Optical Sensors

Prof. Sachin Kumar Srivastava
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 05
Basic Optics for Optical Sensing- III
Reflection and Transmission at interface: Fresnel Equations

Welcome to the 5th lecture of Optical Sensors course. In the third and fourth lectures we discussed, what are electromagnetic waves and we also saw that when the wave is propagating in a certain direction, the electric and magnetic field components are perpendicular to it and they are also orthogonal. So, it becomes kind of right angle triad where electric field magnetic field and the direction of propagation are perpendicular to each other.

And, we saw that when a wave travels in a certain direction, we can have 6 components of electric and magnetic fields in total. And, out of these 6 components, we can have two sets which are called transverse electric and transverse magnetic polarizations and this makes our life easy because it is very important to see, that when you have 6 components and you solve for vector equations -you need to have 6 equations right.

Here you can reduce it to 2 by considering 2 sets of solutions and these are actually orthogonal polarization states, which we saw, and we also saw that any random polarization can be resolved in these 2 parts. Now, we move ahead and see what happens, when these waves travel from one medium to the other medium.

So, today we are going to discuss the reflection and transmission at interfaces, then solve for Fresnel equations. You know, the electromagnetic waves in free space that is the most simple case, where we consider that there is no charge, no magnetization, no polarization and no currents. So, all were 0.

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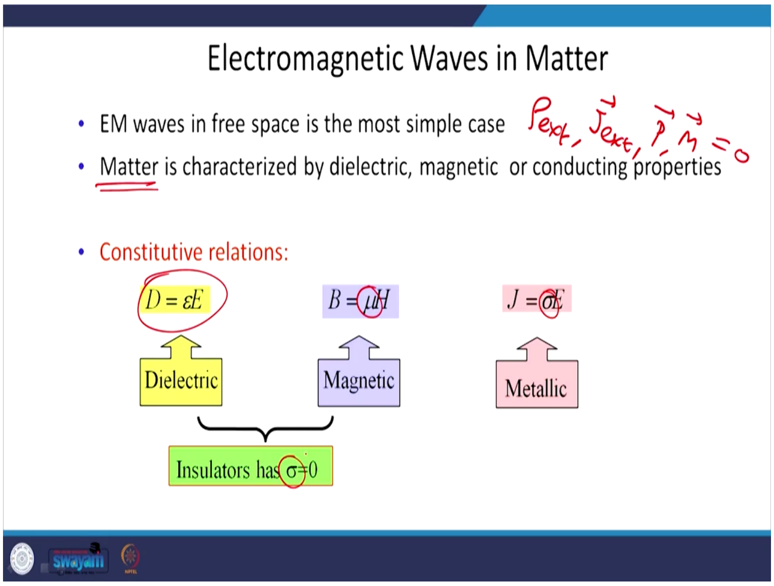

So, basically rho external, J external and P and M, this all was equal to 0 for free space, but when there is a matter, say for example, glass then we need to consider certain properties, which are either dielectric, magnetic or conducting properties.

We discussed about the constitutive relations, while describing the Maxwell's equations and from there, we can see that this one takes care of the dielectric once, this one takes care of the magnetic one, this one takes care of the metallic ones. For insulators, this equal to 0. So, if you have a dielectric or magnetic medium, which is non-metallic, which is non-conductive, then you have sigma equal to 0.

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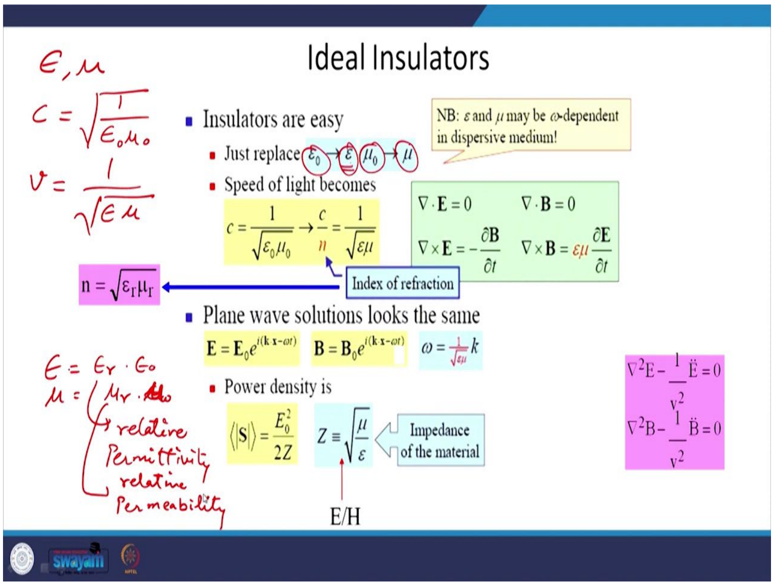

So, you take this kind of material and let us see what happens to insulators. So, in insulators, we know, that there was epsilon and mu, just replace epsilon 0 with epsilon and mu 0 with mu. So, the speed of light which we saw was c equal to one upon under root epsilon 0, mu 0.

Now, c for the medium, let us say, v - it becomes one upon under root epsilon and mu. You can always write epsilon as equal to epsilon r into epsilon 0, mu equal to mu r into mu 0. This is relative permittivity and this is relative permeability.

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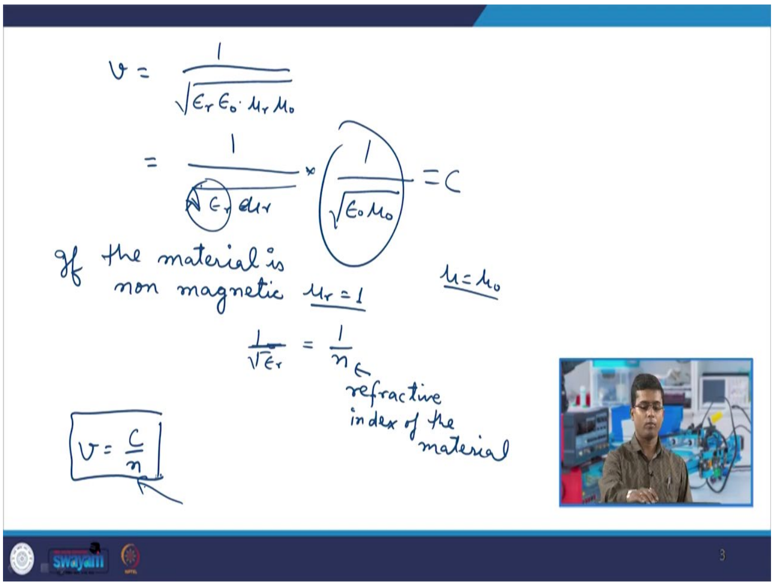

So, what you see here is that, basically, we can write v equal to 1 upon epsilon r epsilon 0 into mu r mu 0. We can take epsilon r and mu r outside. So, it becomes under root epsilon r under root mu r into 1 upon epsilon 0, mu 0, this is equal to c.

If the material is non-magnetic, mu r is equal to 1 - basically mu becomes equal to mu 0. So, 1 by under root epsilon r is equal to 1 by n; n is the refractive index of the material. So, from here we get v is equal to c by n, right!

So, speed of light in a material medium gets slightly decreased and it is by slightly a factor of 1 by n, that is the refractive index of the medium. So, denser the medium- denser means a medium with high refractive index- will have a smaller speed of light. So, the speed of light changes so, you get c by n. You solve for the Maxwell's equations from there you get to these equations for electric field and magnetic field.

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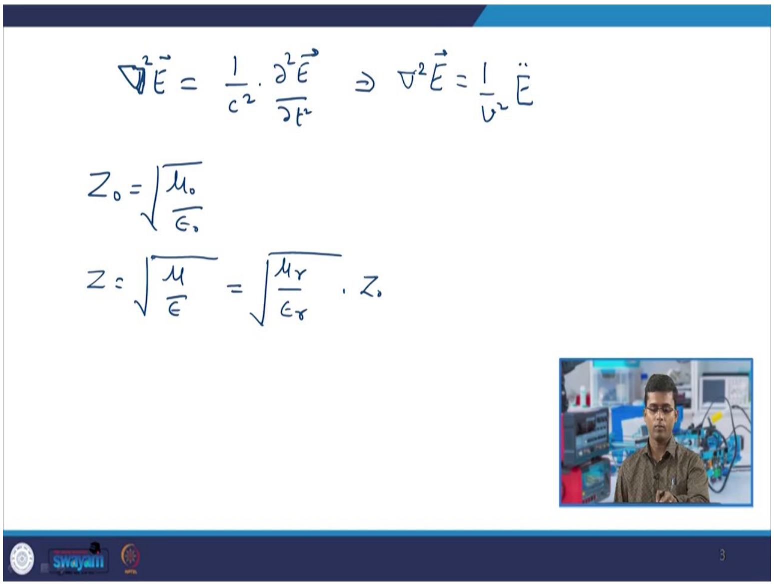

I wrote in the slightly in different form and that was del 2 E equal to 1 upon c square into del 2 E by del t square. You can write it del 2 E is equal to 1 by c square- now, in the present case it will become one by v square into E double dot ok.

So, that is what we have written here and, you can write it this way. You can solve for plane wave like solutions, which are similar like what we have discussed previously. And, now you have here omega equal to k by v. Power density is given by this relation where Z is equal to impedance of the material. If you have Z 0 was equal to under root mu 0 upon epsilon 0 and Z equal to basically mu upon epsilon, which is here you can have equal to mu r upon epsilon r into Z naught.

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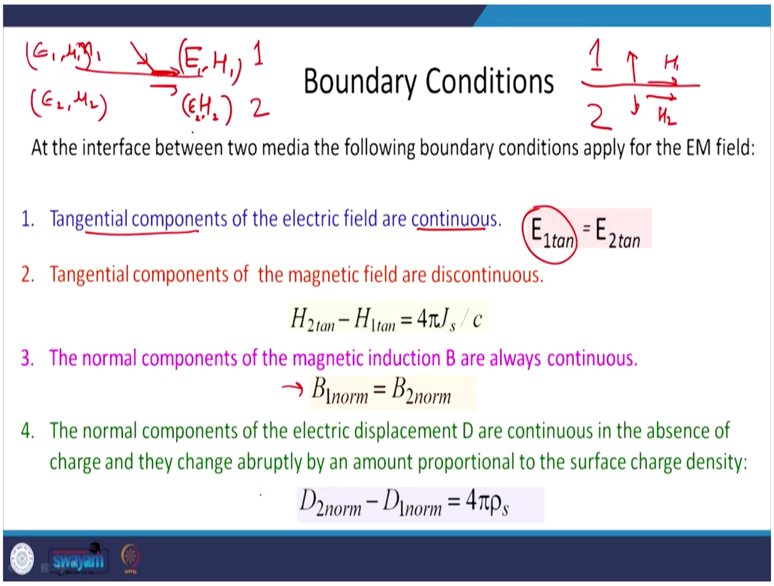

So, you know the impedance of the material. We have certain boundary conditions when a light wave goes from one medium to the other medium. There is an interface. So, suppose I have one medium here, which has a refractive index n 1 or may be epsilon 1 mu 1 from epsilon 2 mu 2, it goes from epsilon 1 mu 1 to epsilon 2 mu 2. What will happen? How will electric fields and magnetic fields E and H in this one will behave to E 2 and H 2? What is relation between them - that is called boundary condition.

So, at the interface of these two media, the following boundary conditions apply to the electromagnetic field. And, what are those? The tangential components of the electric field are continuous. So, tangential component here which is along the surface - component of the electric field which is along the surface in these 2 is the same.

So, electric field tangential component in the first medium and electric field tangential component in the second medium are equal. So, that is why they are called continuous. However, the tangential components of the magnetic field are not continuous - they are discontinuous. What it means - that if you solve for it- this is first one, this is the second one, the tangential component here and here H 1, H 2, H 1 is not equal to H 2, the tangential component - this is equal to 4 pi j s by c. J s was the surface current.

So, if J s is equal to 0 then it will be valid otherwise, it is discontinuous. The normal component of the magnetic induction B is always continuous. What is normal component? Normal component is the component, which is normal to this interface. tangential component is the component of electric and magnetic field, which are along the interface.

Normal component means the components which are normal to the interface and they are here continuous. However, for the displacement they are discontinuous. So, they are continuous only in the absence of charge and they change abruptly, if you have any amount of surface charge density. So, if you have a small charge density then, they are not continuous otherwise they are continuous. So, these are the boundary conditions which we will be using, when we see that light goes in from one medium to other.

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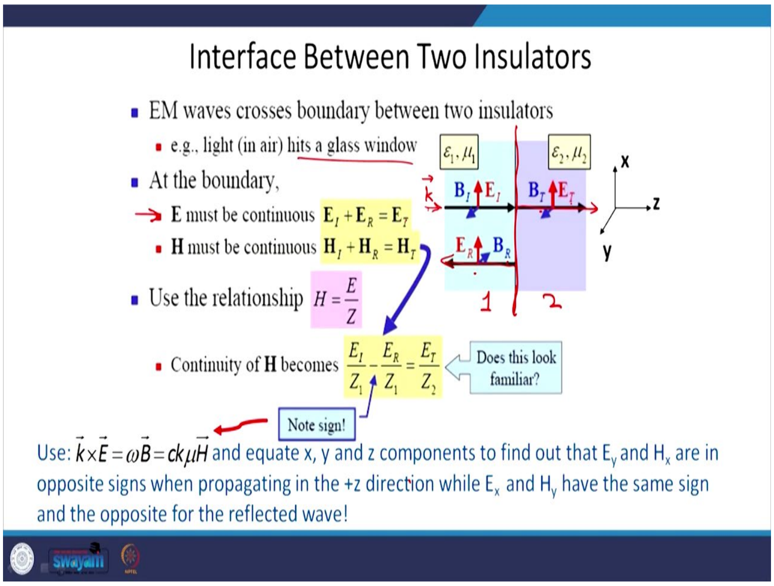

So, let us talk about an interface of two insulators. We have here an interface of two media. This is an interface. Light comes from here. So, that dielectric properties are epsilon1 and mu1 here of the first medium, epsilon 2 and mu 2 for the second medium. The k vector of the incident light is in this direction that is perpendicular to the interface.

So, the B and E components are shown here with red and blue curves. A fraction of this incident light gets transmitted along this direction and a small fraction comes back. So, I have put it here that you can see that a small portion is reflected, I denotes the incident R denotes the reflected ones and T denotes the transmitted ones.

So, you have here reflected and transmitted fields and now you apply the boundary conditions. So, it is similar to the case like light hits a glass window, lights falling on it at 0 degrees. So, boundary conditions are- E must be continuous that E I plus E R is equal to E T, because this is a parallel component, you see this is along the interface. So, it must be continuous. So, the electric fields at this side and electric fields at this side should be continuous. H must be continuous because there are no surface currents.

So, this is also continuous. Using this relationship, we come to this continuity of H, you see that you can use this relation and equate X,Y and Z component to find out what is Ey and H x and you will see that they are having opposite signs. So, why here we have minus? Here this is plus but here it becomes minus. When it is written in terms of E, because when it is propagating in plus Z direction E x and H y have the same sign and the opposite for the reflected wave. So, if it is having same sign, when it is moving in one direction opposite sign, when it is a reflected wave. So, that is why for E R you have minus sign here.

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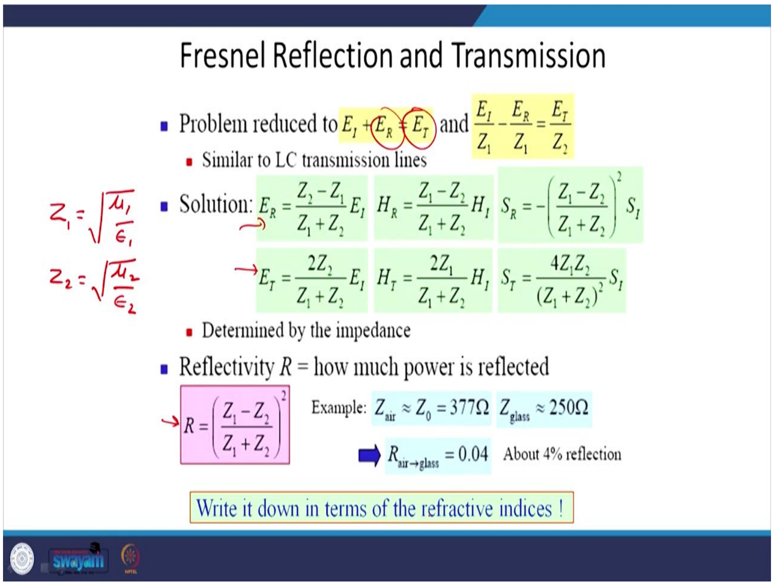

So, you solve for Fresnel reflection and transmission. Now, the problem was reduced to 2 equations in E R and E T. You have to actually calculate the ratio of E R by E I and E T by E I. And, if you solve it, you come to these equations E R E T, H R H T and S is pointing vector. So, now, you know in terms of the impedance - Z is an impedance. Z 1 is under root mu 1 by epsilon 1, Z 2 is under root mu 2 by epsilon 2, you know this.

So, in terms of the impedance you get this formula and from here you can calculate the reflectivity and transmittivity, but we want to write it down in terms of effective indices. Because, when we are dealing with optical surfaces and optical media, we do not want to have these equations in terms of Z, we want to have them in terms of refractive indices n - this is what it looks like.

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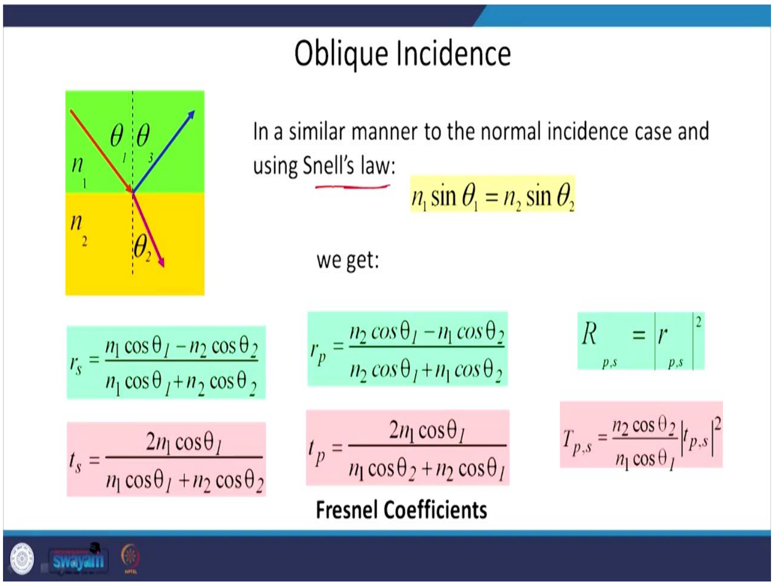

So, in a similar manner, using Snell's law we will prove it now only and if you have this kind of Snell's law, the r s and t s and r p and t p are given by these relations. Let us try to derive them.

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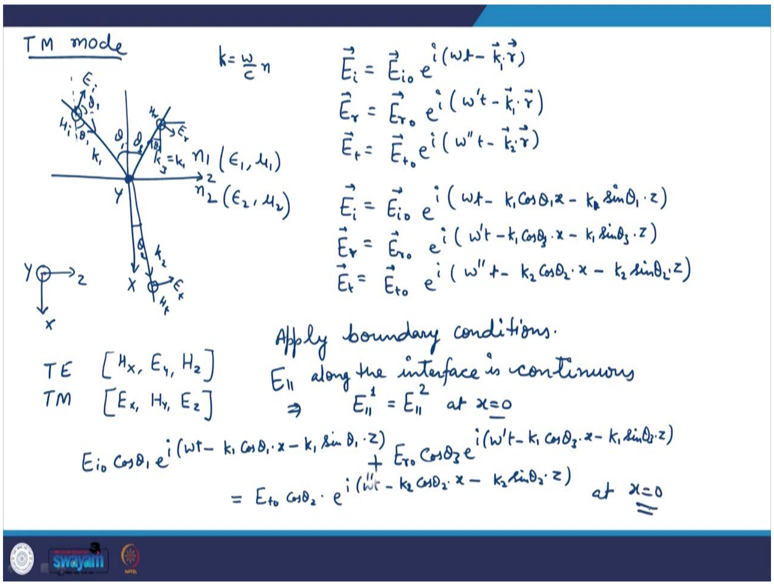

So, let us consider an interface of two media. And, we have refractive indices n 1 and n 2 for media 1 and 2 and it has also epsilon 1 mu 1, epsilon 2 mu 2 and we consider that an electromagnetic wave is incident at angle theta 1, a part of it gets refracted at angle theta 2, and other part of it gets reflected, let us say that this is angle theta 3.

So, if it has wave vector k 1 let us say and this one is k 2 and since it is in the same medium let us again write it, first let us write it k 3, but it will be equal to k 1 only. We consider that this is X axis, this is Z and Y is pointing upwards. So, it is like this X Z Y.

I have already told you that if you have an electromagnetic wave incident at some structure, what you have to do is that you need to solve electric and magnetic fields and they will have 6 components in total, but you can always have 2 independent set of solutions that were called TE and TM solution.

So, for this kind of configuration, you will have H x, E y, H z as T E mode and E x, H y, E z as T M mode. That is defined like, you have this plane of incidence and this plane of incidence change sees the change in refractive index, then the component which is perpendicular to this kind of plane of incidence, if it is the electric field component which is perpendicular then you say it T E mode, and if it is the magnetic field component which is perpendicular to this kind of plane, you call it TM mode.

So, we have this kind of wave and let us say, that we consider here only TM mode and for this kind of mode, the electric field vector lies in the plane of incidence. Let us consider that the electric field is pointing in this direction E i and the magnetic field is perpendicular direction H i. The one which gets transmitted has electric field, E t and the magnetic field H t and suppose in the reflection here, it has E r and H r.

So, let us say that if the equation for the plane wave which is incident is given by this is E i equal to E i 0, e to power i omega t minus k dot r. So, it is k 1 dot r and the one which is reflected, let us say that, E r is given as E r 0 E to power i omega prime t minus k 1, which I told you that it is k 1, because k is equal to omega by c into n and if it is the same refractive index medium the k will be the same, ok.

So, it is k 1 dot r and E t equal to E t 0 e to the power i (Refer Time: 20:27)omega t minus k 2 dot r. So, if a plane wave is incident at an interface of two media having refractive indices n1 and n2, you can have partial reflection and partial transmission through this interface and the electric field components of reflected and transmitted light are given by these relations.

So, if you resolve this k vector parallel and perpendicular to this interface, you can write it like E i is equal to E i 0 e to power i omega t minus k 1 cos theta 1 x minus k 1 sin theta 1 into z. And, similarly E r will be E r 0 e to the power i omega prime t minus k 1 cos theta 3 into x minus k 1 sin theta 3 into z and E t is equal to E t 0 e to the power i omega double prime t minus k 2 cos theta 2 into x minus k 2 sin theta 2 into z. So, we have these kinds of waves. Now we apply boundary conditions.

So, if you apply boundary conditions and what are they the boundary conditions? E parallel along the interface is continuous. So, it means that E parallel in medium 1 is equal to E parallel in medium 2 at x is equal to 0. So, what is the component of E in this direction? If you have theta one here, it will be like this angle theta 1.

If you take this, again, it will be 90 minus theta1. So, it will be sin theta. You can have this actually and this is theta, this is 90 minus theta. So, actually, this will be theta 1 ok. So, you can write that E i o cos theta 1 e to power i omega t minus k 1 cos theta 1 into x minus k 1 sin theta 1 into z minus, because you have theta 3 here. So, this will be theta 3.

So, this will be 90 minus theta 3 and then you have theta 3. So, it will be not minus, it will be plus E r 0 cos theta 3 into e to power i omega prime t minus k 1 cos theta 3 into x minus k 1 sin theta 3 into z equal to Et 0 cos theta 2 e to power i omega double prime t minus k 2 cos theta 2 into x minus k 2 sin theta 2 into z at x is equal to 0.

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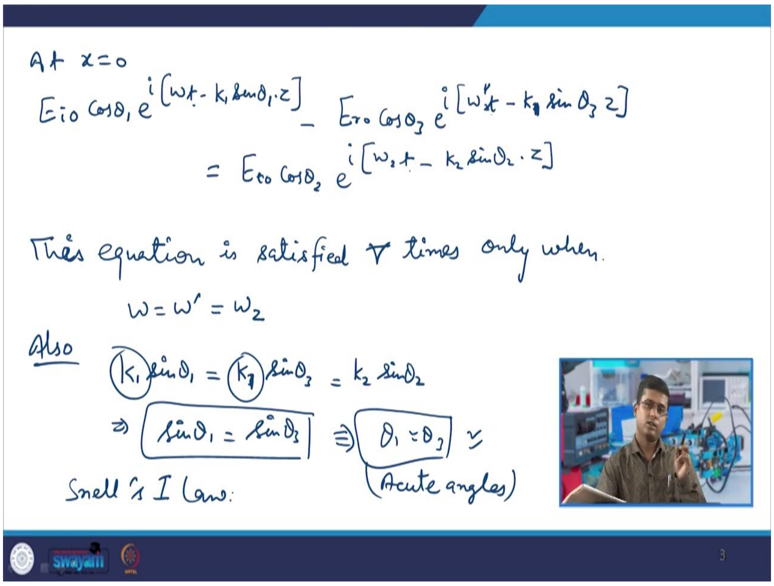

So, at x is equal to 0, E i 0 cos theta 1, that is the tangential component, into e to power i omega t minus k 1 sin theta 1 into z minus E r 0. I am removing the vector sign, because you can put it always, but let us be more simpl, that was omega dash or omega 2 omega dash let us check. So, omega dash omega dash t minus k 1 sin theta 3 into z is equal to E t 0 cos theta 2 e to power i omega 2 t minus k 2 sin theta 2 into z.

This equation is satisfied for all times only when omega is equal to omega dash is equal to omega 2, why? Because for all the values of t to be the same, all the omegas have to be the same. So, this is the first condition that should be satisfied.

Now, let us omit all these things - what you get here? Also, we see that k 1 sin theta 1 equal to k 1 sin theta 3 is equal to k 2 sin theta 2. Why? To satisfy this boundary condition, all this should be equal; that means, since it was k 1, we will have sin theta 1 is equal to sin theta 3, which is not always true but theta 1 is equal to theta 3 for acute angles, we know that all these are acute angles. So, this is true - it is Snell's first law; Snell's first law - law of reflection. So, theta 1 is equal to theta 3; that means, the angle of reflection is equal to the angle of incidence.

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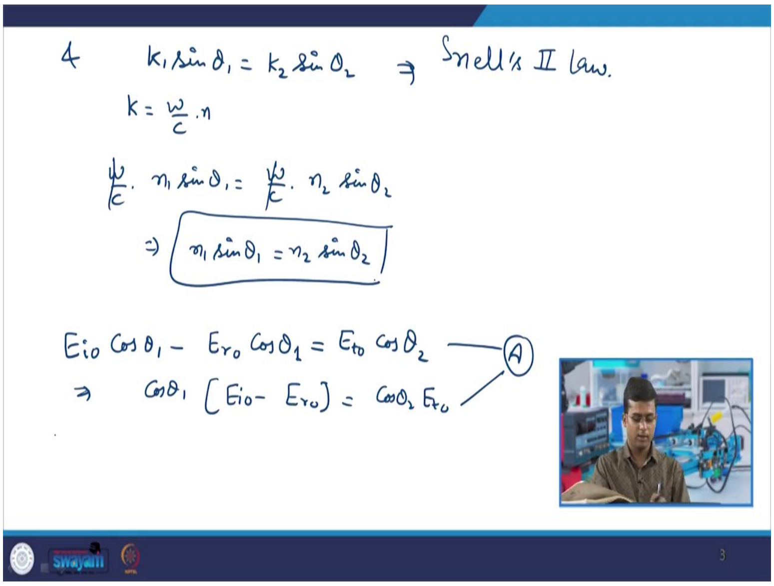

And, what we get is that k 1 sin theta 1 is equal to k 2 sin theta 2, that is Snell's IInd law; you can simplify it a bit. So, it will become n 1 sin theta1 equal to n 2 sin theta 2, because you know that k is equal to omega by c into n. So, you can write omega by c into n 1 sin theta 1 is equal to omega by c into n 2 sin theta 2. This cancels out - you get n 1 sin theta 1 is equal to n 2 sin theta 2, this is another form of Snell's law. So, if you put these things in equation this one, let us say this is equation 1.

So, put all this in equation 1. What you get is E i 0 cos theta 1 minus E r 0 cos theta 2 is equal to E t 0 cos theta 3, which is cos theta 1, actually. This is equation A. You can further simplify it - cos theta 1 E i o minus E ro is equal to cos theta 2 E t o. Basically this is one, what is the second one - the tangential component of electric field?

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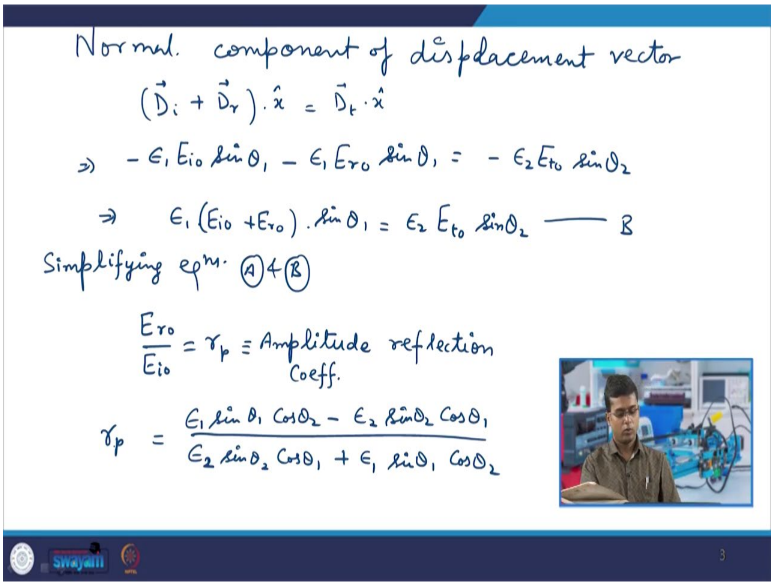

Let us see what happens with the tangential component, tangential displacement vector. So, D i plus D r in this direction must be equal to D t this direction. So, x means the normal component. So, it is not a tangential one, it is normal component. You are having the normal component of displacement - let us see what happens. So, write it like this minus epsilon 1 E io sin theta 1 minus epsilon 1 E ro sin theta 1 is equal to minus epsilon 2 E to sin theta 2, then simply write it is epsilon 1 E i plus E ro into sin theta 1 is equal to epsilon 2 E t o sin theta 2, this is equation B. Now, let us simplify equations A and B, what do you get?

We want to calculate E ro by E i o, which is the amplitude reflection coefficient - r p, amplitude reflection coefficient. We want to calculate r p and that comes out to be epsilon 1 sin theta 1 cos theta 2, minus epsilon 2 sin theta 2, cos theta 1 divided by epsilon 1 sin theta 2 cos theta 1, plus epsilon 2 sin theta 1 cos theta 2. This is r p.

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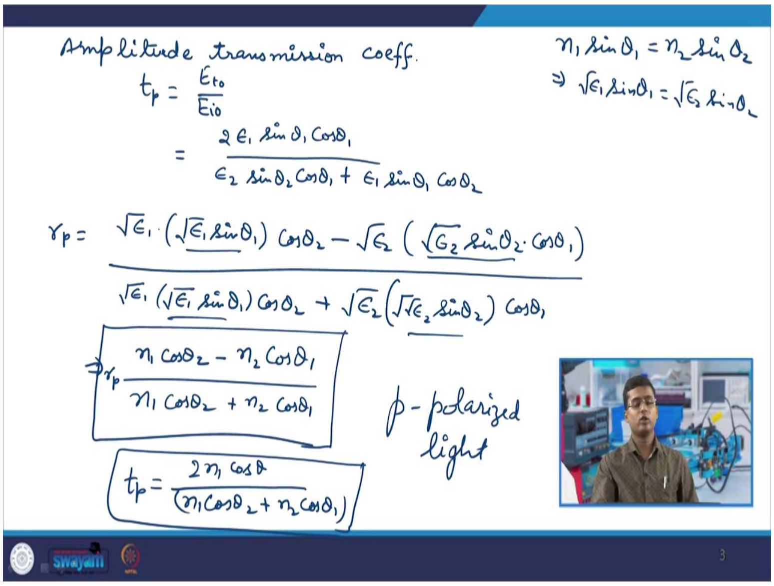

What is the amplitude transmission coefficient? Amplitude transmission coefficient, that is t p is equal to E t 0 divided by E i 0. And, you get 2 epsilon 1 sin theta 1 cos theta 1 divided by epsilon 2 sin theta 2 cos theta 1 plus epsilon 1 sin theta 1 cos theta 2. Now, as we know n 1 sin theta 1 is equal to n 2 sin theta 2 we can write in it in terms of n.

So, r p is equal to it was under root epsilon 1 into under root epsilon 1 sin theta 1 cos theta 2 minus under root epsilon 2, we can write under root epsilon 2 sin theta 2 cos theta 1, epsilon 1, epsilon 1, sin theta 1 cos theta 2 plus under root epsilon 2, sin theta 2 cos theta 1. We know that n 1 sin theta 1 is equal to n 2 sin theta 2.

So, we can write it actually under root epsilon 1 sin theta 1 is equal to under root epsilon 2 sin theta 2. From here you can substitute ,you can take common, these are common this is equal to this you can take it out, this is equal to this you can also take it out what you get is that n 1 cos theta 2 minus n 2 cos theta 1 divided by n 1 cos theta 2 plus 2 cos theta 1. So, that is equal to r p. Similarly, you put all these things in t p and you get t p is equal 2 n 1 cos theta 1 divided by n 1 cos theta 2 plus n 2 cos theta 1. So, these are the final coefficients for p polarized light.

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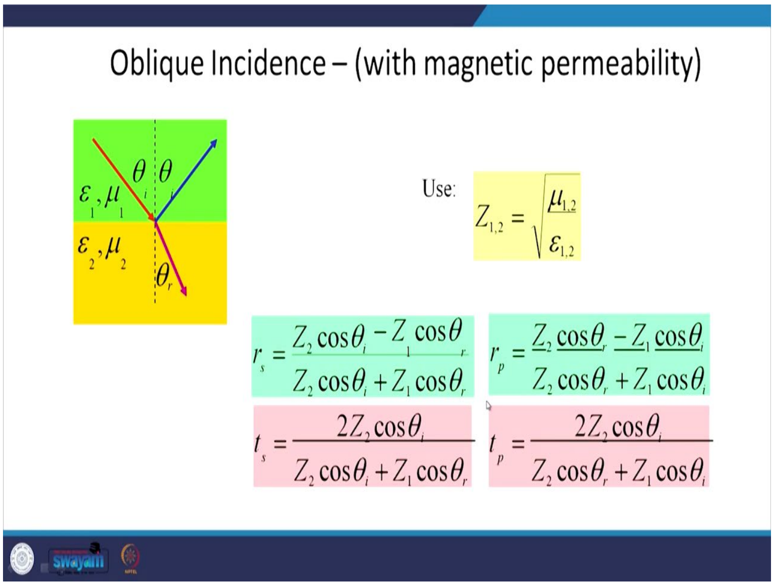

Similarly, you can solve it for s polarized light. This is what we have got here for the r p and for t p. Similarly, you can solve it for r s and t s and we see that amplitude - these are the amplitude reflection coefficients. So, you can basically calculate what is capital R and capital T.

Oblique incidence - if you consider the magnetic permeability, which we have not considered there, then it comes in terms of Z, rather you are having the refractive index and it can similarly be solved for that.

So, to summarize this - we have boundary conditions at the interface of two media and if you want to have a propagation of this wave from one medium to another medium, you must have to consider these boundary conditions and they must satisfy at the boundaries for the electric and magnetic fields. And, the propagation of an EM wave at the interface of two media - we have studied and we have found the final coefficients for p polarized light. Similarly, it can be found for s polarized light; that means, that you have to solve. So, the this was X, this was Z and the polarization was Y.

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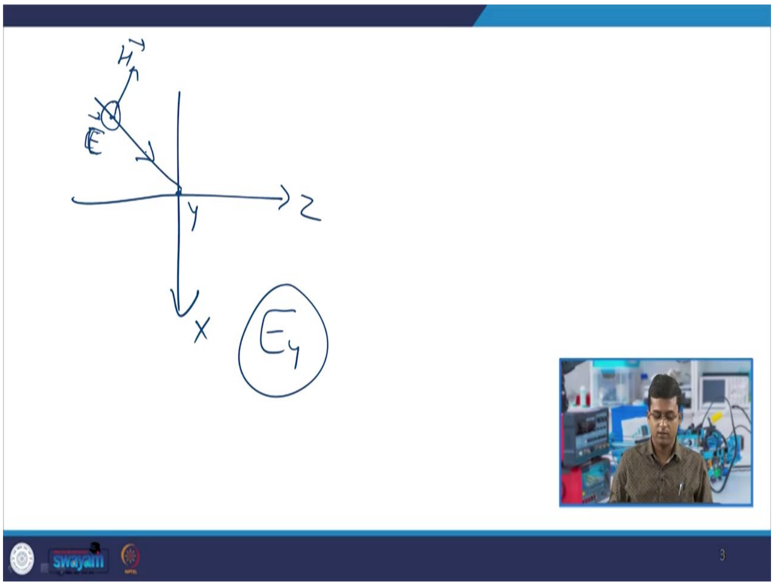

So, the electric field is now in y direction perpendicular to the plane. So, you have to solve for this wave - the electric field is here and now the magnetic field is here. H is pointing in this direction an electric field this, then you have to see what happens at the reflection and transmission. This is a homework for you.

Thank you.