Nuclear Astrophysics

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Lecture – 13

Inverse reactions

Dear students welcome to today's lecture in which I am going to discuss some more interesting features of inverse reactions. We have started discussing the properties of nuclear reactions in terms of cross sections and reaction rates. After that I had spent some time on the mathematical representation of cross sections and rates of nuclear reactions. I started discussing the importance of inverse reactions.

In the previous lecture when I started this topic of inverse nuclear reactions I said at low temperatures predominantly those nuclear reactions which have positive Q values they will dominate the synthesis of elements. As star evolves with respect to time and space, the energy available for the particles also goes up. So, at high temperatures the particles are getting the chance of possessing more energy at high temperatures.

That means energy of the particles is much higher than the Q value of the reaction. We can expect the inverse reactions also to happen. In general at elevated temperatures inverse reactions progressively becomes significant. So, to understand the stellar evolution which is one of the main objectives of this course, it is important to know how these forward and inverse reactions proceed.

So, if you want to understand the stellar evaluation in terms of the forward and inverse reaction what are the parameters we look forward to can you think? See our aim is to understand the energy produced from the stars and the synthesis of elements which overall comprises the stellar evolution. Now it is clear that to understand the stellar evolution we need to understand the rates with which forward and inverse reaction take place.

So, the question is mathematically how to get a better understanding of this whether inverse reactions rate and forward reactions rate will it be sensitive to the Q value. I repeat the ratio of inverse reaction rate and forward reduction rate, whether it will be temperature sensitive to the Q value. If yes can we get some quantitative information in terms of numbers. Can we try to understand not only Q value but the ratio of the reverse and forward reaction rates. How it depends on temperature.

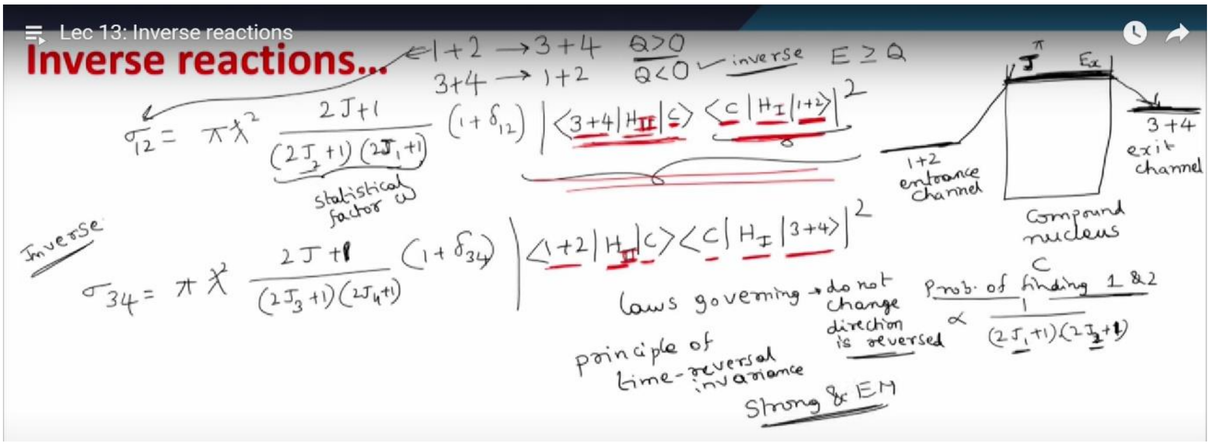
If yes at what temperature it becomes more significant. Qualitative statement I have given that at a high temperature, inverse reactions progressively becomes important. But it is also important to understand this in terms of mathematical expressions and today's lecture I am going to discuss this particular point.

What is it the ratio of the reaction rates when we consider forward and inverse reaction and understanding its relation with the Q value of the nuclear reactions and temperatures of the star. Now for that we need to write the expressions for the reaction rate and if I want to write the expression for reaction rate one needs to write down the expression for cross section. So, with that I will start today's lecture.

So, in the inverse reaction as I said nuclei is of type 1 and 2. When it undergoes reaction they give nuclei of type 3 and 4. At low temperatures Q greater than 0 type of reactions takes place predominantly. And at high temperatures as I said earlier 3 + 4 giving rise to 1 + 2. This is basically Q less than 0. This inverse reaction can happen when the energy available for the particles is greater than or equal to Q value of the nuclear reaction.

One more interesting question why positive Q value nuclear reactions play important role at low temperatures. Because for the energy liberation it is always important to have positive Q value nuclear reactions. So, that is the reason at low temperatures you will come across nuclear reactions with positive Q value.

And as the energy increases and because of various processes when the temperature of the star increases particles will get enough energy to induce reverse reaction as well. And now we are in process of understanding the ratio of the reaction rates. Before that we need to write down the expression for the cross sections. In the last lecture if you remember the cross section of nuclear reaction is there are 4 terms involved.



Number 1 a term which describes the quantum mechanical character of the nuclear reaction I think I have given enough information why pi r square which is the geometrical cross section in classical case should be replaced with pi lambda prime square. Reduced de Broglie wavelength which has the quantum mechanical character because nuclear reaction comes under quantum mechanical processes.

Three more terms are important to include in the expression for cross section. So, number 2 the statistical factor 2 J + 1 divided by 2J 2 + 1 2J 1 + 1. To write this we need to imagine a simplistic case when entrance channel ok 1 + 2 1 + 2 gives rise to a compound nucleus in the exited state whose angular momentum is J and parity pi. This is the compound nucleus.

It has some energy which is excitation energy and with this excitation energy the compound nucleus will decay to 3 + 4 types of nuclei this is called as exit channel exit channel. Now J is the angular momentum or argument of the exited state of the compound nucleus and J 1 J 2 they are the spins of the nuclei of types 1 and 2. So, what is its importance? This is known as a statistical factor statistical factor and this is also is represented by a symbol omega.

So, from now onwards whenever I use the symbol omega please remember I am talking about the statistical factor. Why statistical factor is important? The probability for the reaction to happen. Now I am discussing only the forward reaction. The probability for the forward reaction to happen increases if the number of final states available increases, what does it mean?

1 + 2 is giving rise to 3 + 4 but depending on the conditions in which this nuclear reaction is taking place we can get 5 + 6, 7 + 8, 3 + 6. Any combination of the nuclei types in the exit channel can occur. With what probability that is where this statistical factor comes into picture. Let me refresh this statistical factor concept the probability for the nuclear reaction to proceed in forward reaction goes up with the number of final available states increases.

And you know from quantum mechanics that any energy state with angular momentum J will have 2J + 1 sub states 2J + 1 sub states that is the reason I have written it in the numerator. Now reaction is proceeding through entrance channel that is 1 + 2 and the probability for the particles 1 and 2 to identify in one of the sub states is inversely proportional to the product of the substates of J 1 and J 2.

What I am trying to say here is the probability of finding the particles 1 and 2 is inversely proportional to the number of substates corresponding to particle 1 and particle 2, 2J + 1 and 2J2 + 1. So, J1 is the spin of the particle 1, J2 is the spin of the particle 2. The product of these 2 substates is deciding the probability of finding the particles 1 and 2 in one of the substrates. So, overall the product of these 2 probabilities number 1, 2J + 1, number 2, 1 by 2J 1 + 1 and 2J 2 + 1. This product of these 2 probabilities decides the statistical factor which is represented by omega.

So, this physics aspect also must be considered when you try to write a general expression for the cross section I hope it is clear to you. We have 2 more terms if we need to consider the identical nature of the particles we have to use the symbol delta 1 2 that is Kronecker symbol. Then 4th term which tells us the nature of interaction in the whole process. So, number 1 pi lambda square where the reduced wavelength has quantum mechanical character in terms of h by square root of 2 mu.

And second term is statistical factor, third is this Kronecker symbol concept number 4 because it is a quantum mechanical process as I said in 1 of the previous lectures cross section of any nuclear reaction depends on the nature of the interaction involved, what does it mean? Whether strong force is involved whether electromagnetic force is involved or weak force is involved accordingly the cross section varies.

When you go from weak interaction to electromagnetic interaction to strong interaction the cross section increases tremendously. So, don't you think we have to include this parameter related to the interaction, then how to represent mathematically.

Each step is represented by a matrix element. How many steps are there in this process? In this diagram transition from entrance channel to the exited state number one. So, it is basically 2 step process right and second step transition from the exited state of the compound nucleus to exit channel where particle 3 particle 4 are formed. Which type of interaction is involved in the transition from entrance to compound nucleus?

And which type of interaction is involved from compound nucleus to exit channel? How to represent systematically mathematically. So, it is basically 1 + 2. This H 2 and compound nucleus and C H 1 3 + 4. So, this is basically the probability. You know the square of the wave function in quantum mechanics gives information about the probability. So, here the square, of 2 matrix elements is there.

Number one basically it is the other way round. Let me rewrite this expression because we are not discussing the inverse reaction we are discussing the forward reaction. So, 3 + 4 H 2 C and C H 1 1 + 2, so, here this term involves the transition from 1 + 2 let me take some other colour. So, that it can be clear to you. So, this term represents the transition from 1 + 2 that is exact entrance channel to the compound nucleus and this term tells us the transition from compound to nucleus excited state to the exit channel that is 3 + 4.

This H 1 denotes the interaction in the step 1. H 2 denotes the interaction term in step 2 and this H 1 and H 2 they are called as matrix operators. The operators which represent the interaction type if it is weak interaction if it is electromagnetic interaction or if it is strong interaction. So, together this product of matrix elements and square of it gives us the probability in terms of cross section.

So, overall the cross section is product of these 4 terms. So, regarding the 4th term I repeat the transition from entrance channel to compound channel compound state is represented by this matrix term and this transition is taking place through an interaction that interaction is represented by an operator called as H 1 and the compound state is going to decaying to entrance channel that is 3 + 4 particles.

And this transition is happening through an interaction it could be electromagnetic it could be weak or it could be strong force. Through that interaction is represented by an operator called as H 2 because it is step 2. So, with this analogy can we write expression for inverse reaction. Let us see because ultimately we are looking for the expression for reaction rates of inverse and forward reactions.

With the same analogy I can write down sigma 3 4 is equal to pi lambda square 2J + 1 because there is no change in the compound nucleus for the inverse direction. For the inverse reaction at high temperatures you have to write the statistical factor as 2J 3 + 1 and 2J 4 + 1 and 1 + delta 3 4 Kronecker symbol and here you will write down 1 + 2 this is H 2 this is H 2 and compound nucleus H 1 3 + 4 square.

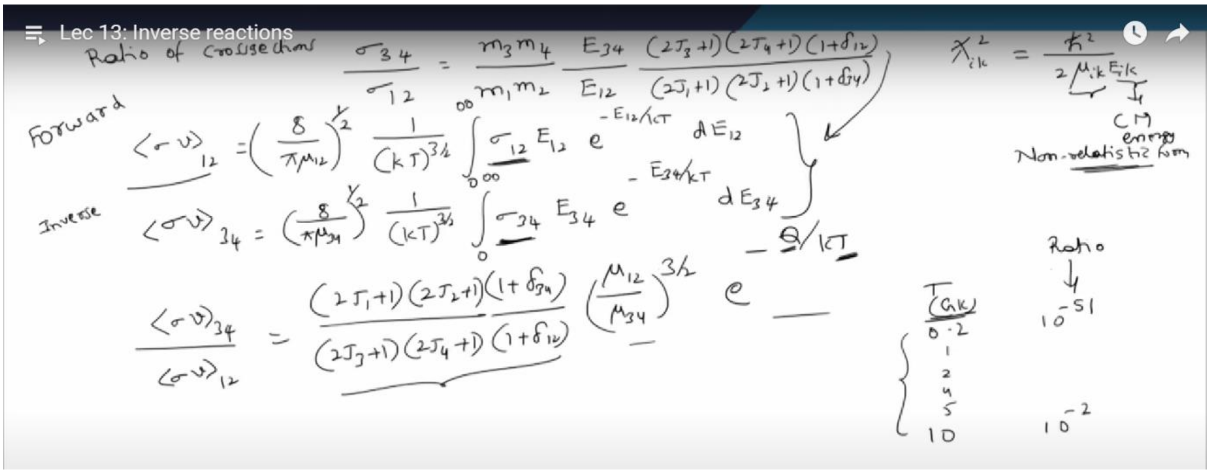
Why I have changed here in this particular case in the inverse reaction the entrance channel contains 3 and 4 nuclear types. They are giving rise to compound nucleus through an interaction which is represented by this operator. And this second term if we see if you see the second term the compound nucleus is going to exit channel 1 + 2 through an interaction represented by matrix operator H 2 H 2.

What you can understand from this expression? From these 2 expressions it is very clear that matrix elements are same matrix elements. Whether it could be forward or inverse reaction. And in general the loss governing laws governing nuclear reactions they do not change. They do not change if the direction is reversed. If the direction is reversed this is called as principle of time reversal invariance.

So, according to principle of time reversal invariance reaction can proceed in both directions. H 2 need not be the same H 1 and H 2 need not be the same. To date there is no experimental evidence which can say that this is violated. Principle of time reversal invariance is not violated We do not have any experimental data at least for strong and electromagnetic interaction.

So, considering the strong and electromagnetic interactions only, can we write down the ratio of cross sections for forward and inverse reactions. So, how does it look like. So, this 4th term as I said it depends of nature of the interaction and these matrix elements are identical except for the reverse order. So, the processes can happen in opposite direction without any problem and this is called as I said principle of time reversal invariance.

So, in the next slide let me write down the ratio of cross sections and no violation has been observed experimentally for at least strong and electromagnetic interaction strong and electromagnetic interaction.



Now if we take the ratio of the cross sections ratio of cross sections how it looks like my dear? Sigma 3 4 divided by sigma 1 2. Please remember the matrix elements will get cancelled. What are the 3 remaining terms, statistical factor omega, pi lambda square and the Kronecker symbol. So, constraining these 3 terms the ratio of the cross section will look like this. So, it is basically m 3 m 4 because I am expanding the expression for lambda and m 1 m 2 E 34 divided by E 12 2J 3 + 1 2J 4 + 1 1 + delta 1 2 and 2J 1 + 1 2 J2 + 2 1 + delta 3 4.

So, to get to this relation the quantity lambda ik square has been represented by replaced with the expression of h cross square divided by 2 mu i k E ik where this is the reduced mass. This is the centre of mass energy. Remember this expression is in non relativistic form. The interesting thing is that though it is a 2 step process in the ratio of the cross sections there is no appearance of the compound nucleus property.

I mean the property of the compound nucleus is gone in the ratio the cross sections and this shows that if I can measure the cross section in one direction I can also measure the cross section in reverse direction provided all these terms are known to you.

We have written the cross section ratio. Let us go for the reaction rate. For that let us write down the reaction rate for forward reaction and reaction rate for the inverse reaction. Then go for the ratio and you see at what temperature inverse reactions plays important role. So, that is the purpose of this lecture. So, in order to arrive at the analogous ratio for the reaction rate let me write down sigma V 12.

The general expression for the reaction rate either you can use mu 1 2 or m 1 2 does not matter and 1 by k T to the power of 3 by 2 and 0 to infinity sigma 12 E 12 and e to the power of minus E 12 by k T and dE 12. Similarly for the inverse reaction sigma V 34 can be written as 8 by pi reduced mass when third and 4th type is considered 1 by 2 and 1 by kT to the power of 3 by 2 0 to infinity sigma 34 E 34 e to the power of the derivation and the rational behind this kind of expression already have discussed in detail in previous lecture.

Now when you have the reaction rates for forward and reverse inverse reaction and when you have the ratio of the cross section because in the reaction rate I have to write down the sigma 12 and sigma 34 also. Take the ratio of this reaction rates and consider the ratio of the cross sections one can write down the reaction rate like this sigma V 34 sigma V 12 is equal to 2J 1 + 1 2J 2 + 1 divided by 2J 3 + 1 2J 4 + 1.

And here you cannot ignore the Kronecker symbol case 1 + delta 12 and the ratio of the reduced masses mu 12 divided by mu 34 to the power of 3 by 2 e to the power of minus Q by k T e to the power of minus Q by k T. So, because the quantity is proceeding the exponential preceding the exponential order are of normal unity the terms coming before the exponential if you see that ratio if you take it is normally close to one.

Now I am giving a very important statement. The ratio of the reaction rates depends on only the exponential term. In the exponential term you have 2 quantities Q value and temperature. Now with this dependence of reaction rates ratio in the numerator we have inverse reaction in the denominator you have the forward reaction the ratio depends on the e to the power of minus Q by kT.

So, if I increase the Q value which one will dominate. If I increase the temperature which one will dominate. You can easily have a guess. Just I can tell you one in some interesting number if I go for temperature 0.2 to 10 then the ratio considering a specific Q value it ranges from

10 to the power of minus 51 to 10 to the power of minus 2 1 2 4 5 if you take all these values of temperature in ah giga Kelvin.

And the ratio if you take it ranges from 10 to the power of minus 51 to 10 to the power of minus 2. Inverse reaction divided by forward reaction rate that means at high temperatures inverse reactions play a very important role. I hope all of you have understood what I am trying to say and in the next lecture I will discuss more about this based on the reaction rates. So, the purpose of today's lecture is to understand the dependence of the ratio of the reaction rates when we consider forward and inverse reaction.

How it depends on the Q value of the nuclear reaction which is the energy liberated in the nuclear reaction that energy liberated in 1 nuclear reaction decides together with all other Q values of the nuclear reactions over all the energy produced from the stars. So, that is why it is important to understand how the 2 reactions proceed forward and inverse and with respect temperature how ratio looks like.

Thank you very much for your attention I am looking forward to meet you in the next lecture when I want to discuss more about the inverse reaction, thank you so much.