

Optical Sensors
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Lecture - 09
Basic Optics for Optical Sensing- VII
Absorption and dispersion: Drude model for conductors, Bulk Plasmons

Welcome to the 9th lecture of Optical Sensors course. In the previous lecture, we discussed evanescent waves and we also saw how to use them for sensing applications. Till now we have been studying materials which have only real value of the dielectric constant so, it is like transparent materials and dielectrics.

Now you will see what happens with metals. So, today we will discuss Absorption and dispersion and from there we will end up with Drude model for conductors and if time permits, we will discuss bulk Plasmons.

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Absorption

$\epsilon_s = n^2$
 $\epsilon = (n_r - ik)^2$
 $\epsilon = n_r^2 - k^2 - 2in_rk$

When the refractive index is complex: $n = n_r \pm ik$

k - The extinction coefficient !. The sign depends on the choice: $e^{\pm ikz}$

The relation to real and imaginary parts of the dielectric function:

$\epsilon_1 = \epsilon_r = n_r^2 - k^2$
 $\epsilon_2 = \epsilon_i = 2n_rk$

$\epsilon_r = n_r^2 - k^2$ and $\epsilon_i = 2n_rk$

→ A plane wave will be decaying as: $E(z) = \exp(ik_0 n_r z) \exp(-k_0 k z)$

The intensity is decaying as: $|E|^2 = \exp(-2k_0 k z)$

Beer's law

The absorption coefficient is defined as: $\alpha = 2k_0 k = \frac{4\pi k}{\lambda_0}$

$I_0 \rightarrow$ $I_0 \exp(-\alpha d)$

If we ignore reflections at the boundaries!

I. Abdulhalim, BOSMI Lectures

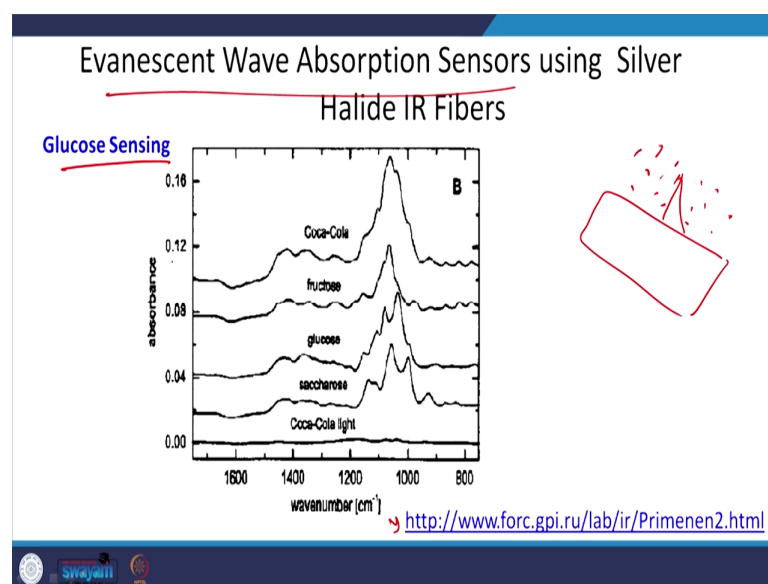
When the refractive index is complex basically- it will have a complex component also. Till now we had epsilon s equal to n square; now we have complex ones. Since n is complex, epsilon will be complex. So, epsilon will be square of n r minus i K. You can choose the sign of this and depending on the sign of K, it can become negative or positive.

If you make a square of it, it becomes epsilon equal to $n^2 - \frac{\kappa^2}{\omega^2}$ plus $i\frac{2\kappa}{\omega}$. So, basically it is epsilon one or epsilon real. we will be using one of it. So, this is plus $i\frac{\kappa}{\omega}$; i^2 is minus 1. So, epsilon r will be $n^2 - \frac{\kappa^2}{\omega^2}$ and epsilon i will be $\frac{2\kappa}{\omega}$ - this is what we get.

So, if there is a plane wave and it is incident on an interface of a dielectric and metal then it will be decaying as this. And the intensity will decay like E^2 so, it will be like this. So, you want to see how much light gets absorbed. So, we define absorption coefficient alpha, that is given by this relation. So, it is a simple experimental setup like, you have a medium, all you know is that it is absorbing. What you do is that you shine light on it. So, incident intensity is I_0 what you get is I so, I will be $I_0 \exp(-\alpha d)$; that is called Lambert Beer law. So, from there you can determine this alpha the depends on d here - this intensity. So, if you have larger d than the intensity will decrease rapidly more, ok.

But this condition we arrive only when we have ignored the reflections at the boundaries. If any material has complex refractive index, it will be absorbing in that particular wave length range; because all this refractive indices are wavelength dependent - that we will see.

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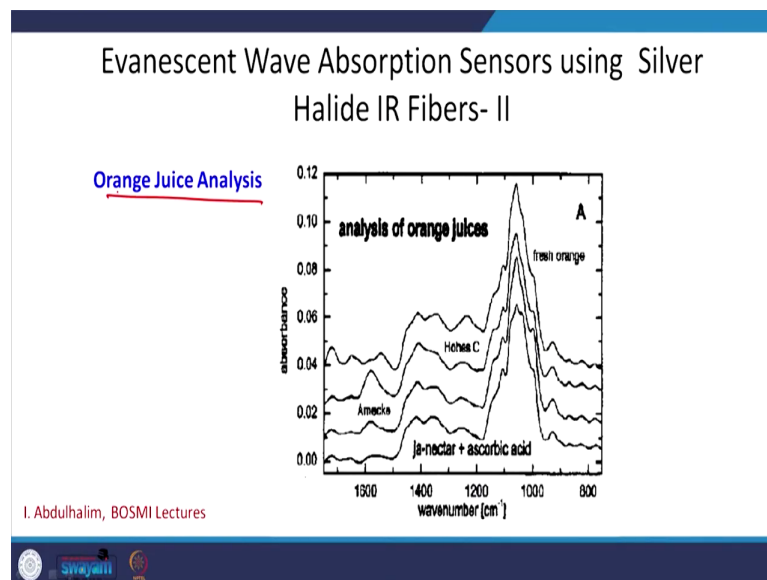


We know evanescent wave sensors. You know, that you have certain configuration - it has evanescent wave, when you put a material here which is kind of absorbing. So,

basically you can use it for evanescent wave absorption sensors. For example, here I have shown you evanescent wave absorption sensors using silver halide IR fibres.

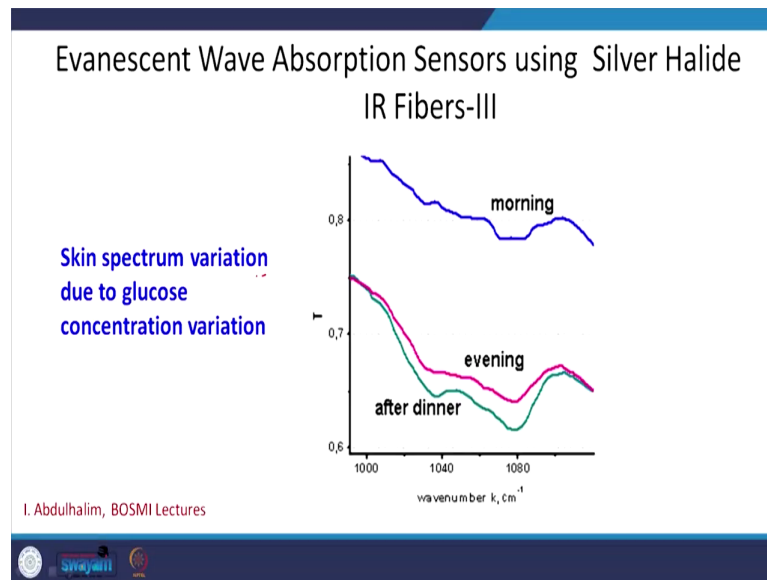
In particular, we are studying glucose sensing and from this reference, you can see that the absorbance as a function of wave number for different sample solutions. So, here we have Coca Cola light, Coca Cola and their fructose glucose and saccharose. And you see that in Coca Cola the absorbance is high; so, if we translate this absorbance value with respect to glucose concentration, you can say that the concentration of glucose in Coca Cola is higher, while in light Coca Cola it is relatively much smaller.

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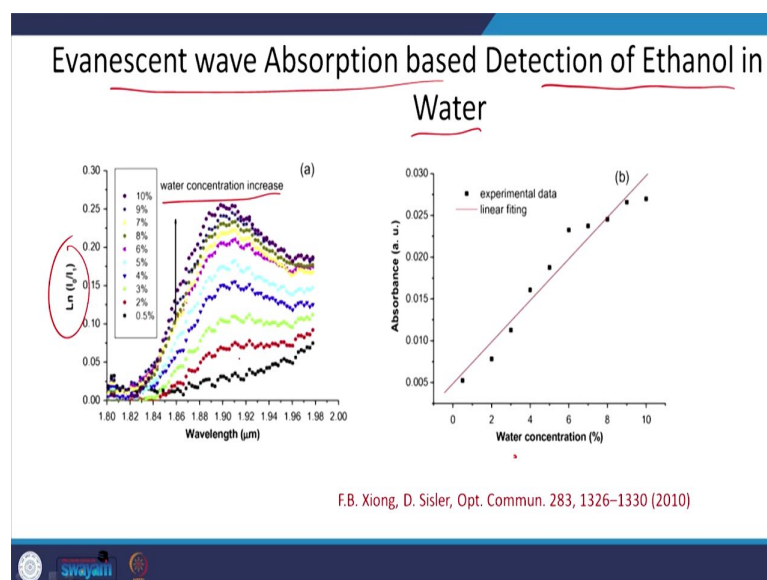
Let us see another example, where they were doing orange juice analysis. And it was shown that if you have fresh orange juice the absorbance is maximum here and if nectar has something then it starts decreasing. So, based on the absorbance, you can say that if the juice is fresh or not.

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Similarly, people have monitored the spectrum of the skin at different times of the day and they have found that because of change in the glucose concentration, the transmission or reflection changes. Whatever you measure - you measure either absorption or reflection or transmission; here they have measured the transmission as a function of wave number and you can see that it starts decreasing with evening.

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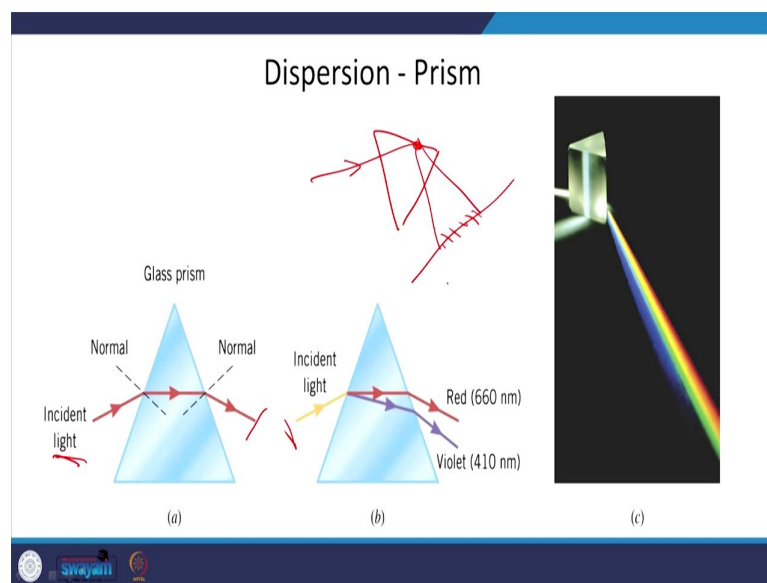


Here is another example, where they detected ethanol in water using fiber optic evanescent wave sensor and for different water concentrations, they recorded the

absorption spectra took the logarithms - I told you that this is Beer's law and from there they calculated absorption coefficient. And now they plot the absorption with respect to water concentration and it was found that with an increase of water concentration in ethanol there is an increase in absorbance. It is first increasing and then decreasing. We will explain it later, when we see a comparative study on surface Plasmons. But for now, we see that you have a peak somewhere and that the peak absorbance is changing with respect to change in water concentration in ethanol. So, if you have larger water concentration - if you get larger absorbance; that means, ethanol is impure ok. So, you want to avoid that.

So, till now we have seen what an evanescent wave is, what is the role of absorption here for making evanescent wave absorption sensors. And we discussed a few sensors based on evanescent wave absorption technique. Now let us see what is dispersion. It is also very important optical parameter which allows us to do various sensing mechanisms so, we need to study this also.

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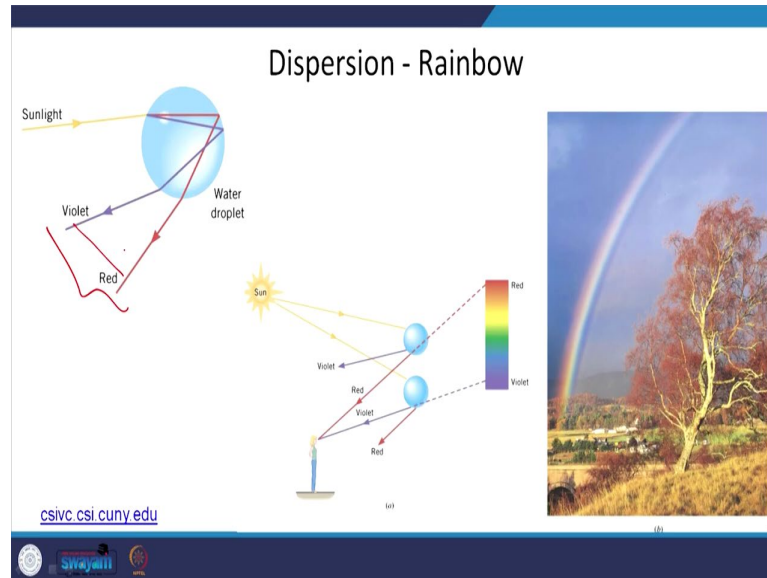


When you shine a prism or any material with white light what you get is 7 colours - it splits into seven colours - like here in this picture. And why it happens because all the wavelengths in this polychromatic light have different speeds in the material medium.

So, for example, if you have an incident light which is monochromatic you will have only one ray coming, but if you have polychromatic light; that means, white light what

you see is that all the components will get dispersed and that is because of speed of different wavelengths is different in the particular material medium. So, for example, here you see that the red is less deviated, and the violet is more deviated.

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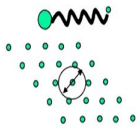
Another example is rainbows; where you see that when it just rained and after that when there is sunrise you see a rainbow like this. Why it appears - because we have dispersion. So, the sunlight which falls on the water droplets, because now the weather is humid, there is lots of moisture in the atmosphere - you see that the violet and red components and all the components in between - they get separated. So, if you have a droplet here, sunlight is falling on it somewhere what you see is that it gets dispersed.

But this is not exactly what you see you - see other way around you see the red one on the top and violet ones on the bottom. And that is because of this thing - that because we do not perceive the violet light from the drops which are at top - at higher altitudes and that the ones which are at the bottom - the red ones we do not see. So, basically what we see is, this - that the red one is on the top and the violet in the bottom. So, actually we see kind of inverted ones because of this altitude thing ok.

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Dispersion – Lorentz Model

Oscillating bound electron model (Lorentz model):



$$m_e(\ddot{r}(t) + \gamma\dot{r}(t) + \omega_0^2 r(t)) = -q_e E_{eff}(t)$$

inertia
damping
restoring force
applied force

Now use:

$$E_{eff}(r, t) \approx E(r, t) + \frac{4\pi P(r, t)}{3}$$

Mossotti (1850) equation (CGS)

and

$$P(r, t) = N\mu_a = -Nq_e r(t)$$

Depolarization field

So, we know what dispersion is, now. Let us try to solve it analytically and find an expression for it. To understand it analytically, we consider a material medium as a spring problem. And what we see is that the electrons are attached to the lattice in a fashion that it can have oscillations along the lattice, and oscillating electron is actually bound. So, if you write the equation of motion for this electron, which is oscillating along the around this lattice, hat you see is that you have a component of inertia and then a damping; why damping? Because it is kind of damped oscillator and then this is the restoring force. What constitutes the restoring force? You see that in equilibrium, you have a positive core and then the electrons. Now somehow you disturb the equilibrium. Suppose you put the electron close to the core, what will happen? Core will try to further bring it closer while the other electrons - they will try to push it, because they want to create the equilibrium.

So, these two forces altogether are called restoring force. So, this is the force, and this is the force which was applied. And if you solve it similar to (Refer Time: 13:03) Mossotti equation using this; P is the polarization; mu is the dipoles - what you get is that-

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Dispersion – cont'

Then: $\dot{P}(t) + \gamma \dot{P}(t) + (\omega_0^2 - \frac{4\pi N q_e^2}{3m_e})P = \frac{N q_e^2}{m_e} E(t)$

Use: $P(t) \propto \exp(i\omega t)$ and: $D = \epsilon E = E + 4\pi P$ (CGS)

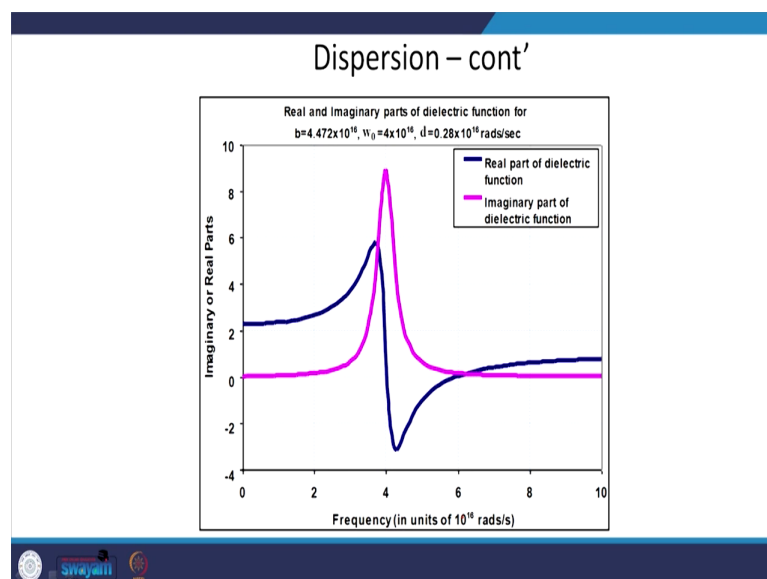
To get: $\epsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$ *Oscillator strength*

Where: $\omega_{0s} = \sqrt{\omega_0^2 - \omega_c^2}$ *Resonance frequency is shifted by frequency of oscillating dipoles*

$\omega_c^2 = b^2/3 = 4\pi N q_e^2 / 3m_e$

this relation in dipole moment and P is proportional to exponential of i omega t. And you know D is equal to epsilon e - you put them together to get this relation; where b is the oscillator strength. So, you know the oscillator strength and the resonance frequency actually here is shifted by frequency of the oscillating dipoles which is given by this relation, omega c is given by this relation which is 4 pi N q e square upon 3 m e; m e is the mass of the electron and q e is the charge of the electron.

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So, if you plot the dispersion curve, basically the real part shows a kink here while the imaginary part shows this pink curve for this particular values of b ω_0 and d .

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RI Dispersion in Glasses

Glasses are usually transparent in order to be used for imaging, hence their refractive indices are real and follow one of the following formulae:

→ Cauchy:
$$n(\lambda) = n_\infty + \frac{a}{\lambda^2} + \frac{b}{\lambda^4} + \dots$$

→ Sellmeier:
$$n(\lambda) = \sqrt{1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3} + \dots}$$

$$n(\lambda) = \sqrt{A + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3} + \dots}$$

What happens to glasses? In glasses, the refractive indices are real and follow one of the formulae either they follow Cauchy, or they follow Sellmeier relations. So, n is a function of some constant and then λ^2 λ^2 to power 4 something like this. Or, again, you can have it like λ^2 by λ^2 minus 1 and then there are these coefficients which are basically different for different materials.

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RI Dispersion in Glasses II

For BK7:

Coefficient	Value
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	$6.00069867 \times 10^{-3} \mu\text{m}^2$
C_2	$2.00179144 \times 10^{-2} \mu\text{m}^2$
C_3	$1.03560653 \times 10^2 \mu\text{m}^2$

SF11

See one of the useful websites:
<http://refractiveindex.info/>

So, here is an example. You see that the refractive index varies with wavelength for different materials here. At higher wavelengths, it is almost constant - the variation is not that prominent. You can choose the coefficients, like I told you and these coefficients define the material. So, if you have, say, silica it will be different if you have BK7 it will be different, if you have SF11 - that is another glass, then you can have different of coefficient for this.

So, if you want to use any material and want to know what the refractive index of that is, you can go to this site which has a database of materials refractive indices. You can download also the excel files and text files from there and you can use it for simulations.

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Dispersion of Free Carriers

In space with free charge carriers such as free electrons in metals or ions in the ionosphere no restoring force and no damping hence the equation we derived for the oscillating dipole becomes:

$$\ddot{P}(t) = \frac{Nq_e^2}{m_e} E(t) \rightarrow P(t) = \frac{-Nq_e^2}{m_e \omega^2} E(t)$$

Use:

$$D = \epsilon E = E + 4\pi P \rightarrow \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Where the plasma frequency:

$$\omega_p^2 = 4\pi Nq_e^2 / m_e = b^2 \quad (\text{CGS})$$

$$\omega_p^2 = Nq_e^2 / \epsilon_0 m_e \quad (\text{SI})$$

Now let us see what happens to dispersion of free carriers - free carriers means free electrons in metals or ions in the ionosphere, where there is no restoring force - they are free. No damping - they are free. So, the equation we derived for oscillating dipole reduces to this - all the other terms vanish.

And if you solve for P, this is the second order differential equation in t, you get this relation. And if you use this, you arrive to this important relation, where omega p is the plasma frequency, given by this relation in CGS and then in SI.

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Dispersion of free Carriers – cont'

From the relation for bound electrons:

$$\epsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$$

Substitute $\omega_{0s} = 0$ for free carriers to get:

$$\epsilon(\omega) \approx 1 - \frac{b^2}{\omega(\omega + i\gamma)} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \xrightarrow{\gamma=0} 1 - \frac{\omega_p^2}{\omega^2}$$

Drude Term

Which is the same function obtained without damping

From the relation of bound electrons, we had this. Now this becomes equal to 0. So, you get this relation for gamma is equal to 0. This is called Drude term which is the same function obtained without damping. So, if there is no damping you get this term, ok.

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Dispersion of Metals

Better described by a combination of Drude term and two oscillators:

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\Gamma_p\omega} + \sum_j \frac{f_j \omega_j^2}{\omega_j^2 - \omega^2 + i\Gamma_j\omega}$$

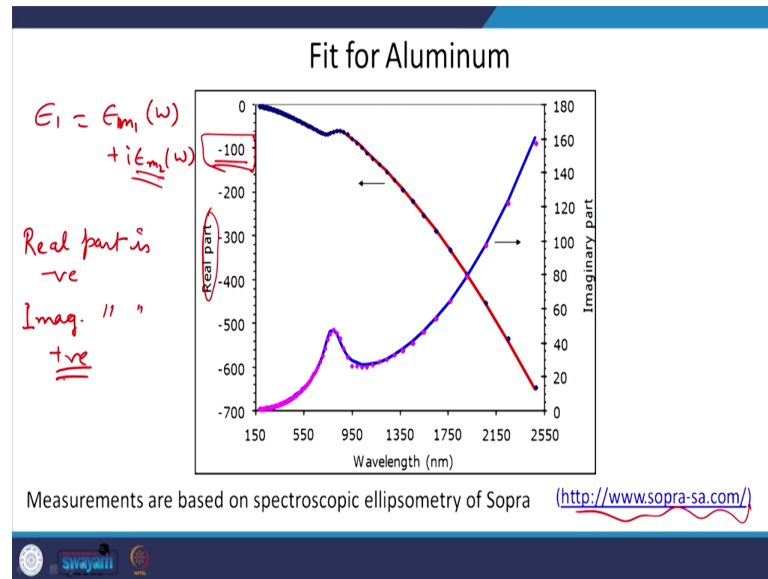
Metal	ϵ_∞	$\omega_p(eV)$	$\Gamma_p(eV)$	f_1	$\omega_1(eV)$	$\Gamma_1(eV)$	f_2	$\omega_2(eV)$	$\Gamma_2(eV)$
→ Ag	2.302	8.807	0.082	0.439	4.602	0.761	1.098	6.008	2.393
→ Au	3.952	8.542	0.070	-85.116	2.6	3.712	90.046	2.789	4.27
→ Al	0.952	13.335	0.117	8.119	1.579	0.407	9.908	2.006	1.942

What is plasma frequency? I will come to that. Let us see what happens to the dispersion of various metals. We see here that for different metals, say silver, gold and aluminium - why we are choosing only these materials? Because they have large number of free

electrons and they are noble metals and what is important about them is that they can be used for plasmonic sensing.

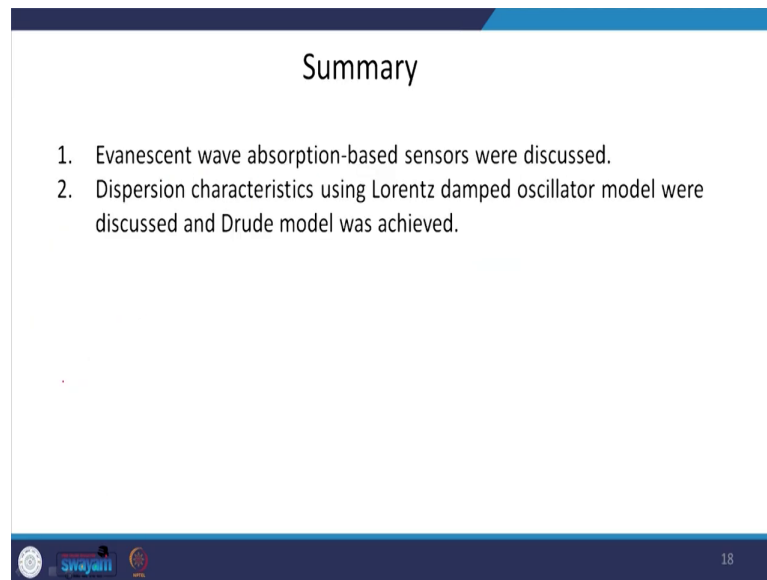
We are basically concerned about plasmonic sensing. So, if you can put all these values there and you can see, what are the dielectric functions.

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Basically, this is from SOPRA database. You can use it for this, and we show that the real part and imaginary part of epsilon or like this for aluminium. So, this is epsilon 1; epsilon 1 is epsilon m 1, m is for metal - omega plus i epsilon m 2 omega. The epsilon m 1 is the real part and epsilon m 2 is the imaginary part. What you say here is that the real part is negative. That is something very important - you see, the real part is negative, while their imaginary part is positive; positive imaginary part means that it is absorbing, ok. Real and imaginary part - I will come to that - what it means, when we will discuss Plasmons.

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Summary

1. Evanescent wave absorption-based sensors were discussed.
2. Dispersion characteristics using Lorentz damped oscillator model were discussed and Drude model was achieved.

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So, today we discussed what are evanescent wave absorption sensors. The dispersion characteristics using Lorentz damped oscillator model were discussed and Drude model was achieved. So, till now what we see is that, you consider a material as oscillators, having oscillators which are electrons and the ions there.

And these are damped oscillators - you solve for the equation of motion and then you arrive to the equation which describes the dielectric constant of this particular medium. If it has imaginary part, it will be absorbing; if it does not have imaginary part only real part, then it will be transmitting. ω_p is the frequency below which, the material becomes absorbing and above it becomes radiative or transmitting.

Thank you.