Optical Sensors
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Lecture – 07
Basic Optics for Optical Sensing – V
Total Internal Reflection – Sensors

Welcome to the 7th lecture of Optical Sensors course. In the last couple of lectures, we discussed propagation of an electromagnetic wave at an interface and there we solved for the Fresnel coefficients. And, from there we saw that when the angle of incidence is equal to the Brewster angle, we get total transmission in p-polarized light and there is no reflection. We discussed the reasons behind that, and we also discussed how to use it for sensing applications.

That was the case when the wave was travelling from medium of a smaller refractive index to the medium with larger refractive index - like rarer to denser medium, say air to water. Now, we will discuss what happens when the electromagnetic wave travels otherwise. To say, that it moves from a denser medium to rarer medium, say water to air. So, today we are going to discuss what is called Total Internal Reflection and we will try to use them for sensing applications.

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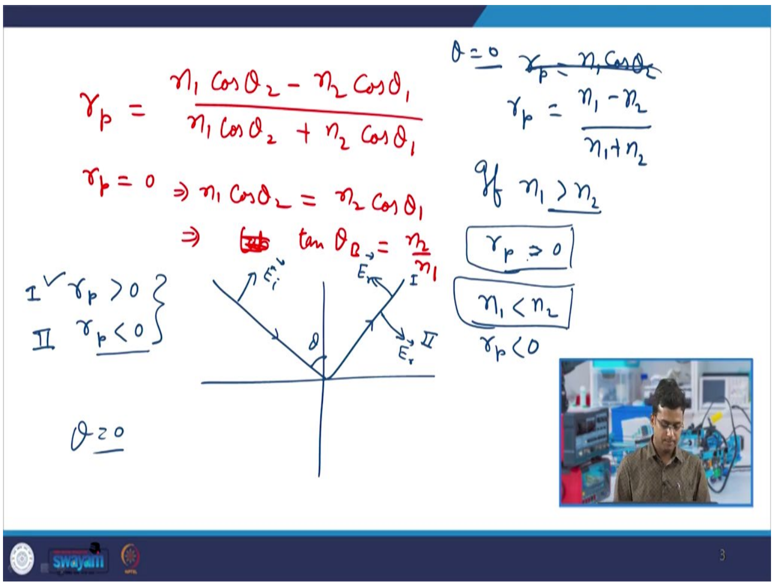

But, before we move ahead, I want to say that what we derived was r p is equal to n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1 - that is what we derived and we saw that r p was equal to 0 when n 1 cos theta 2 was equal to n 2 cos theta 1 and from that we came to the conclusion that tan theta B - that was the Brewster angle, n2 by n 1. So, we have arrived until here and now, we see that sometimes r p can be greater than 0 or r p can be smaller than 0. What does it mean? See, if it is greater than 0; that means, that for the electric field, the phase has not changed. When it is smaller than 0; that means, the direction of the electric field has changed. So, if my wave was incident at an angle theta with electric field pointing in this direction, this was E i.

In the reflected wave, if it is the same direction like this, then this case is valid. If r p is smaller than 0; that means, so this is case first, and in the second case, when it is smaller than 0, you will have electric field vector here. So, that is the meaning. I mean, the electric field changes its direction when you have r p less than 0. What it physically means? To understand it let us say that theta is equal to 0.

So, r p will be n 1 minus n 2 divided by n 1 plus n 2 for theta is equal to 0. If n 1 is greater than n 2, then r p will be equal to greater than 0; if n 1 is less than n 2, r p will be less than 0. It means that the electric field changes its direction when it is moving from a medium of smaller refractive index to a medium of larger refractive index. If the electromagnetic wave is moving from a medium of high refractive index to low refractive index, then r p is greater than 0 and there is no change in the sign of the electric field.

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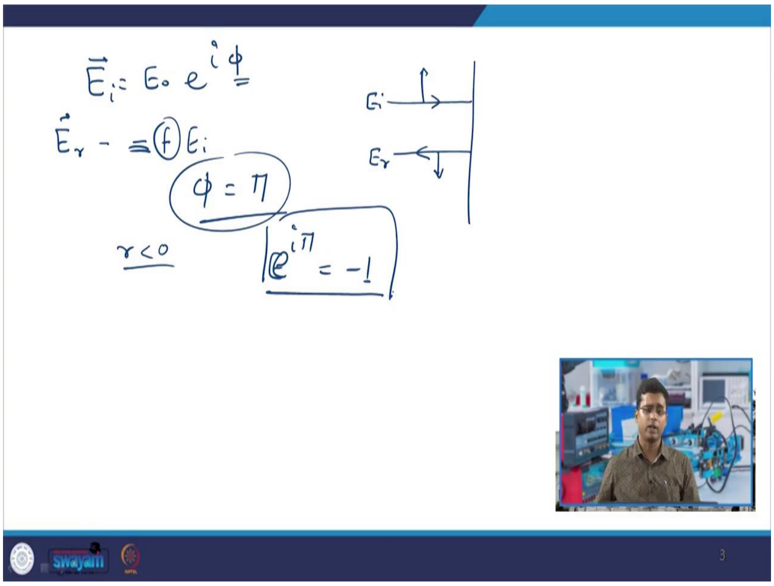

What happens to the electromagnetic wave? So, if you have an electromagnetic wave E is equal to E 0 e to power i phi, if it is E i and if E r is, say, minus of a function E I; why is this minus coming - because phi is equal to pi. So, it is like this. - this is an interface, wave is going in this direction moving here. So, the reflected once for r smaller than 0, what you have is that if the electric field is in this direction the reflected one will be having it in the other direction it is like this. So, if it is E i you have E r. So, this phase difference of pi is happening - e to power i pi is equal to minus 1; if it is moving in the other way then it will be positive.

So, we know until here. Nnow let us see what happens when we have a medium of high refractive index and the wave moves to a medium of low refractive index.

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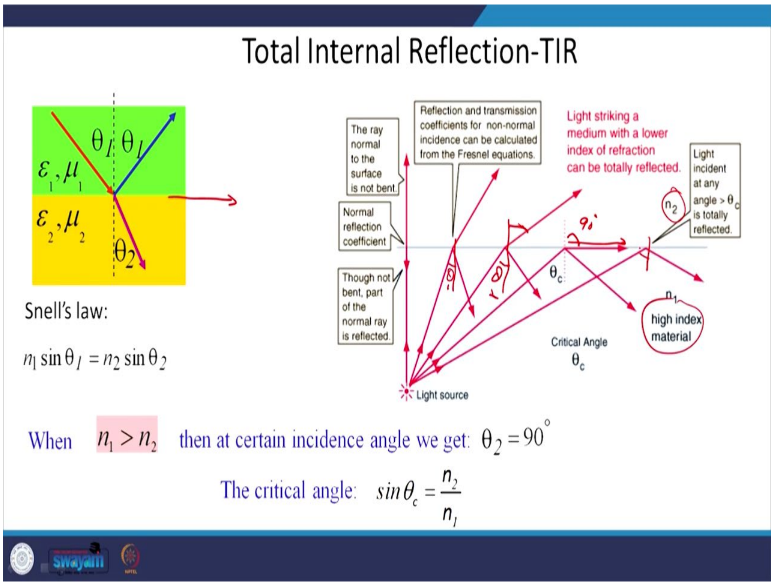

So, let us say that we have an interface where a wave incident at angle theta 1 gets refracted at angle theta 2 and it has dielectric constants epsilon 1 mu 1 and epsilon 2 mu 2. So, we see here - in this particular picture, that this is a high index material and this is a low index material and what happens. When the light source is here, a ray normal to the surface will not bend. So, it will go into the medium second medium, that is that angle theta is equal to 0 to the interface.

Now, if it goes at some angle, say theta 1, then what happens that a part of it will get get reflected into the same medium and a small part will go into the other medium that is reflection and refraction both happening there. Now, if you keep on increasing the angle say from theta 1 to theta 2, what happens that the ray bends more away from the normal. So, when you start increasing the angle of incidence, the ray refracted into the higher medium - this angle gets larger. So, it is moving away from the normal.

We arrive some particular condition of theta c, a critical angle - that is called critical angle; for this particular angle of incidence, all the rays which are incident at this interface will have an angle of 90 degree is refraction. But, it will be travelling along the interface into the second medium not in the first medium - it will be travelling into the second medium. When you further increase the angle of incidence greater than theta c then it comes back to the same medium - this is called total internal reflection or TIR.

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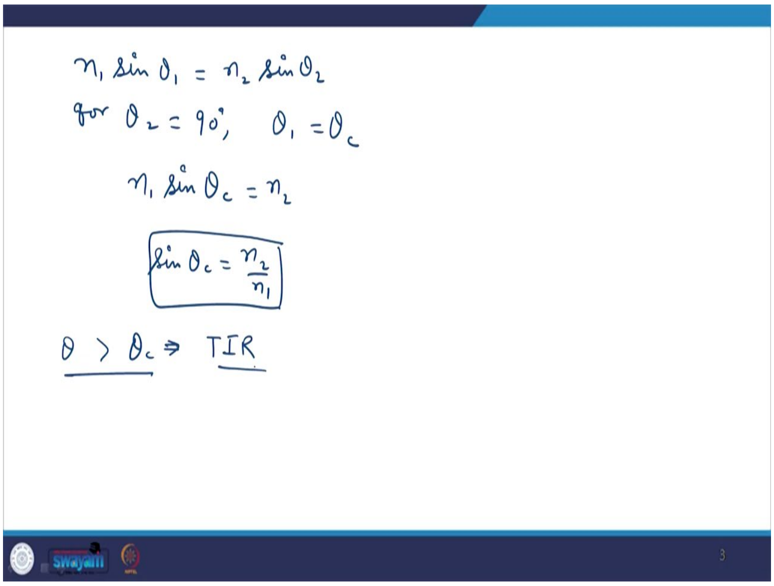

So, if you want to calculate critical angle, we know that the formula was n 1 sin theta 1 was equal to n 2 sin theta 2 and when for theta 2 is equal to 90 degree, theta 1 is equal to theta c. So, what we get is n 1 sin theta c is equal to n 2 and sin theta c is equal to n 2 by n 1. This is the condition for critical angle. So, if theta is greater than theta c, this implies that there will TIR - total internal reflection. We will see happens to the that. So, we arrive to this condition when we have critical angle.

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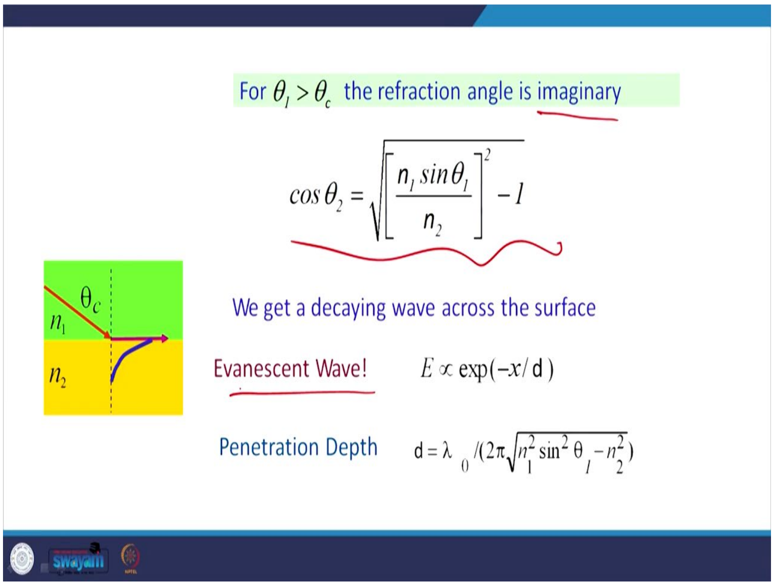

And, for theta greater than theta c the reflection angle is imaginary. How? Let us try to see it.

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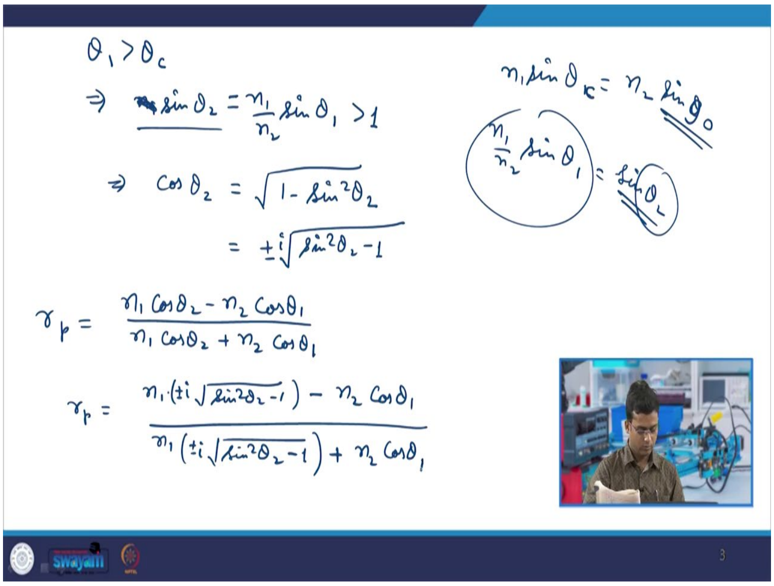

We have arrived here. So, let us put n 1 sin theta 1 greater than theta c. This implies that n 1 sin theta 2 is equal to n 1 by n 2 sin theta 1 is greater than 1, because this should be larger than 1. Here we do not have n 1 because n 2 is here. So, cos theta 2 will be under root 1 minus sin square theta 2 that is equal to plus minus i sin square theta 2 minus 1.

So, in this case when n 1 sin theta 1 was n 2 sin 90, theta 1 was equal to theta c sin 90. So, sin 90 is 1. So, if I write n 1 by n 2 sin theta 1 that is equal to sin theta 2; so, it was 90 for 90 it was 1. For angles more than 90, means if theta 2 is larger, than it has to be larger. If it has to be larger, then it is greater than 1. But we know that the value of sin theta cannot be greater than 1, but in this case, it is it - means that theta 2 is imaginary. So, that is what we arrive here.

So, you have theta 2 - this value, which is imaginary. What happens to the electric field? Let us try to see. So, this happened to the angle. The reflection coefficient r p was equal to n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1. Let us try to see what happens to each of these. So, r p becomes equal to n 1 into plus minus I under root sin square theta 2 minus 1 minus n 2 cos theta 1 divided by n 1 into plus minus i under root sin square theta 2 minus 1 plus n2 cos theta 1.

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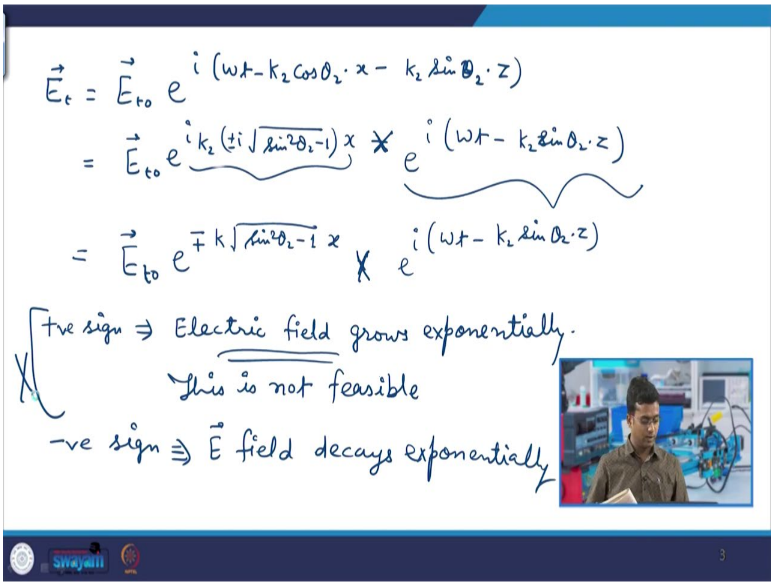

Then, this term is complex - refraction coefficient is complex. What it means, we will come to that. Let us try to see what happens to the transmitted field. E t, if you remember, was E t0 e to power i omega t minus k 2 cos theta 2 into x, minus k 2 sin theta 2 into z - this was our electric field. You put the value of cos theta 2, you will have E t 0 e to power i; you can take k 2 - both are k 2 so, I can take k 2 common; we will have plus minus i under root sin square theta 2 minus 1 into x into e to power i, we can separate it, i omega t minus k 2 sin theta 2 into z.

So, we have two components, since this has plus minus i, I separated it out and now you have one term which is similar to the previous case and then you have different term. Let us simplify it further: E t 0 e to the power plus minus I, that is minus 1. So, it will be minus plus k under root sin square theta 2 minus 1 into x into this thing - e to power i omega t minus k 2 sin theta 2 into z.

Out of these two signs - it can have a positive sign or negative sign, what it means? Positive sign means that the electric field is growing exponentially, which means - this is not feasible. Why? because, the electric field you have incident - it can happen only in a gain medium while what you have is a simple medium and you do not have a medium where the electric field is growing exponentially like that. So, it is not feasible. So, what is feasible?

The negative sign; this means the E field decays exponentially. So, there are two conditions one like it is can grow or it can decay, but this is not feasible. So, you will not take this into consideration, you take only this into consideration.

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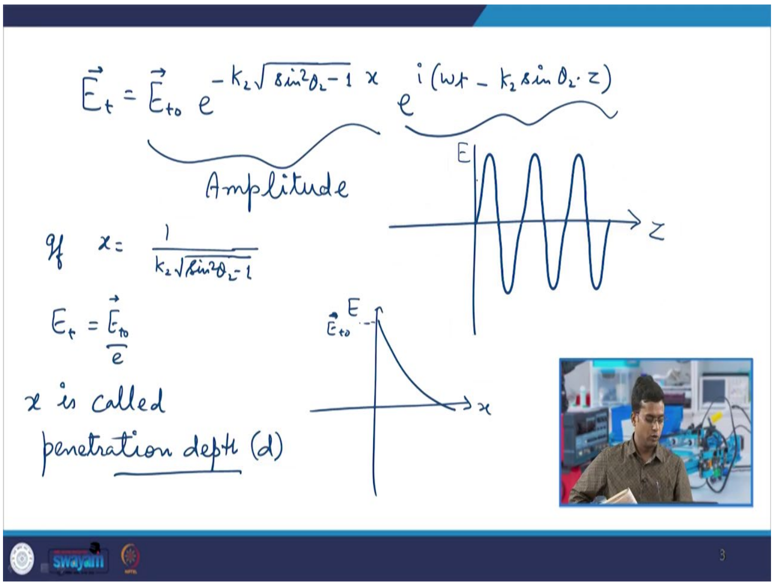

So, what we get is E t is equal to E t0 e to the power minus k 2 under root sin square theta 2 sin square theta 2 minus 1 into x, e to the power i omega t minus k 2 sign theta 2 into z. So, we get this solution. If you remember, our consideration was like this. Here it was x, and this was z, and what we see is that now we have a wave which is a plane wave. So, you have a plane wave traveling in the z direction which has an amplitude - this is amplitude, which is decaying exponentially.

So, it means that we have a wave which travels like this and its amplitude decays. So, its amplitude if you plot the amplitude with x the electric field E it decays exponentially. So, here you will have E t 0 value and then it is start decreasing. So, if x is equal to 1 upon k 2 sin square theta 2 minus 1 then E t is equal to E t0 by e; then x is called penetration depth that we represent as ‘d’ or ‘lambda’.

So, we get a wave that is called evanescent wave. So, we get a wave which is traveling along the interface and when it travels along the interface its amplitude decays exponentially into the media and the penetration depth is given by this relation. We will arrive to this relation. We arrived here that to the value of x where the amplitude decays to 1 by e is called penetration depth.

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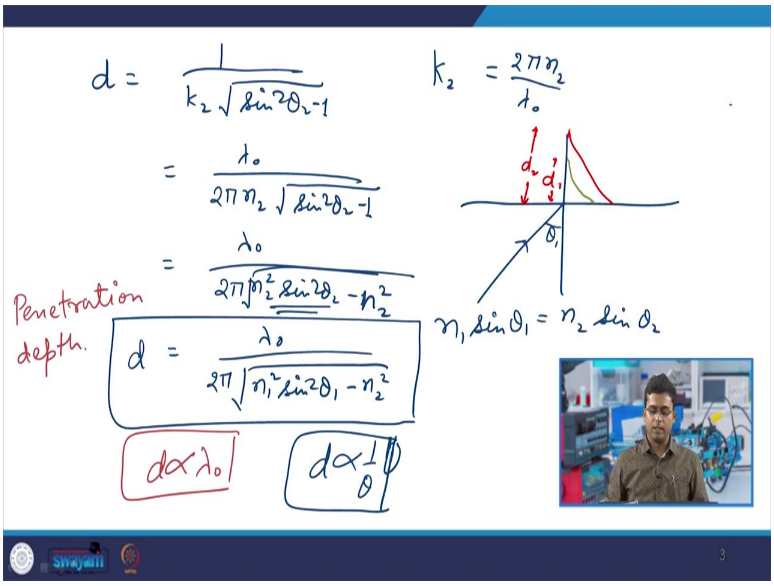

And, this is given by d is equal to 1 upon k 2 under root sin square theta 2 minus 1. Let us try to solve it further. I have k 2 is equal to 2 pi n 2 by lambda 0. So, we will have lambda 0 upon 2 pi n 2 sin square theta 2 minus 1. You take n 2 inside, you will have lambda 0 by 2 pi under root n 2 square sin square theta 2 minus n 2 square.

Now, you know that we do not need to know what is theta 2, because if the wave is going like this at theta 1 and you want to know the penetration depth you do not care what is theta 2. So, what we do is that n 1 sin theta 1 is equal to n 2 sin theta 2 - we substitute here. Here you have n 2 square. So, you get lambda 0 by 2 pi under root n 1 square sin square theta 1 minus n 2 square. This is called penetration depth.

You see here that d is proportional to lambda 0 - that is first thing; second thing is d is inversely proportional to f (theta). So, there are two important implication of this relation. One is that d is proportional to lambda 0, what does it mean? It means that the penetration depth will increase for light of larger wavelengths..

So, if you have the penetration depth like this and if you take a red one which has larger wavelength, the penetration depth will be larger for the same angle of incidence. So, if it was d 1 for green one, d 2 will be for red one. So, for red one it will be larger, that way you can penetrate more and more into the medium when you increase the wavelength.

This can also happen with theta. So, for smaller value of theta when you are very close to the critical angle you will have large penetration depth - close to infinite. As you increase theta from critical angle to 90 degrees, it will start decreasing slowly. So, since we arrived here, now let us see the what we can do with it.

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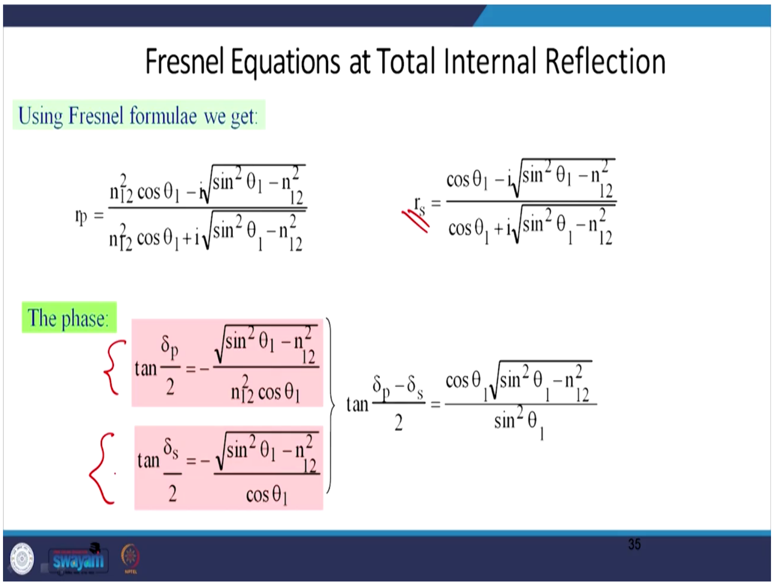

Also, the Fresnel equations at the total internal reflection change. So, you can always calculate what is r p that we did already, and you can also see what happens to r s - that is the s-polarized light; you can also what happens to the phase of the wave, how it changes. So, these are the relations you should derive at home - that is the homework for you.

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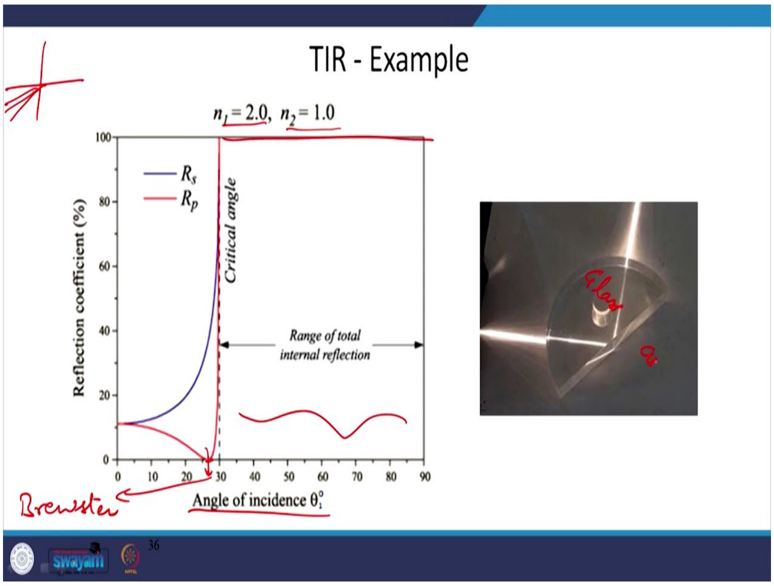

Let us see what happens. So, if you plot it - the reflection coefficient with an increase in angle of incidence, if you keep on increasing the angle of incidence, what happens in total? We saw that for r s, there was no dip - that there was no Brewster angle and when we arrive to p-polarized light, we see that there is a dip and that is due to Brewster angle. And, we keep on increasing the angle of incidence then there arrives the condition that is called total internal reflection.

Critical angle – this is the critical angle for this configuration. So, when n 1 is equal to 2 and n 2 is equal to 1 and after the critical angle there is no light. So, there is 100 percent reflection. There is no transmission. It is like this, you get 100 percent. So, the reflection becomes maximum.

This is this is a region, which you call the region of total internal reflection. So, you can see, for example in this image, that when the light is falling on this, this is because of total internal reflection from, this is suppose a glass and this is air, and you can see that the change it all gets reflected for angles larger than the critical angle.

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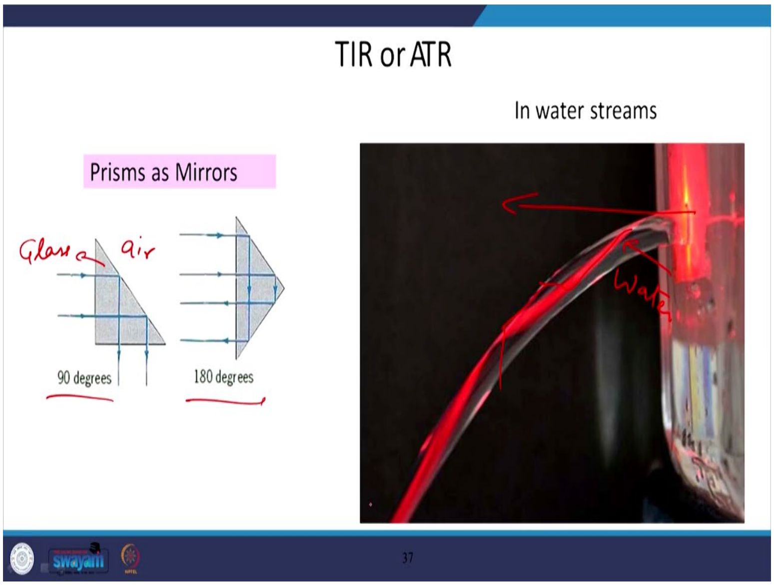

It can also be used as mirror. You know this interface is - again this is glass and this is air and you can see that if it comes to ninety degrees it can have a phase change of 180 degrees. In water streams also, light gets trapped because of the total internal reflection.

This is a stream of water and then you have air surrounding this water and a laser light is incident from the back side and it rather penetrating like this, it gets bent, again and again and again and again. So, this is called total internal reflection or attenuated total reflection. Why attenuated I will come to that.

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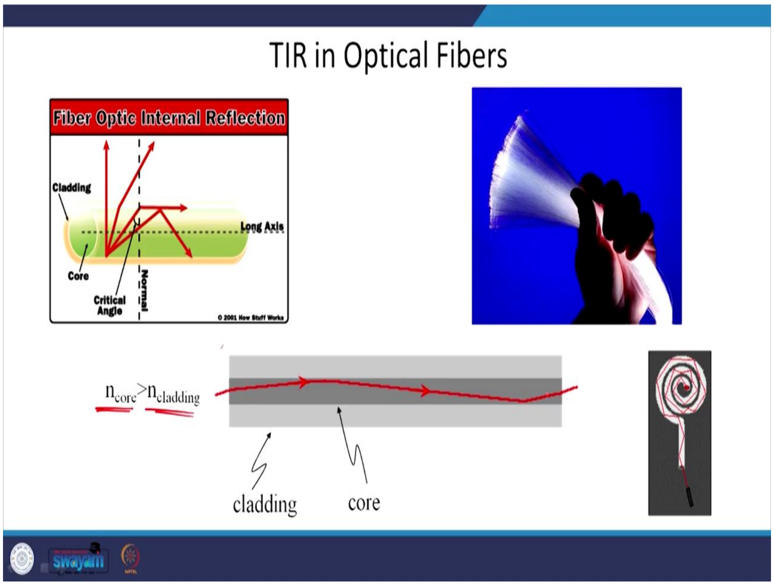

It can also be used in optical fibers. An optical fiber is basically a cylinder of glass which is surrounded by another glass which has a refractive index slightly smaller than the refractive index of the core glass. For example, this is a core and then surrounding it is a cladding and, the refractive index of the core is slightly larger than the refractive index of the cladding. One can see that the rays again which have an angle larger than the critical angle because of this slight change in the refractive index from higher to lower, they get total internally reflected.

All the rays other than this are just radiated out. So, these are called radiation modes of the fiber, while those which get guided into it are called guided modes of the optical fiber. So, light can travel through an optical fiber because of this basic principle of total internal reflection. You can see a bundle of optical fiber which can send light from here to remote places and it can also be bent. So, that is a very beautiful use of optical fibers in communication these days.

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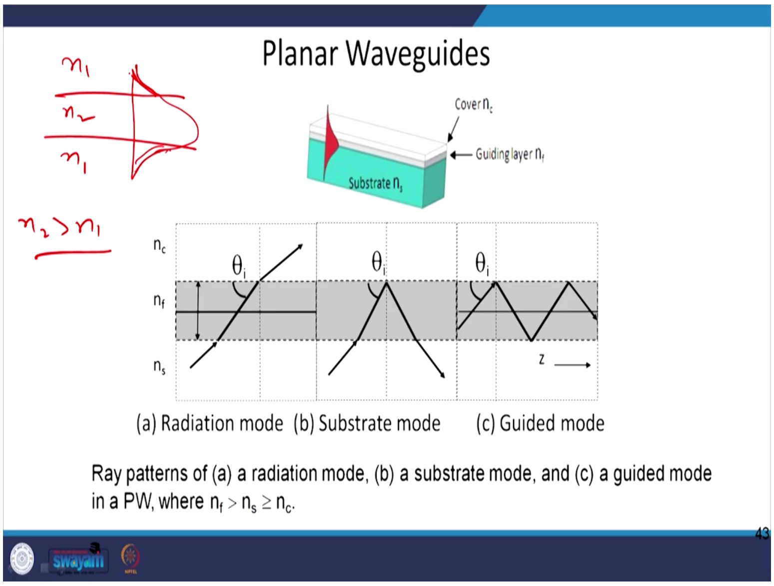

We can also make planner wave guides where because of the total internal reflection suppose it is a waveguide where you have n 1 and 2, n 2 is greater than n 1. So, for angles larger than the critical angle, you can get the light in it and then it will have evanescent field which is travelling and here you will have a mode while here you will have a field which is decaying exponentially because of the evanescent wave.

So, that is all for the present lecture. We will see how to use it for sensing in the next lecture.

Thank you.