Optical Sensors

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Lecture – 06
Basic Optics for Optical Sensing – IV
Polarization by reflection – Brewster angle sensor, Total Internal Reflection

Welcome to the 6th lecture of Optical Sensors course. In the last lecture, we discussed the propagation of an electromagnetic wave at an interface and we solved for the reflection and transmission coefficients for a p-polarized wave; p-polarized means TM polarized wave where, the electric field components are in the plane of incidence.

And, I gave a homework for the waves, which have electric field component perpendicular to the plane of incidence. So, I hope that you have solved for the reflection and transmission coefficients for that. Now, we move ahead and we discuss certain properties associated with this and want to see what happens to it.

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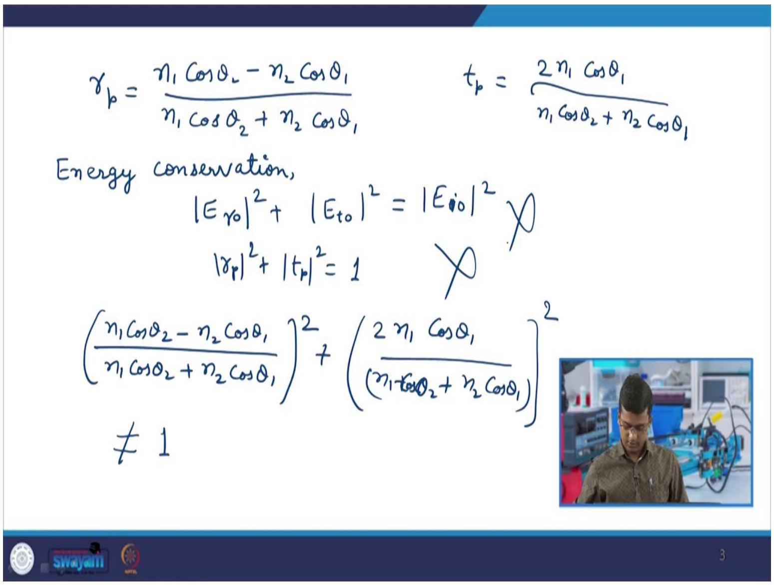

So, let us see the reflection coefficient, that was n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1, this was the reflection coefficient. And the transmission coefficient t p was equal to 2 n 1 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1. So, we had these two.

Now, if we solve the energy conservation - according to energy conservation, E r 0 square plus mod E t 0 square should be equal to E i 0 square; which means that r p square plus t p square is equal to 1. Let us try to solve it. So, it becomes like n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus square plus 2 n 1 cos theta 1 n 1 cos theta 2 plus n 2 cos theta 1, this all squared. So, what do you get? You will get - this denominator is the same.

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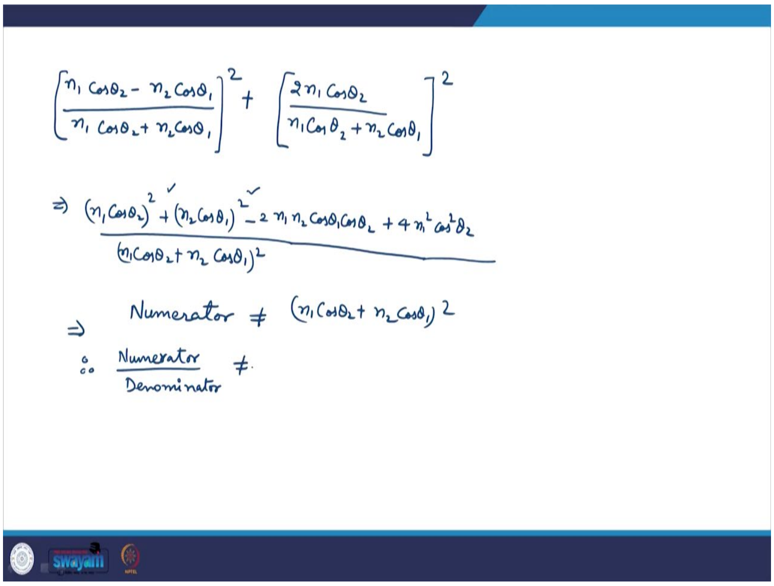

So, you can have n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus 2 cos theta 1 whole square plus 2 n 1 cos theta 2 divided by cos theta 2 plus n 2 cos theta 1 square. So, you can see that - in the denominator you have n 1 cos theta 2 plus n 2 cos theta 1 whole square. And here you have n 1 cos theta 2 square plus n 2 cos theta 1 square minus 2 n 1 n 2 cos theta 1 cos theta 2 plus 4 n 1 square cos square theta 2.

So, numerator - you can see that is not equal to n 1 cos theta 2 plus n 2 cos theta 1 square; because you have this - a square term, b square term, but you do not have plus 2 ab term so, this is not equal to this. Hence, numerator divided by denominator is not equal to 1 and you see that this does not come equal to 1. So, where are we wrong? I mean there should be something wrong with it. It does not come 1. So, what is the problem? Why are we not getting this? So, something is wrong here. The energy of an electromagnetic wave is given by the Poynting vector. So, we have to we have to solve for the Poynting vector. So, let us solve for the Poynting vector.

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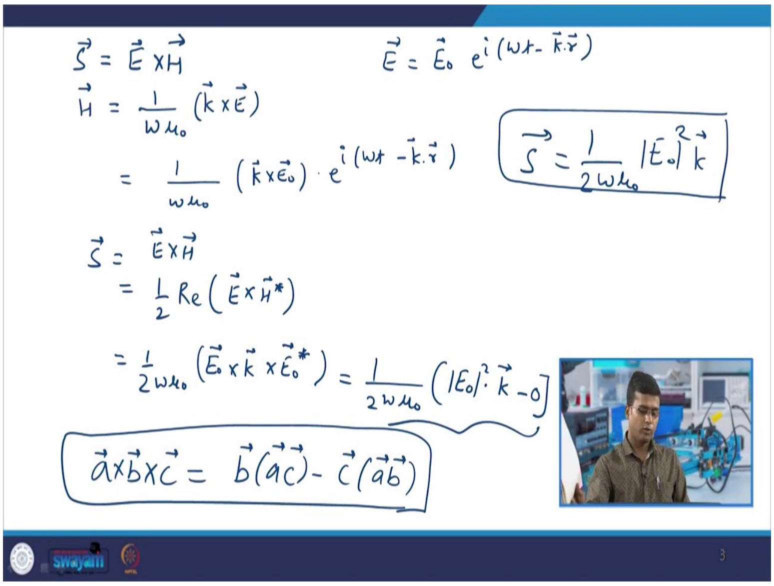

So, the Poynting vector S; S was given by E cross H and H was given by 1 upon omega mu 0, k cross and E and E is equal to E 0 e to power i omega t minus k dot r, you know this. So, this can be written as 1 upon omega mu 0 k cross E 0 into e to power i omega t minus k dot r, right. What does it give?

So, if we want to calculate S, that is E cross H, that is equal to half of real of E cross H star - you can solve it for this and that is 1 upon 2 omega mu 0 E 0 cross k cross E 0 star that is 1 upon 2 omega mu 0 mod E 0 square into k minus 0. You know this identity, I told you about this identity. a cross b cross c was equal to b ac minus c ab. So, you use this identity, I told you on that day to prove it. So, go back and prove it and try to see what happens. So, we end up with this relation.

So, that is S is equal to 1 upon 2 omega mu 0 mod E square k. S gives the direction of flow of energy. So, we solve for it and, we go to see what happens.

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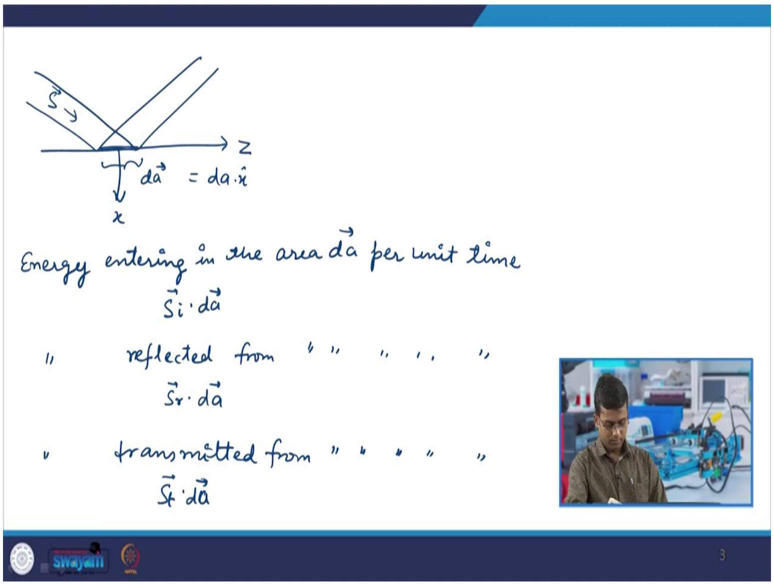

So, this is our interface. A beam is coming like this, it is getting reflected from here and this is the area. So, it was x direction if you remember, this is z direction this is S now flowing in this direction and this is da, the area where it is falling that is da into x. So, energy entering in the area da per unit time - S i dot da. Similarly, energy reflected from area da per unit time is - S r dot da. Energy transmitted from area da per unit time is S t dot da.

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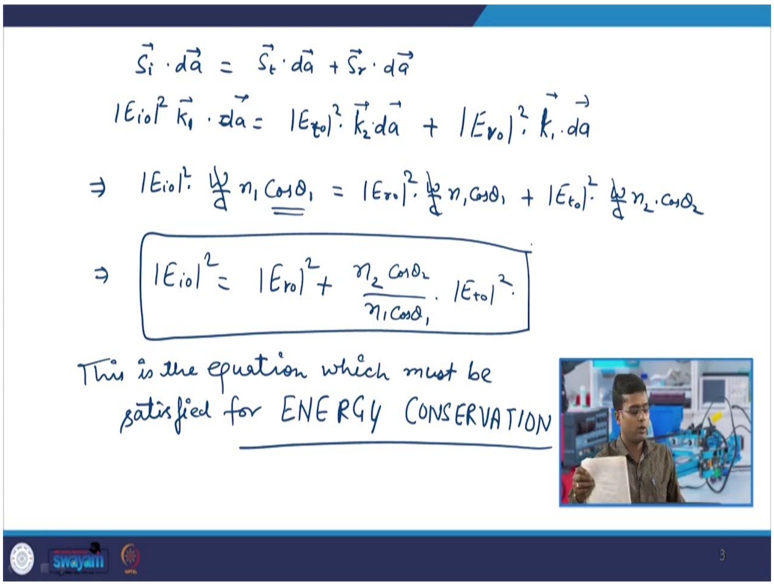

So, in principle, S i dot d a should be equal to S t dot d a plus S r dot da; that is mod E i 0 square k 1, k 1 is equal to into da is equal to mod E r 0 square into k 2, k 2 it is also k 1 no, this one is k 2 dot da r t actually plus mod E r 0 square dot k 1 dot da. k is small actually sometimes I write it seems like capital, but it is small k.

So, you have actually mod E i 0 square into k 1; k 1 is omega by c into n 1 cos theta 1, you take the theta 1 component is equal to mod E r 0 square omega by c. Again n 1 cos theta 1 plus mod E t 0 square omega by c now n 2 cos theta 2; because it was a vector, that is why you have cos theta 2 and I have removed da now, ok.

So, we arrive to this relation. Omega by c cancels out from here. Now, you have mod E i 0 square is equal to mod E r 0 square plus n 2 cos theta 2 divided by n 1 cos theta 1 into mod E t 0 square. This is the equation, which needs to be satisfied for energy conservation. If you simply want to say that mod r p square and mod t p square is equal to 1, that is wrong interpretation. You must have it in terms of the Poynting vector because that is the vector which gives you the energy flow. And from here we arrive to this relation which gives you the actual equation which must be satisfied to prove the energy conservation.

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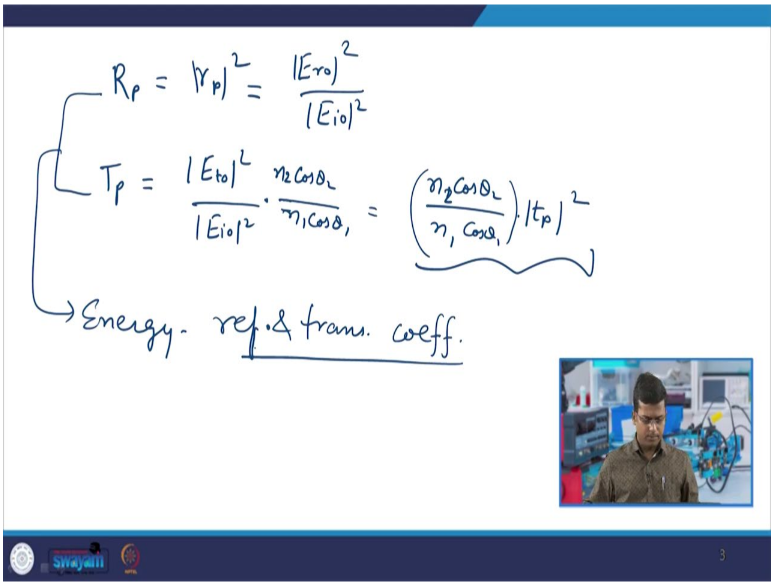

So, from her if you want to calculate what are the R p - is equal to simply mod r p square and that is equal to E r 0 squared divided by E io square and T p - is equal to which is mod E t 0 square divided by E i 0 square into n 2 cos theta 2 divided by n 1 cos theta 1 and this is into n 2 cos theta 2 divided by n 1 cos theta 1 into mod t p square; this is the correction. So, it is not like that mod t p square is not equal to T p. You have to have this coefficient also.

So, now we have arrived here and we know the energy term. These are called energy reflection and transmission coefficients reflection and transmission coefficients. If you want to have total energy you have to get sum of it, ok. So, now, we know that if an electromagnetic wave is incident at an interface of two media at an oblique angle incidence, then its reflection and transmission is governed by these Fresnel equations and from there you can calculate the amplitude of the reflected and transmitted wave.

And, from all this exercise which we have done, you can arrive to the condition where you can calculate the energy transmission and reflection coefficients. And, let us see what happens at certain special cases when this reflection and transmission goes 0.

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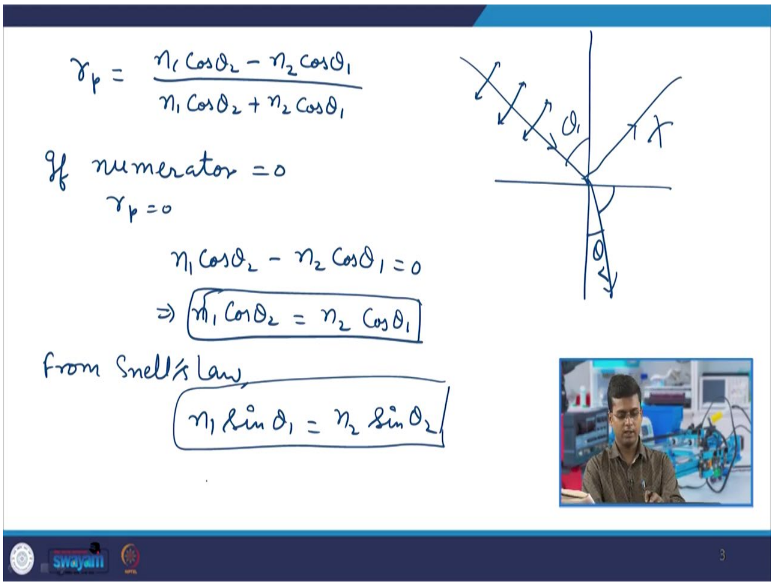

So, we know that r p is equal to n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1. If numerator is equal to 0, then r p is equal to 0. What does it mean? It means that, for p-polarized light: p -polarized light means that the electric field component which is in the plane of the incident light will not be transmitted. So, there would not be any reflection for this electric field.

Let us see what happens. So, n 1 cos theta 2 minus n 2 cos theta 1 becomes equal to 0 this means n 1 cos theta 2 is equal to n 2 cos theta 1 - this is one condition. From Snell's law, we know that n 1 sin theta 1 equal to n 2 sin theta 2. So, if we divide equation 1 to equation 2, what will happen?

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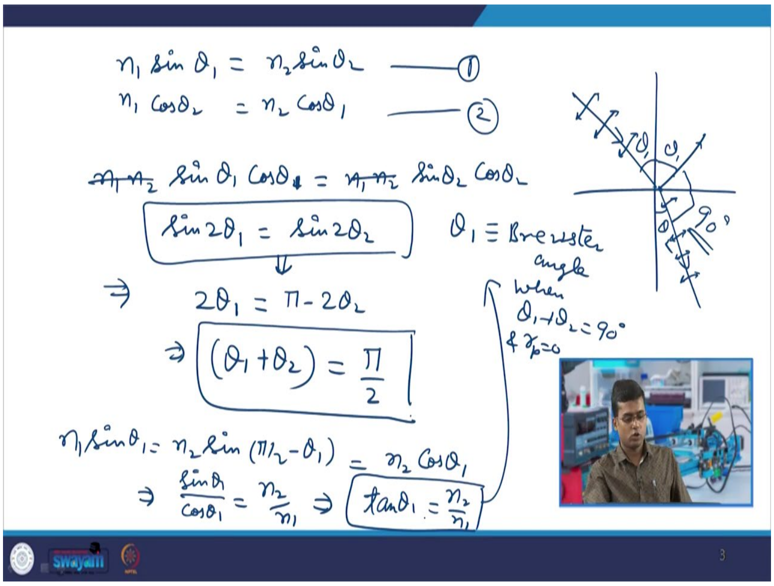

So, let us see if we divide it then what will you get. Again, I am writing n 1 sin theta 1 is equal to n 2 sin theta 2 and you have n 1 cos theta 2 equal to n 2 cos theta 1. So, you can have it like this is, suppose, equation 1, this is equation 2. You can have n 1 n 2 sin theta 1 cos theta 2 is equal to n 1 n 2 sin theta 2 cos theta 2. So, n 1 n 2 n cancels out - you have sin 2 theta sin sine 2 theta 1 is equal to sin 2 theta 2; when is it this possible? This implies that, 2 theta 1 is equal to pi minus 2 theta 2 to satisfy this. This implies that theta 1 plus theta 2 is equal to pi by 2.

What does it mean? It means that, this angle theta 2 when light was incident - theta 1 plus theta 2 should be 90 degree. So, if you have this wave which is traveling and if you have theta 1, you have theta 2 then you have all the light like this which is coming here, all the light gets transmitted - no light gets reflected. So, if it is theta 1, basically this angle should be 90 degree. So, theta 1 plus theta 2 is equal to pi by 2. If you have n 1 sin theta 1 is equal to n 2 sin pi by 2 minus theta 1 then; that means, is equal to n 2 cos theta 1.

So, it becomes like sin theta 1 by cos theta 1 is equal to n 2 by n 1; this implies that tan theta 1 is equal to n 2 by n 1. This theta 1 is called Brewster angle. So, this is called theta 1 is Brewster angle, when theta 1 plus theta 2 is equal to 90 degree and r p is equal to 0. So, when r p is equal to 0 it is satisfies this condition - this is Brewster's condition and it is called Brewster's angle.

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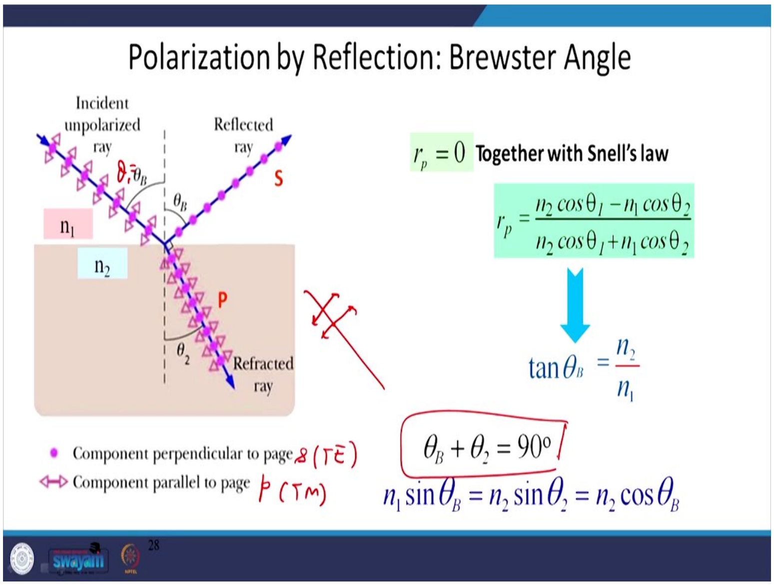

So, what happens? At Brewster angle r p goes equal to 0 and you see that we can have this ray like this. You have electric field components in the plane and electric field components out of the plane. So, these ones - components perpendicular to the page and components parallel to the page. So, this one is parallel to the page - this is called p polarization, and this is called s polarization or TE. And this is TM - Transverse Magnetic.

So, you see that when the angle of incidence theta 1 is equal to theta B - the Brewster angle, all the p polarized light gets transmitted, it does not get reflected. Only s polarization gets reflected. And this was the condition we derived, and it was equal to 90 degree.

So, why it happens so? What is the reason behind this kind of transmission - complete transmission of light at Brewster angle, if you have p polarized light then all the light goes into the transmission medium: does not reflect back into the same medium. And this happens only when, this angle is 90 degree - means theta B plus theta 2 theta 1 plus theta

2 is equal to 90 degree. If this is 90 degree this is totally 180 degree from here to here. So, this is 90 degree what is the reason? So, the here is the Physics.

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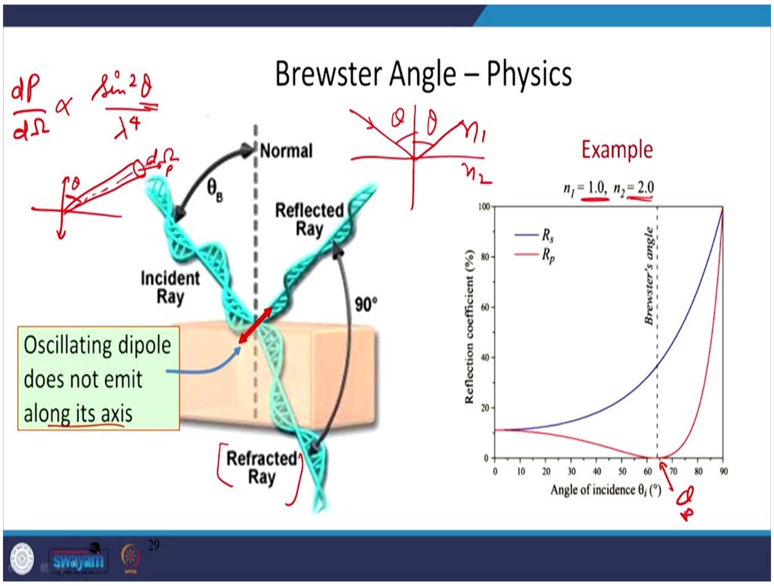

You know that when light is incident on any medium I told you that how it propagates. So, when light gets transmitted through a medium what happens actually, that it excites dipoles. And the dipole oscillations: we discussed that power d P by d omega was proportional to sin square theta by lambda to the power 4, where this theta was the angle made from the axis of the dipole. So, if this was the axis of the dipole and you want to measure here at an angle theta and this was the solid angle if you remember d omega and wanted to see how much power it is here it was making from the axis actually.

So, at theta is equal to 0, there is no power radiated and the maximum radiation goes in 90 degree to the dipole oscillations. So, if it is incident in such a way that reflected ray and transmitted refracted ray - these make 90 degree angle; that means, that the dipole is oscillating in such a way that it does not produce any radiation in this direction. So, all the light comes in that direction. That is a basic physics behind this. So, an oscillating dipole does not emit along its axis, all the light is going, that is why, into that ray. So, this is something very important. This is a very basic concept and most of the people forget; I also sometime forgot about this.

So, how does it look like when you have an interface, say, from n 1 to n 2 light is going like this and you increase the angle theta, you keep on increasing the angle what will happen? You want to measure the reflected light. That is how you perform the experiment for Brewster angle. This is also theta 1.

So, what happens actually that, when you start from 0 degree of angle of incidence for this particular condition when you have n 1 is equal to 1 and n 2 is equal to 2, the energy reflection coefficients R s and R p have been plotted here. And you can see that R s goes like - this this blue curve; while R p shows a dip here and this is because of the Brewster angle. So, at the Brewster angle theta B all the light gets transmitted and the reflection is 0.

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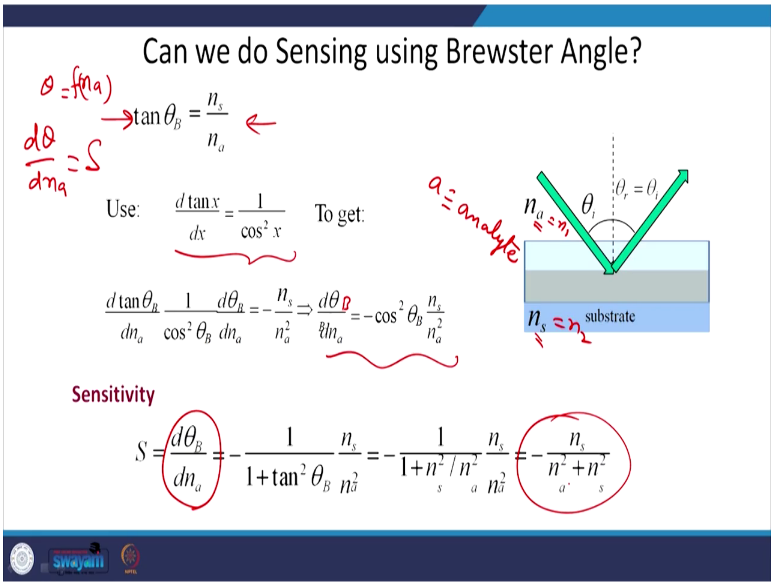

So, can we do sensing using Brewster angle. How?

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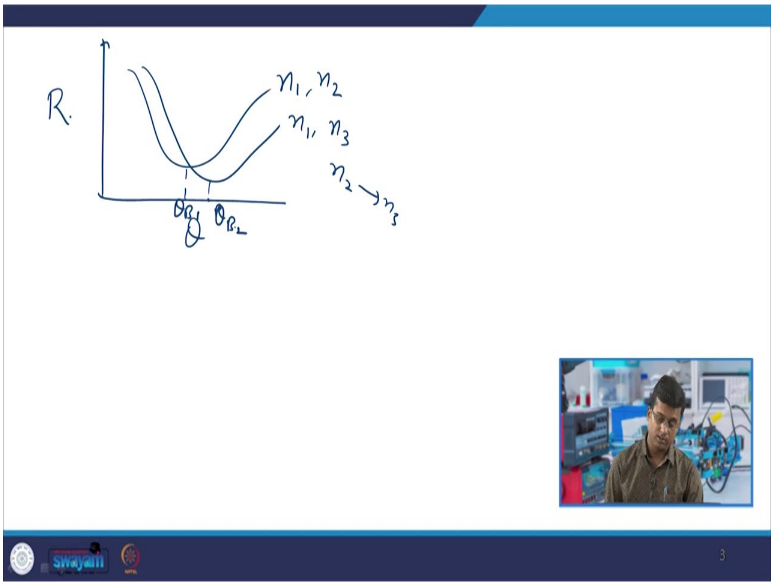

How do I say that - if you remember that when we were discussing sensors, it was shown that it has certain characteristic. Here I have theta and here I have reflected power and I want to see what happens? Say, suppose this is condition for n 1, n 2. Now, I just changed from n 2 to n 3. Does it shift? Can I measure the shift?

The basic idea behind that is can we use it for sensing applications? That is the basic question and the answer is yes, we can use it. So, tan theta B was given by n s by n a. Now, I have put it n s was actually n 2 and this is n 1. And now why I write a and s is like - s is for substrate and n a is for analyte, that is how I write. Analyte is the medium which we are trying to sense. So, I have already a substrate, suppose I have glass slide; I put a drop of water and I want to see the Brewster angle. If you have this relation, we know that if we are measuring theta as a function of n a, then d theta by d n a will be the sensitivity. We want to calculate the sensitivity.

So, we can use it for sensing as I told you here that if we move from n 2 to n 3, this Brewster angle theta B 1 will be theta B 2. So, it can be done in a principle. But if it works as a sensor, what will be its sensitivity? That is the question. So, what you do is that, you take the derivative of this and derivative is given by this relation. You know that d tan x by dx is equal to 1 by cos square x that is sec square x. So, you can write d tan theta by dn a into 1 upon cos square theta B d theta by dn a.

So, you arrive to this relation. Somehow these have changed, actually, this has to be theta B. And what we see is that we get this relation ultimately that the sensitivity depends only on the refractive indices n s and n a.

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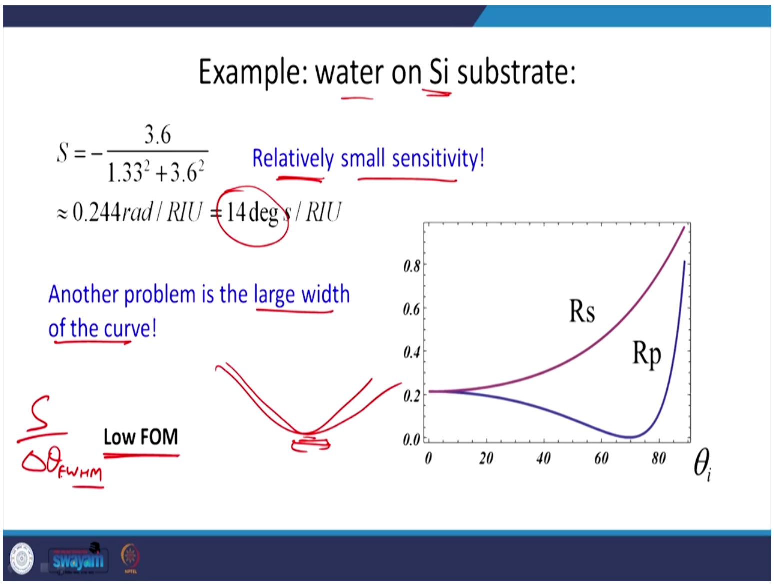

So, to take an example, we have put water on silicon substrate and you know that the sensitivity is around fourteen degrees per reflective index unit, which is very small. I will show you that the sensors have very high sensitivity. This is very poor sensitivity. Also there is a problem with the large width of the curve. I told you that if there is a very small change, suppose this curve is like this and then you have another curve which is like this, it is very difficult to find what is the change in the curve parameter due to the shift in the refractive index.

So, it has very small figure of minute. I hope that you remember how you define figure of merit - that was sensitivity was divided by delta, here it will be delta theta. So, if the delta theta means FWH:, the theta FWHM is broad that means the figure of merit will be poor.

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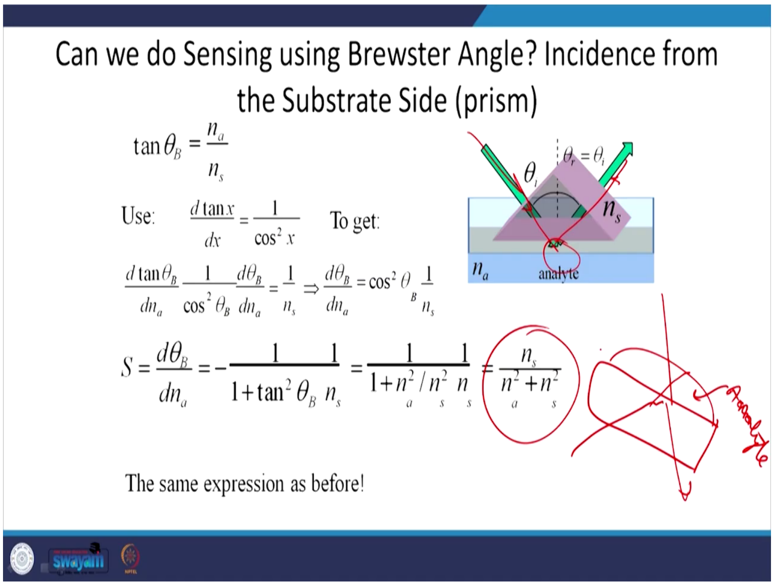

If we want to use it other way around like on a substrate we were putting a drop and we were shining from here. So, here was the analyte, which you do not prefer most of the time. Can we put light from here and see what happens and measure the reflectivity here? That can be attained by using a prism.

So, the analyte is from the other side and you shine from the prism side that goes here gets reflected from here and here is your analyte. So, you arrive to the same equation - only there is a minus sign in the previous one. So, the sensitivity is the same, it does not make any difference.

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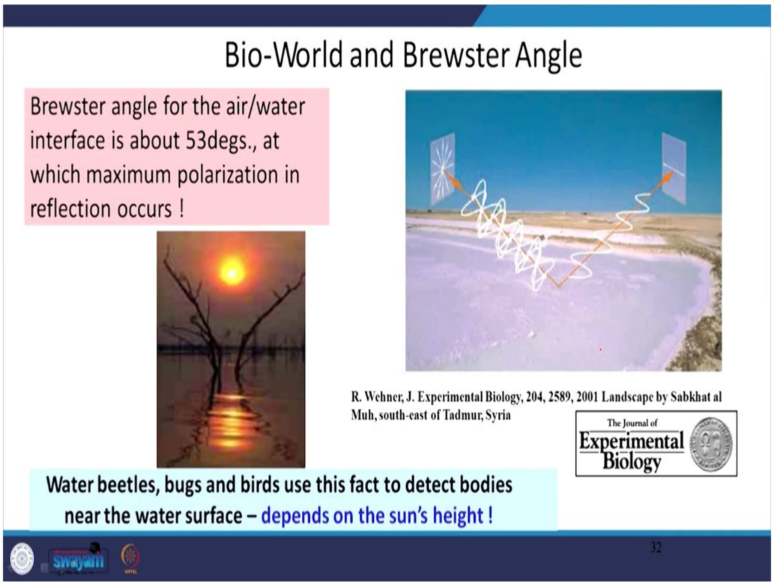

Brewster angle for the air water interface is about 53.5 degrees at which maximum polarization by reflection occurs. So, you can see that when the angle is large, say, during the morning or evening times you have maximum polarization in reflection. So, sometimes you do not see, even, reflection only if you are using a p-polarized light.

Similarly, what can be shown here is that when you are looking at a grazing angle, at an angle about 53 degrees or so, you see only small reflections coming from. So, that is used basically in photography, that if you want to avoid speckles, you use these kinds of filters at this particular angle. Water beetles, bugs and birds use this fact to detect bodies near the water surface and, you can cut down the glare from reflections at Brewster angle.

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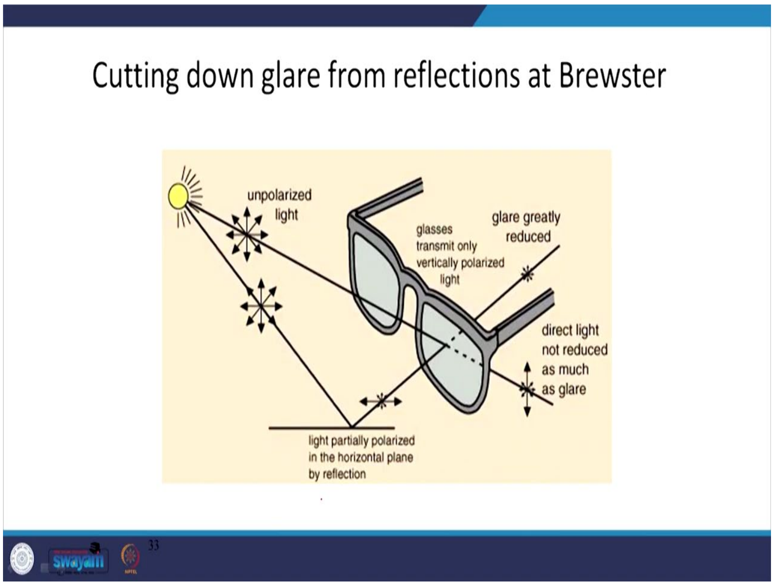

So, we studied polarization by reflection at Brewster angle and the potential use and limitations of Brewster angle for sensing applications we are presented.

Thank you.