

Optical Sensors
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Lecture – 08
Basic Optics for Optical Sensing-VI
Evanescent sensors, Absorption and dispersion:
Drude Model for conductors

Welcome to the 8th lecture of Optical Sensors course. In the last lecture, we studied the basic phenomenon of total internal reflection and derived the expression for the electric field and the penetration depth for evanescent waves which travel along the interface when you have total internal reflection. And we saw that, there was a plane wave which had the amplitude decaying exponentially while it was travelling along it,- the interface.

In the present lecture, we will try to see how we can use this phenomenon for sensing applications and if time permits we will also see the basic characteristics of say; for example, absorption and dispersion of metals and we will try to solve for the Drude model for conductors. So, till now we have been studying the dielectrics or insulators and now slowly, we will move into the metals and we will try to see metal optics. But before we could do that, let us try to see what we have got in TIR.

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$$d = \frac{\lambda_0}{2\pi \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}}$$

if $\sin \theta_1 = 1$

$$d_{\min} = \frac{\lambda_0}{2\pi \sqrt{n_1^2 - n_2^2}}$$

Smallest value of penetration depth

$\lambda_0 = 500 \text{ nm}$
 $n_1 = 1.5$
 $n_2 = 1$

$$d_{\min} = \frac{500}{2 \times 3.14 \sqrt{2.25 - 1}}$$

$$= 70 \text{ nm}$$

70 nm

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Also, if you remember that the penetration depth was $\lambda_0 \sin^2 \theta_1 / 2 \sqrt{n_1^2 - n_2^2}$. If $\sin \theta_1$ is equal to 1, d is equal to $\lambda_0 / 2 \sqrt{n_1^2 - n_2^2}$. This is the minimum value of d ; that is the smallest value of penetration depth. It was the largest when we had θ_1 equal to θ_c .

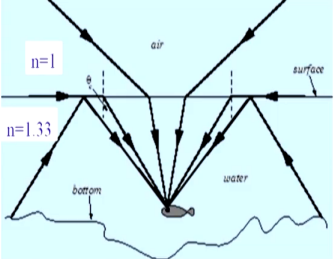
If you have say λ_0 is equal to about, let us give it a value, about 500 nanometers and n_1 is equal to say 1.5, n_2 is equal to 1. The minimum value of d is equal to 500 divided by 2 into 3.14 or π , you can keep it π and under root 2.25 minus 1 which will roughly come equal to about 70 nano meters.

So, you can see that this is of the order of the wavelength of light, it is very small; it can be hundreds of nanometers to few microns, but not more than that. So, it means that if you have a medium, the wave which is travelling along the interface, it can penetrate up to 70 nanometers say for example. So, it will be like 100 of nanometers only.

So, this is very small, it is not like a penetrating more and more; but it can be up to few microns if you can design it suitably. So, this is such a small distance.


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TIR, Fish Vision and Diamonds Brilliance



The fish sees the sky if it looks upwards but sees the bottom of the pond if it looks at angle larger than 48.8 degs.

Question: explain the brilliance of diamond in air knowing that for diamond $n=2.42$ and explain why it is less shiny when it is in water?



There are other uses, say for example, fish vision. So, if you have a fish and if it sees the sky upwards, it will seem like it can see the sky. But if it sees at an angle say around 49 degrees, it does not see the sky; it sees the bottom of the pond. So, it can see far away

objects, but that at the bottom of the pond, not at the sky. So, this is the problem with the fish, ok.

So, if you come at an angle which is larger, basically the fish cannot see, right. And because of this total internal reflection only the diamonds are so shining, so bright. So, if you have a diamond and say if its reflective index is around 2.42; it has air which is surrounding it, then you have large brilliance.

Now, if you put it in water, then it becomes less shinny. So, I am giving you a question what happens to the brilliance? why it is less shinny when it is in water? So, this is a homework for you. Now, we come to the basic question; can we do sensing using TIR?

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Can we do Sensing using TIR?

$n_1 \sin \theta_c = n_2$
 $\sin \theta_c = \frac{n_2}{n_1}$

Use: $\frac{d \sin x}{dx} = \cos x$

$$\frac{d \sin \theta_c}{dn_a} = \cos \theta_c \frac{d \theta_c}{dn_a} = \frac{1}{n_p} \Rightarrow \checkmark$$

Sensitivity

$$S = \frac{d \theta_c}{dn_a} = \frac{1}{n_p \cos \theta_c} = \frac{1}{n_p \sqrt{1 - \sin^2 \theta_c}} = \frac{1}{n_p \sqrt{1 - n_a^2 / n_p^2}} = \frac{1}{\sqrt{n_p^2 - n_a^2}}$$

Incident light, Reflected light, n_p , n_a , Field distribution, Analyte

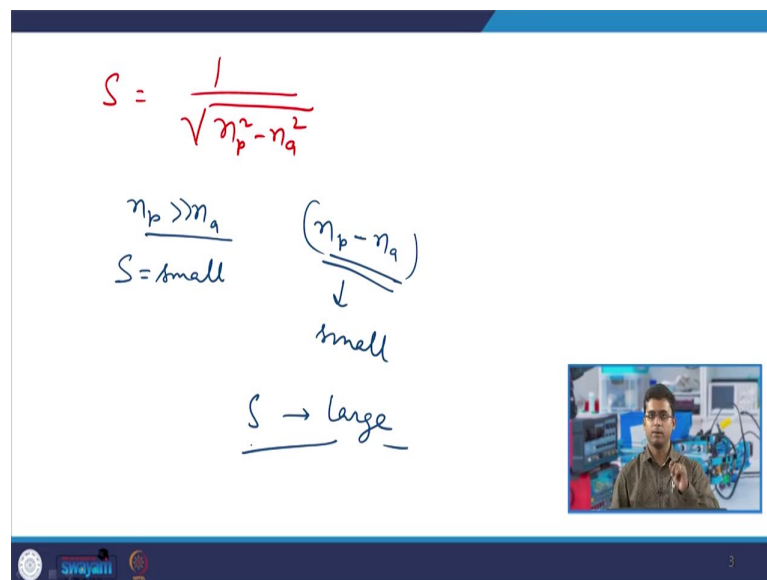
So, if you remember, $\sin \theta_c$ was equal to n_2 by n_1 , right. So, it was - $n_1 \sin \theta_c$ was equal to n_2 . So, $\sin \theta_c$ is n_2 by n_1 that is what we got.

Let us say that we use a prism here and n_a is the refractive index of the analyte, n_p is the refractive index of the prism; light is incident from the top, because of refraction it goes this way, it gets total internal reflected and you get the reflected light. You put a layer which specifically binds to the analyte. So, this all, say, has a refractive index n_a . The field distribution, I told you already that, it is exponentially decaying here. So, you have this relation, if you replace n_1 and n_2 by n_a , n_p and n_a .

Now, you take a derivative of this, you arrive to this and then we have sensitivity by this relation. I told you in the beginning of this course - in the first lecture, how we define sensitivity; sensitivity was change in the parameter which we are measuring with respect to the change in the parameter which is being interrogated.

So, here we are changing theta and we are trying to measure it in terms of the refractive index. So, theta is being interrogated. That is why it is $d\theta$ by dn_a . And you arrive to this equation which is $1/\sqrt{n_p^2 - n_a^2}$. What it means? It means that, the sensitivity will be large.

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The slide contains the following handwritten content:

$$S = \frac{1}{\sqrt{n_p^2 - n_a^2}}$$

Below this, on the left, it says $n_p \gg n_a$ and $S = \text{small}$.

On the right, it shows $(n_p - n_a)$ with a downward arrow pointing to the word "small".

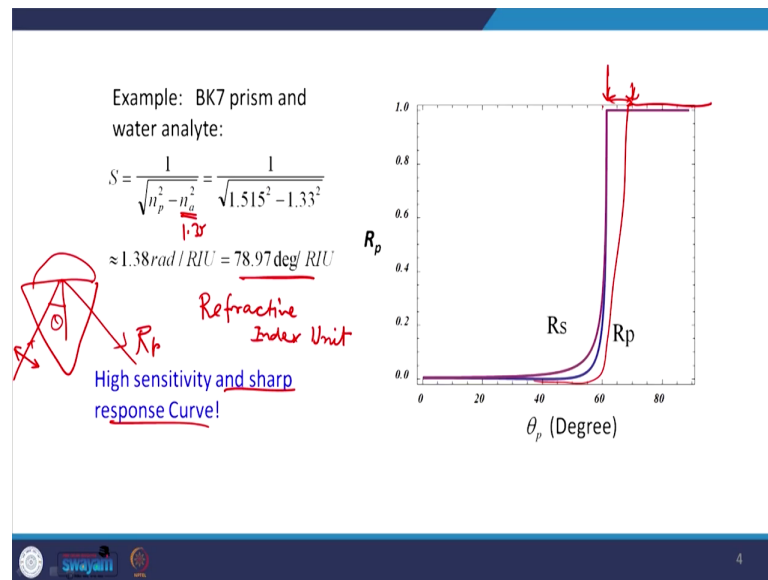
At the bottom, it says $S \rightarrow \text{large}$ with a horizontal line underneath.

In the bottom right corner, there is a small inset video of a man in a lab coat speaking.

So, we will have S equal to $1/\sqrt{n_p^2 - n_a^2}$; it means that, if n_p is greater than n_a , much larger than n_a , S is a small.

Sensor will have a small sensitivity if n_p minus n_a is large. If the difference is small; n_p minus n_a is small, S is large. So, for high sensitivity of the sensor, one needs to have a small contrast between the prism and the analyte refractive index.

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Let us take an example. We have a BK 7 prism here and we have put water droplet here; light is incident from one end and it gets reflected here and we are changing the angle theta and we try to measure the reflectivity of the p polarize light, -this is my setup.

So, the sensitivity is defined by this relation and we come out to have a sensitivity about a 79 degrees per RIU; that is the refractive index unit. So, this means that in change of refractive index one, 79 degrees change happens in the theta critical. So, it has high sensitivity and sharp response curve. So, this is a very good sensor. This has very sharp response - you can see around theta c.

So, we are trying to measure this. If you change say n_a , if you put n_a is equal to say 1.33, now you added some sugar; so, it became 1.35. What will happen? It can move, this curve will slightly move here. So, the position will change. So, you want to measure the position of this. By measuring the change in position, you can say, how much change in refractive index occurs. That is how we can use it for sensing applications.

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Evanescent Wave Sensors

The sensing is performed by the evanescent tail of the modal field in the cover medium. This sensing operation consists of measuring the change of the effective index of a propagating mode when a change of the refractive index takes place in the waveguide cover. Or due to absorption!

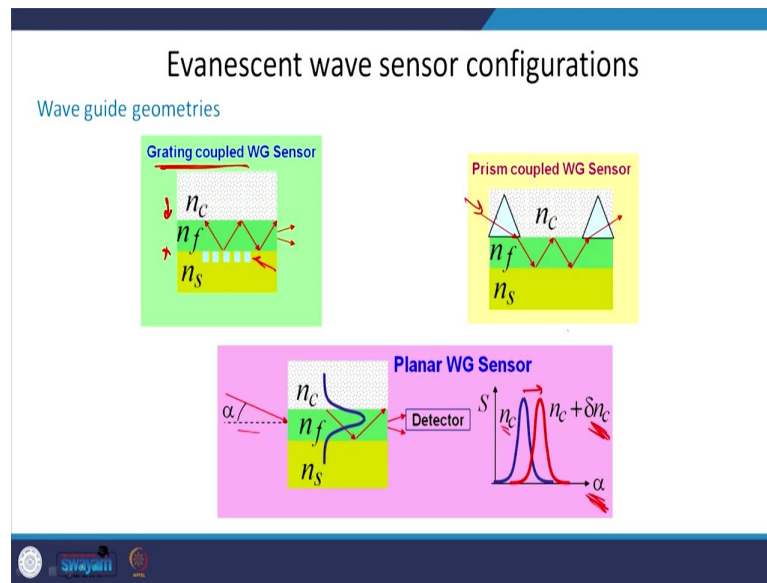
$n_g > n_s$ $n_g > n_c$

That was one part; I mean, we used the critical angle for sensing. Beyond critical angle there was the phenomenon of total internal reflection. But what happens to the evanescent waves? This is something which is basically interacting with the material of the refractive index of the analyte; that is what it depends on and it is getting changed also because of that. So, the sensing is performed by the evanescent tail of the modal field in the cover medium. So, suppose I have an interface here and light is incident in this direction; what will happen that, you have a wave which has exponentially decaying amplitude and it moves along this interface.

So, if you make a wave guide, like I discussed; if you have two media having refractive index slightly smaller than this. So, n_g is greater than n_s and n_g is also greater than n_c . So, what happens actually that, when light travels from here to here; you have an evanescent wave in both the cover and the substrate. So, the wave is travelling like this, the evanescent wave is also travelling like this and the field is decaying exponentially in both the media.

This another example: if you have air and glass interface at certain angle you get all the light reflected, because of total internal reflection. And then there will be a wave which will be travelling along this interface.

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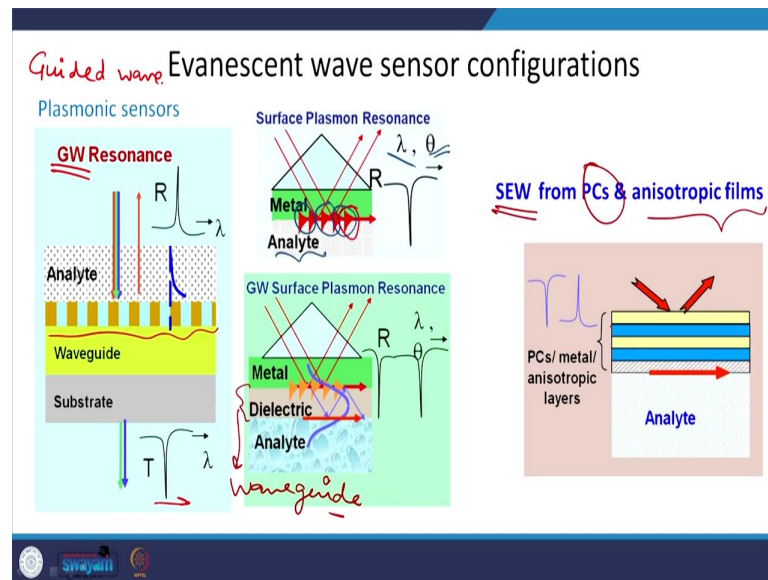


There are various configurations for evanescent wave sensing; one is grating coupled waveguide sensor, you have a waveguide where the coupling of light is done by gratings.

So, here either you do by grating or you do by prism. Why we need a grating or prism? Because this is very small - the dimensions of the waveguide - the core of the waveguide is very small and you want to couple light to this. So, the convenient method says either using prism; so, you have a prism from where light leaves out to the core or you have a grating which basically sends all of it and excites many modes of this waveguide. And then you use it for sensing.

In a planar wave guide sensor, you have this kind of arrangement where you have light incident at an angle, and you can measure it with the absorption or sensitivity with respect to this angle only. If you change the refractive index from n_c to $n_c + \delta n_c$, n_c is the refractive index of this cover; you can basically have the peak moving. And by measuring the change in this peak position in terms of α , you can say how much change in δn_c occurred - that is how you use it for sensing.

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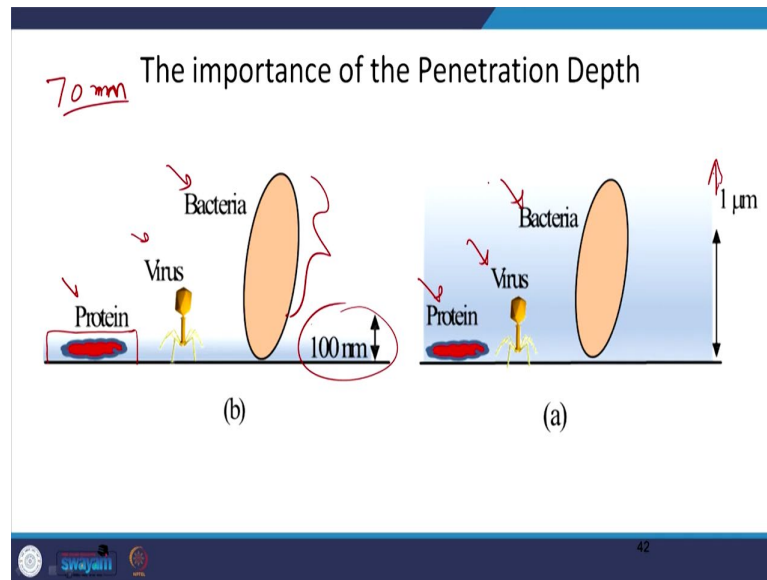


Then there are other configurations on plasmonic sensing; so for example, there is something called guided wave resonance. Guided waves, where you have a periodic structure with a varying refractive index. It is both dielectric and then you have a Whatever gets guided, does not come out of the substrate or the guided mode couples back to the grating and then it gets reflected. So, you get a peak in the reflected spectrum or a dip in the transmitted spectrum. By measuring the change in the position of this deep or peak, you can say how much change in the refractive index occurred. Or we use it for Surface Plasmon Resonance sensors; we will discuss in detail this later. Here also you can see that these are kinds of waves which are evanescent in nature or excited at the interface of this metal and the analyte here. These are evanescent waves and this senses change in the refractive index of this analyte.

So, either you measure the change in λ or measure the change in θ and this is how you see the reflected light basically. By measuring the change in λ or θ you can see how much change in refractive index occurred. Then you combine these two and then there are guided waves Surface Plasmon Resonance sensor, where after the metal you put the dielectric layer which becomes a wave guide. Under suitable conditions, it can excite certain modes in the wave guide and this can be used for sensing applications.

Also, you have surface evanescent waves using photonic crystals or anisotropic films; basically, you can modulate this anisotropic films or photonic crystal structures and you have dip and peak like structures and then you use it the evanescent field for sensing. So, there are various configurations and we will discuss few of them in detail.

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But, I was discussing the penetration depth before we started seeing what are evanescent wave sensors; and I told you that, for the example we had about 70 nanometer - we got the penetration depth.

So, if you want to say have, suppose you have a penetration depth of say about 70 or 100 nanometer here for example, and you want to detect say protein or virus or some bacterium as analyte medium; what you see is, this protein - is all had overlapped with a evanescent field.

Virus has partial overlap and also the bacterium. So, this much portion of the bacterium, it not seen by the evanescent field. While if you increase it, the penetration depth, say like this - to here may be; then this will see all, it can detect protein, it can detect virus, it can see bacteria. So, you designed the sensor in such a way that it detect the particle a species. So, the both are useful.

Suppose you want to detect protein or at the that sample also have the bacteria and virus and you want to equipped them; you make the penetration depth is small. So, it does not

sense the bacteria at all, it senses the protein. If you want to detect bacteria, then this configuration with a small penetration depth will not work for it. So, one has to design the TIR sensor in such a way that, the penetration depth is large. So, that the evanescent field sees the whole cell, bacteria cell.

So, this is called this penetration depth engineering, and this is important - it depends on the kind of analyte which we are going to sense.

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Analytical Approach towards Evanescent Field Sensing

$$\epsilon = \begin{cases} n_w^2 & r \in V_w \\ n_{a,s}^2 & r \notin V_w \end{cases}$$

Assuming a particle is added to the analyte, it creates a variation in the dielectric function:

$$\delta\epsilon = \begin{cases} n_p^2 - n_a^2 & r \in V_{int} \\ 0 & \text{otherwise} \end{cases}$$

Handwritten notes:
 n_p \Rightarrow Ref. Index of the particle
 $\delta\epsilon$ After the particle was added
 The wave vector will change by: $\delta k = k_f - k_i$ and the field E_i to E_f
 Wave vector before the particle was added

Let us try to understand analytically what happens to the evanescent field sensing; I mean, why it is important to have large volumes or something to have evanescent high sensitivity. So, how the sensitivity varies that we saw for critical angle and what are the other parameters which it depends on.

To solve that let us say that, we have this configuration, where you have a waveguide say that we call confinement regions say n_w . And this waveguide is fabricated on a substrate which has refractive index n_s and on top of the wave guide we have an analyte which has refractive index n_a . Because of the total internal reflection, we have evanescent field there and this evanescent field covers the volume that is called the interaction volume inside the evanescent field region. So, the dielectric constant can be defined in these regions as this; thus when this is in the volume region of the waveguide, it is n_w square; if it is not in the volume region of the waveguide, then it is either n_a square - that is the analyte or n_s square. So, here it will be n_s square or when you say epsilon.

Now let us see that what happens if I put a particle here. If I put a particle here, what happens that, the dielectric constant or the refractive index of the analyte region will change slightly. So, let us say that it changes by delta epsilon. So, n_p is the dielectric, do not confuse it with the prism refractive index; this is the refractive index of the particle.

So, it will be n_p^2 minus n_a^2 ; actually, it will be like difference in this region, otherwise it will be 0. So, when it is in the interaction region, there is a slight change in the refractive index. When there is a change in the refractive index, there will be change in the wave vector also. And let us say that, k_i and k_f are the wave vectors - initial and final wave vectors, and E_i and E_f are the initial and final electric fields.

So, k_i means wave vector before the particle was added and k_f is after the particle was added and same happens to the electric fields E_i and E_f .

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Evanescent Field Sensing

1st Order Perturbation approach

Wave equation before the perturbation: $\nabla \times \nabla \times E_i = k_i^2 \epsilon E_i$ ✓ (1)

The wave equation after the particle is added: $\nabla \times \nabla \times E_f = k_f^2 (\epsilon + \delta \epsilon) E_f$ ✓ (2)

Multiplying by E_i^* and integrating over the entire volume and subtracting yields:

$$(k_i^2 - k_f^2) \int_V E_f \cdot E_i^* dr = k_f^2 \int_V E_i^* \cdot \delta \epsilon E_f dr$$

Let us take first order perturbation approach. So, we already know that, the wave equation was $\nabla \times \nabla \times E$ right, from there we got the wave equation. So, wave equation is like this and after the particle is added, you will have this delta term, it is like this. Now we multiply these equations with E_i^* and integrate over the entire volume and then subtract from one, say this is 1 this is 2; you apply this in 1 and 2 and you get to this relation, ok.

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Up to the first order in: δk

$$\delta k \approx \frac{k_i}{2} \frac{\int_V \delta \epsilon E_i^* \cdot E_f dr}{\int_V \epsilon E_i^* \cdot E_i dr}$$

The shift in the wave vector is equal to the overlap integral normalized by the mode energy integral. Sensing in the evanescence region!

So, in the first order in delta k you arrive to this equation that, a small change in a delta k is this. So, the shift in the wave vector is equal to the overlap integral normalized by the mode energy integral sensing in the evanescence region. So, you can see that this is something which is called overlap integral. So, the larger the overlap integral, the better will be the sensitivity; because the better will be the change in the delta k.

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Main Parameters Affecting the Sensitivity

$\frac{\delta k}{k_i} = \frac{\delta n_{eff}}{n} = \frac{\delta \lambda}{\lambda} \approx -\frac{1}{2} \frac{\int_{int} \delta \epsilon E_i^* \cdot E_f dr}{\int_V \epsilon E_i^* \cdot E_i dr}$

Field Strength

$\frac{d\alpha}{\lambda}$

$FOM = \frac{S}{\Delta(FWHM)}$

Sensors geometry, nano-structures, localization, band gaps, material parameters

Interaction volume

Evanescence region, lateral propagation length, porosity

So, if I want to divide it by k i, we arrive to this equation; where the field strength gives the sensor geometry, nano structures, localization, band gaps, and material parameters.

While the interaction volume in the evanescence region depends on lateral propagation length and porosity also. So, if you have a sensor which has larger penetration depth, it will have larger sensitivity. So, here is a something, like you have two curves and a sensor is suppose, we are measuring R_p and we have two curves where this is one is λ_1 , this one is λ_2 and you see that λ_1 is smaller than λ_2 .

That means, I am talking about the evanescent wave sensor; so, you can see that at this λ_2 the sensitivity will be larger. If you remember, I told you that penetration depth d was proportional to λ . So, at λ_2 the penetration depth will be larger; that means, the sensitivity will be larger from here. So, it means that for red wavelengths, since the penetration depth is change, the interaction volumes will be larger. I mean the electromagnetic field will interact more with the analyte medium and that is why the sensitivity will be larger; while you have the smaller sensitivity at blue wavelengths, where the penetration depth is small and the interaction volume is also small.

So, this is something which is very important while designing a sensor. That is why when we have this evanescent wave sensors, now we try to move more and more towards the NIR region, because then the sensitivity becomes better. But then there are also other things which we need to take care of and one of that is detection accuracy; what happens to detection accuracy and all that.

So, when we move ahead, we will see that, it is not just the sensitivity it is also detection accuracy which is important and what happens to the detection accuracy at larger wavelengths. So, if it is also increasing, the overall performance of the sensor will not be affected; because the figure of merit was defined as the sensitivity upon $\Delta FWHM$.

So, at $\Delta FWHM$ is also large, then over all figure of merit of the sensor will not change. So, you will see, these things that what happens to the detection accuracy also.

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Absorption

When the refractive index is complex: $n = n_r \pm i\kappa$

κ -The extinction coefficient !. The sign depends on the choice: $e^{\pm i\kappa z}$

The relation to real and imaginary parts of the dielectric function:

$$\epsilon_r = n_r^2 - \kappa^2 \text{ and } \epsilon_i = 2n_r\kappa$$

A plane wave will be decaying as: $E(z) = \exp(ik_0 n_r z) \exp(-k_0 \kappa z)$

The intensity is decaying as: $|E|^2 = \exp(-2k_0 \kappa z)$

The absorption coefficient is defined as: $\alpha = 2k_0 \kappa = \frac{4\pi\kappa}{\lambda_0}$

$I_0 \longrightarrow \boxed{d} \longrightarrow I_0 \exp(-\alpha d)$ If we ignore reflections at the boundaries!

So, that was it for the TIR based sensors and evanescent wave-based sensors. In the next class, we will see the absorption and dispersion characteristics of conductors and then we will try to solve for plasmonic sensors.

Thank you.