier. So, this data again reflects the fact that Coulomb barrier plays a very important role. So, here we can give some general statements regarding the burning stages within the stars, how? As I said earlier the consumption of those nucleus with the lowest Coulomb barrier takes place first.

Once the consumption is over, once all the nuclei with lowest Coulomb error consumed then what will happen? The gravitational contraction take place between the remaining nuclei until the temperature rises to initiate the burning of those nuclei with second least Coulomb barrier. Hope you are following what I am trying to convey.

We have a set of nuclei, in that those nuclei with the smallest Coulomb barrier will consume first then gravitational contraction takes place until the temperature rises which is sufficient to ignite the nuclei with the second least Coulomb barrier. Those nuclei will start undergoing reaction with each other which are responsible for the energy production at that stage and also nucleosynthesis.

This energy liberated in the nuclear reactions having second least Coulomb barrier stabilizes the star against further contraction. This gravitational contraction is taking place continuously but the moment temperature is sufficient to ignite second set of burning that energy liberated in those nuclear reactions will stabilize the star and stops the star for contacting further because of the gravitational interaction.

So, the relation between the stabilization and the gravitational contraction because of the nuclear reaction has to be very clear to you. And these well-defined epochs as different burning stages, do you recollect what are these different burning stages? Hydrogen burning, helium burning, carbon burning, then silicon burning, s-process, r-process, all these burning stages the underlying mechanism is the same thing, the Coulomb barrier is playing extremely important role here. So, the reaction rates are extremely sensitive to the Coulomb barrier.

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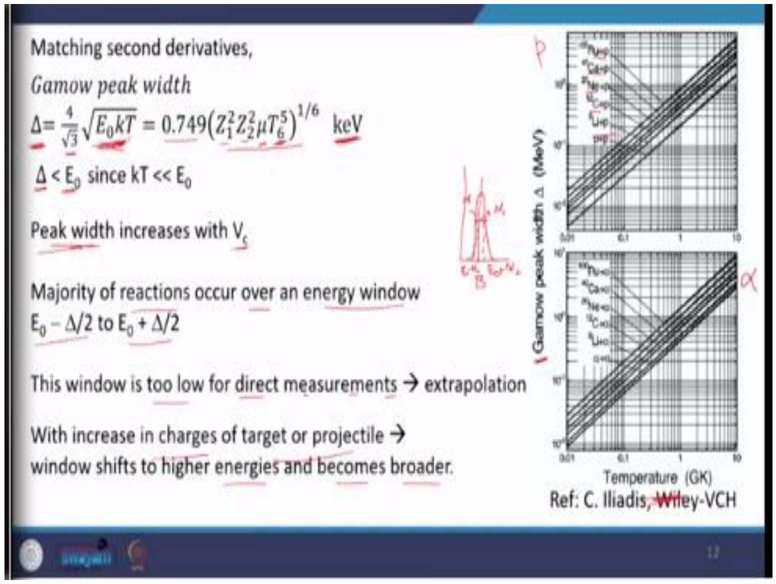

Now, the second question, we have found out the centroid of the peak by taking the first derivative. Now in this previous slide you take the second derivative of this and you take the second derivative of this and equate each other then you can get an expression for the peak width delta. So, our aim is to find out relation for Gamow peak width, it is $\Delta = \frac{4}{\sqrt{3}}\sqrt{E_0 kT}$.

Numerically you can find out the value by knowing the reduced mass, z_1, z_2 and temperature in MK and finally the peak width is $0.749(Z_1^2 Z_2^2 \mu T_6^5)^{1/6}$ in keV. So, now we are in a position to calculate the centroid of the Gamow peak and the width of the Gamow peak, what is the relation between these two? Can any time the peak width be less than the E_0 or greater than E_0 that we have to understand?

Remember the peak width is always less than the centroid because this peak width is having terms E_0 and kT. And already I have explained you kT is always less than E_0 in charged particle induced non-resonant reactions. So, because kT is much less than E_0 of course Δ, that is peak width will be less than E_0 from this expression of $\frac{4}{\sqrt{3}}\sqrt{E_0 kT}$. So, peak width, however it increases with coulomb barrier, earlier I have shown you the diagram where with increase in Coulomb barrier the width of the peak is increasing though area is decreasing.

Now the majority of the reactions occur over an energy window $E_0 - \Delta/2$ to $E_0 + \Delta/2$. So, between the $E_0 - \Delta/2$ and $E_0 + \Delta/2$ in this particular range majority of the nuclear reaction is taking place. So, you might have realized the importance of this Gamow peak and also peak width.

Now this window is too low for direct measurements, many times the energy range of this peak width is too low for direct measurements to carry out. That is the reason researchers carry out nuclear reactions at available energies then do the extrapolation. And when you do the extrapolation, it is always preferable to extrapolate the S-factor and then you find out the cross section.

Because the probability of getting uncertainty is less when you extrapolate S-factor because of its less energy dependence on energy compared to cross section's dependence on energy. So, let me show you some data regarding the variation of Gamow peak with temperature. This is induced by protons and this set of reactions is induced by alpha, you see the charge is increased. And you can see that with a decrease in temperature Gamow peak width is also decreasing and also this is sensitive to the charges of target and projectile.

So, with increase in charges of target and projectile say from p + p, Li, C, Ne, Ca and Ru you see the window is shifting to higher energies and becomes broader. The window is becoming broader and it is shifting towards the higher energies. So, the shifting towards left or right depending on various parameters this relation is extremely important to understand and to get convinced yourself, I hope these diagrams are helping you.

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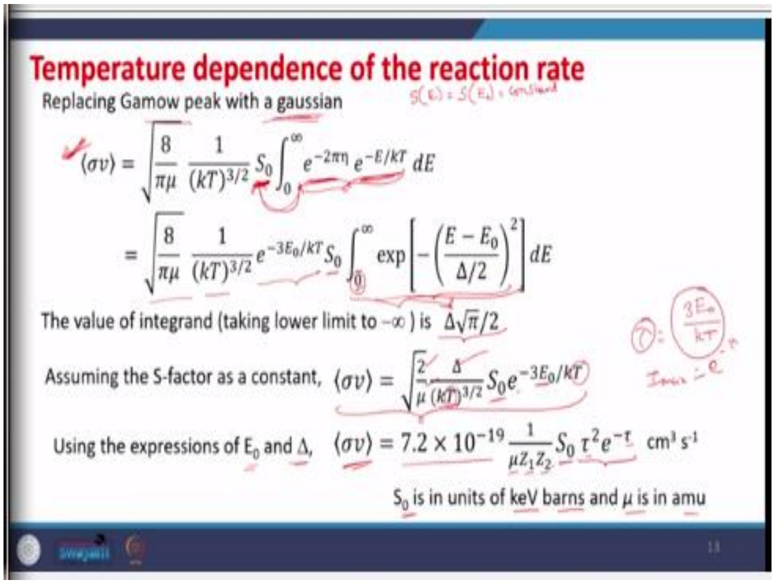

Now the most important relation that we are looking for. If you remember I have started the discussion on non-resonant reactions induced charged particles. By showing the reaction rate equation and highlighting the fact that temperature of the star is changing very frequently. And each and every temperature gives rise to a different value to the reaction rate. And when you are dealing with large number of nuclear reactions within the stars, it is not possible I mean it is very time taking process to calculate the reaction at all these temperatures.

And that is the reason that motivated us to come up with an analytical expression for reaction rate on temperature. After discussing the concepts of centroid of the Gamow peak, width of the Gamow peak, maximum intensity of the Gamow peak, now the time has come to come up with an analytical expression for temperature dependent nuclear reaction rate. How do we do it? Let us see.

Please replace the Gamow peak with a Gaussian for better approximation then this is the well-known formula for the reaction rate, very standard formula where I have taken $S(E_0)$ as a constant, S€ = $S(E_0)$ as constant, this is another assumption. So, by now how many assumptions we have done? One, the Gamow peak has been assumed to be exact Gaussian function, number two, S-factor is not at all changing with the energy.

For simplicity, for the sake of convenience we have started with these two assumptions, do not forget practically corrections corresponding to these two assumptions have to be done, whether there will be a change or not that is a different thing. But at this stage it is sufficient to, I mean it makes sense to take these two assumptions. First assumption Gamow peak assuming as a perfect Gaussian function, second constant value of the S.

So, that we can take the S-factor outside the integrand. So, you see earlier S was inside, is not it? Now I have taken it outside because S is a constant, now the only thing remaining is $e^{-2\pi\eta}$ and $e^{-E/kT}$ and the product of these two terms gave rise to Gamow peak as I discussed earlier. Now this formula we are taking help from to come up with analytical expression of rate and temperature relation.

So, S_0 is as it is, these two are as they are; now the product has been represented as I_{max} taken outside. And the previous slide has shown this expression, which has this peak width formula, so this is inside the integrand. Now, without introducing any new value I can always replace this lower limit to minus infinity, you can always check yourself. So, the value of integrand will not change if I replace 0 to -∞ but it makes my job easier to evaluate the integral.

So, it will be $\Delta\sqrt{\pi}/2$, this is the value of the integrand. So, you use this value and come up with the final expression for reaction rate and assume the S-factor is a constant. So slowly our job is becoming easy to come up with the relation for temperature dependent nuclear reaction rate.

$$\langle\sigma v\rangle = \sqrt{\frac{2}{\mu}}\frac{\Delta}{(kT)^{3/2}}S_0 e^{-3E_0/kT}$$

Now let us use the expressions for E_0 and Δ which we are aware of. And come up with a numerical expression for reaction rate, $\langle\sigma v\rangle = 7.2 \times 10^{-19}\frac{1}{\mu Z_1 Z_2}S_0\,\tau^2 e^{-\tau}$ in $cm^3\ s^{-1}$, τ is something which you are aware of that is $3E_0/kT$. Now this S_0 is in the units of keV b and μ is in amu as usual.

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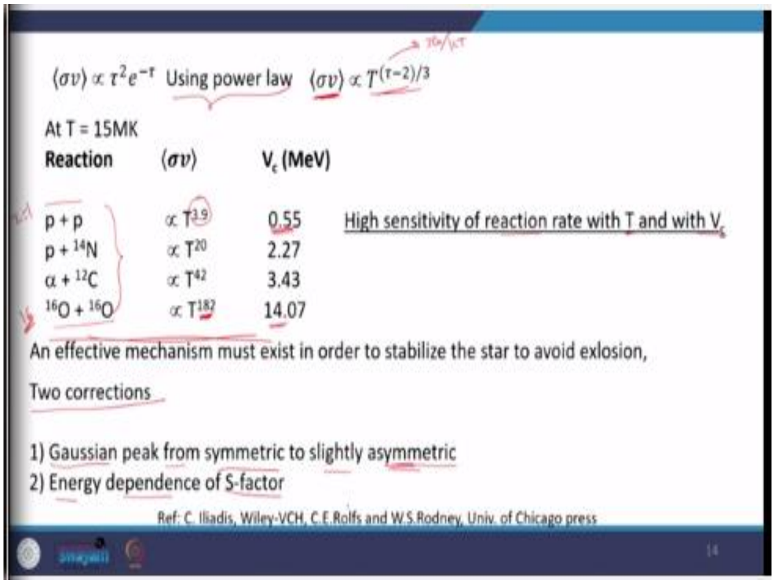

So, from this you can say that reaction rate is proportional to $\tau^2 e^{-\tau}$. And using some mathematics which I am not discussing here, I strongly suggest you to do yourself. Using some power law or mathematical concepts, you can derive the relation between reaction rate and the temperature. Now let me give you some important numbers for the same set of reactions starting from lowest Coulomb barrier to highest Coulomb barrier of course for starting from z = 1 to 16.

I think this is sufficient to get an idea, the reaction rate you see $T^{3.9}$ and varying up to T^{182}, please see the values of Coulomb barrier almost 1 keV to 14 keV only. But you see the temperature dependence, you can see the dramatic change in the reaction rate with small change in the temperature which depends on the Coulomb barrier finally.

So, how Coulomb barrier, temperature and reactions are reaction rates are related to each other? This should help you to have a clear idea. So, this data reflects the high sensitivity of reaction rate with temperature and also with Coulomb barrier. Now when it is clear that the reaction rate is highly sensitive to the temperature and of course on Coulomb barrier, there must be a mechanism for the stabilization of a star.

Now we are into the actual part of nuclear astrophysics, for star to be stable for millions and billions of years there has to be a mechanism considering the fact that temperature is extremely sensitive.

It means temperature is playing an extremely crucial role in deciding the reaction rate which is further deciding the energy produced from the stars and also the synthesis of elements.

If there is no specific mechanism to stabilize the star, there is no way for the star to explode; I mean star has to explode there is no other way, fine. So, this tells us various beautiful mechanisms happening within the star which are trying to stabilize for a long time. So, the temperature dependent reaction rate mathematical expression must be clear to you. Now after this as I said ideally you need to include 2 corrections, what are those? Gaussian peak from symmetric to slightly asymmetric and energy dependence of the S-factor.

See the analytical expression which I have derived clearly assumed the S value as a constant but there is always some change in the S-factor with respect to energy that has to be taken into account though that change is less when compared to the change in the cross section. And also the asymmetric nature of the integrand of reaction rate which is little bit away from the actual Gaussian function also to be considered.

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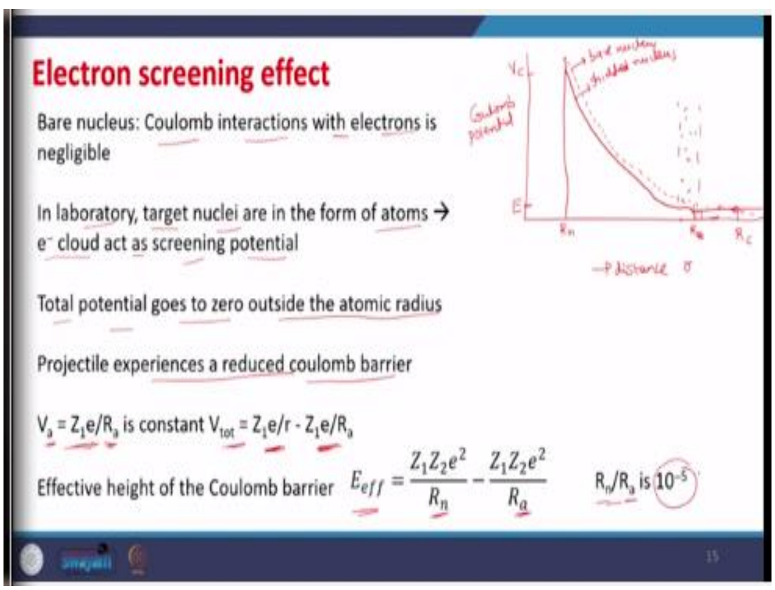

Next I am going to discuss one very interesting feature of the nuclear astrophysics. Within the stars it is the nucleus which is undergoing reaction with another nucleus, electrons are not present surrounding the nucleus. But whereas in the laboratory when projectile is reacting with the target,

target is full of atoms, so you have electron cloud surrounding the nucleus. Till now whatever you have done we have assumed it is a bare nucleus.

So, for better understanding let me draw one interesting diagram to understand the effect of electron screening. So, it is basically distance r on x axis and Coulomb potential on y axis, so this is the energy of the projectile and says this is the Coulomb barrier and this is one type of potential and this dotted one is the bare nucleus till now we have been considering. You see it never reaches to 0 but it comes close to 0.

Now the incident projectile is coming with some energy say E and the point at which it is touching the bare nucleus potential is say classical turning point R_C and this is the electron cloud in the atom. This is atomic radius you can say and this is a nuclear radius, so this is a bare nucleus and this is the shielded nucleus. So, we have considered bare nucleus that means Coulomb interactions with electrons is negligible, but in laboratory when you do the experiment target nuclei there in the form of atoms which have electrons surrounding the nucleus.

So, the cloud of electrons act as a screening potential, the net effect of this electron cloud presence is to reduce the Coulomb barrier little bit. So, the moment Coulomb barrier comes down cross section will increase, so this enhancement in the cross section because of the electron screening effect is it significant or not? That we will discuss. So, you see the total potential goes to 0 outside the atomic radius, from this diagram it is clear.

And because of this the projectile sees a reduction in the Coulomb barrier. Now the electrostatic potential less than the atomic radius is almost constant, from this formula it is clear. So, the total potential can be calculated as $V_{tot} = Z_1e/r - Z_1e/R_a$, so this gives rise to effective height of the Coulomb barrier which contains R_n and R_a. So, the effect is basically equivalent to the ratio of R_n and R_a which is of the order of 10^{-5}.

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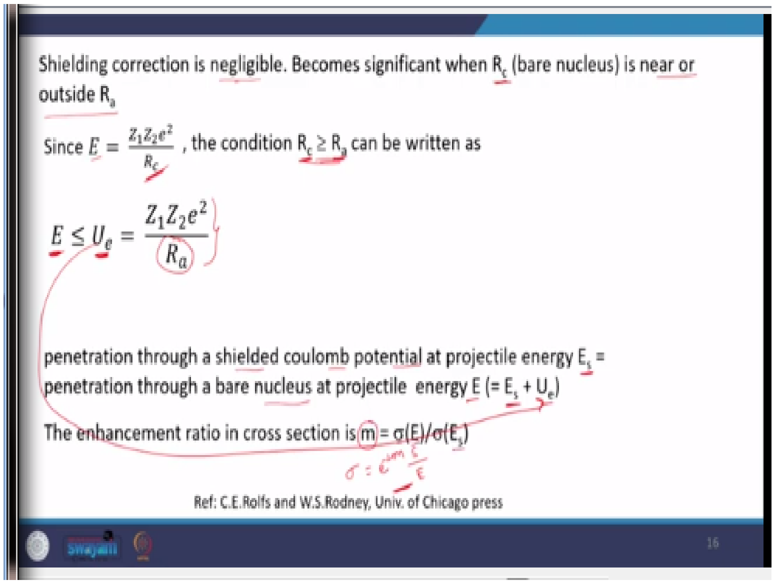

Now it is clear that shielding correction is quite negligible but it becomes significant when classical turning point is in near or outside atomic radius. If R_C is within the atomic radius it does not matter but if it is outside the atomic radius sometimes it becomes significant, let us try to understand this. Because we know that this expression for E is in terms of classical turning point is this expression the condition that R_C for bare nucleus classical turning point is greater than or equal to atomic radius can be written as like this.

Energy is less than some potential given by this expression, where we have R_a using this condition. Now this penetration through a shielded Coulomb potential having a projectile energy E_s, assume it is equivalent to penetration through a bare nucleus at energy which is a sum of E_s and also this U_e, which I have defined here as, $U_e = \frac{Z_1Z_2e^2}{R_a}$. So, please listen to this and understand carefully. Because of this using the well-known formula of $e^{-2\pi\eta}$ and S-factor divided by energy that well known formula. The enhancement in the cross section can be taken as σ(E)/σ(E_s).

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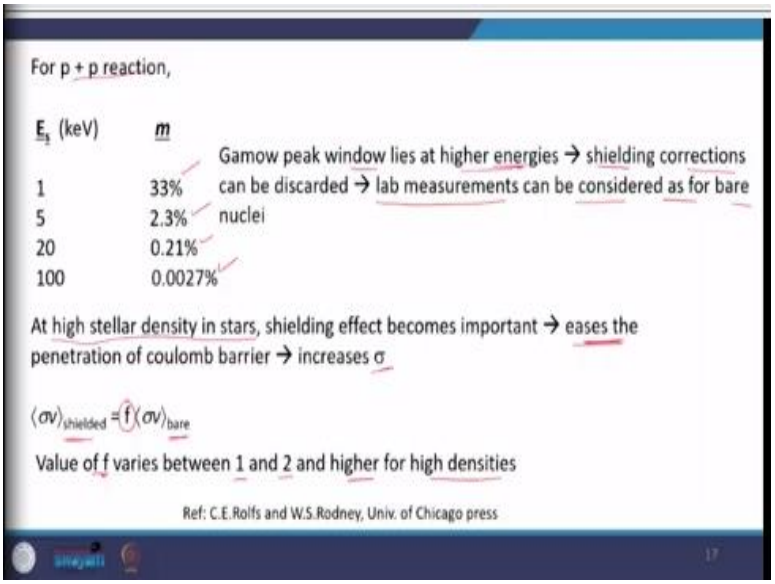

For example, if you take the p + p reaction the ratio is 33% for 1 keV and 2.3% for 5 keV and as you increase the E_s value the ratio becomes very less. So, we know that Gamow peak window lies at higher energies when compared to these values of E s that is why we can always ignore the shielding correction. And the lab measurements can be considered as if they are happening for the bare nuclei. So, this is an important statement we need to understand. Electron screening in principle can be neglected based on these logical arguments.

However, at very high stellar density in stars where nuclei are surrounded with large number of electrons, clusters of electrons are there surrounding the nuclei. Then it can see the impact of electron screening on the cross section provided the density is very high. And the positive point is that because of the electrons present surrounding the nuclei the penetration will become relatively easier and cross section will become high. And for shielded nuclei and for bare nuclei how the reaction rates are represented by linking with some factor f where the f varies between 1 and 2 and this is higher for higher density.

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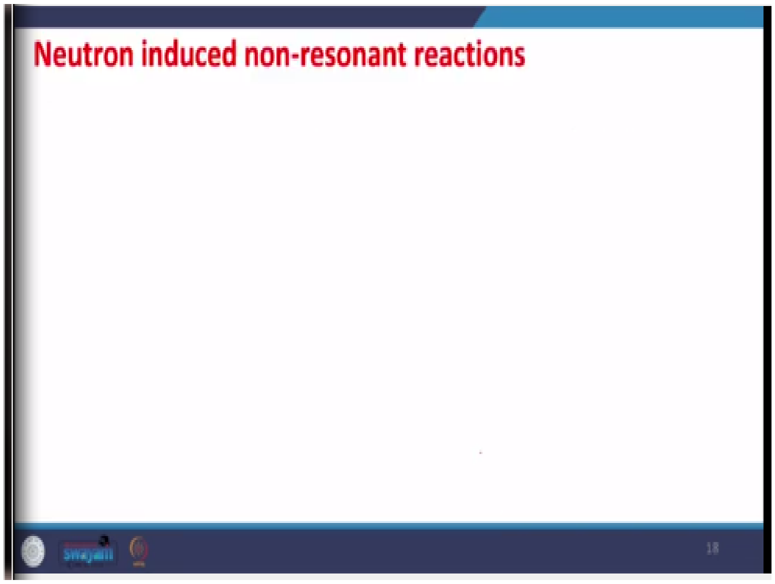

So, to summarize today's lecture, what I have discussed? How to calculate the peak width; how the centroid peak width depends on temperature and also the area of the peak. And the temperature dependent reaction rate I have derived, hope it is clear to you. And then I have discussed one important effect in nuclear astrophysics that is electron screening effect which happens because of the presence of electrons surrounding the nucleus leading to the reduction in the Coulomb barrier increase in the value at the cross section. So, in the next lecture I will discuss non-resonant reaction induced by neutrons; see you soon, thank you so much.