Optical Sensors

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Lecture - 03
Basic Optics for Optical Sensing - I
Electromagnetic Waves in Free Space & Poynting Vector

Welcome to the third lecture of Optical Sensors course. On the previous two lectures, we discussed the basics of sensors and bio sensors, and what are the components, and what are the performance parameters; I mean before we start to make any sensor, we must know what are the ways to make a sensor and what are the things we have to take care of.

For example, I told you about the sensitivity, detection accuracy, and limit of detection, response time, all these parameters you have to take care of when you are really concerned about sensor. Now, when we go to optical sensors, we have to understand what is optics, what are basically these waves or rays or particles, we will see. And, what are their fundamental properties which can be modulated and can be used for making a sensor. So, from now onwards, we will study the basic optics and use it for optical sensing.

Today we are going to discuss Electromagnetic Waves in Free Space and Poynting Vector. Here we will see that, as I told you, light basically is an electromagnetic wave. So, if I have an electromagnetic wave, how it propagates through a medium, how it propagates through free space, and what will be the direction of flow of energy, will it be a transverse wave or longitudinal, we will see all these things.

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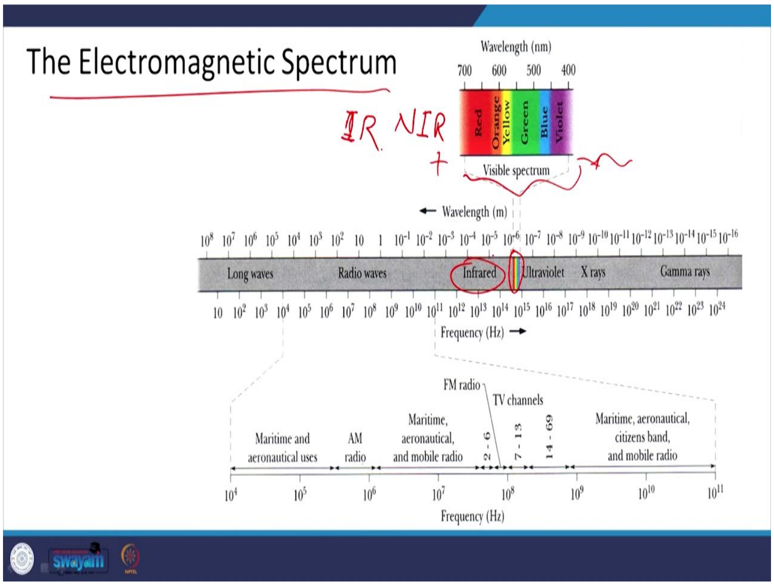

The electromagnetic spectrum that is the broad spectrum, it starts from gamma rays to long waves, and here a very small patch of it say only about 300 to 750 nanometers that is called visible spectrum. So, if you are talking about optical sensors, we will be essentially talking about this range plus little bit of say from violet to say NIR or may be IR or maximum IR infrared red, and infrared, not more than that. So, we are talking about this is small patch of the electromagnetic spectrum.

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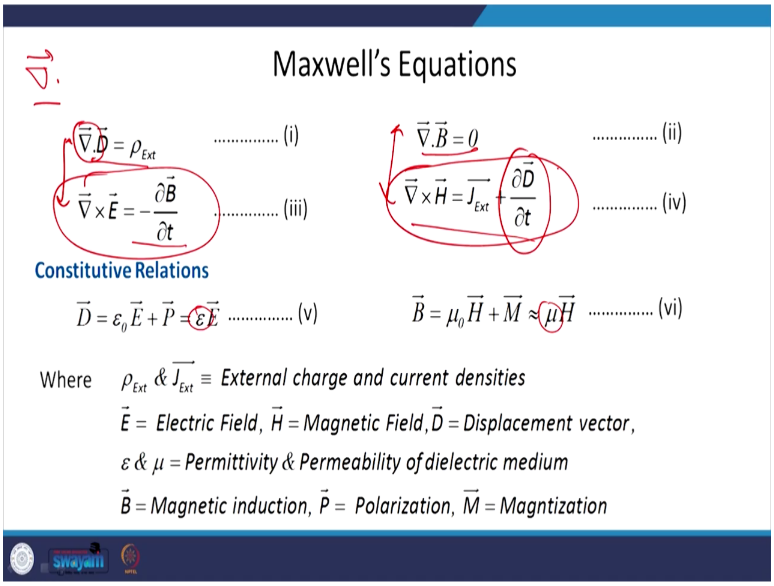

So, here are Maxwell's four equations, del dot D is equal to rho external; del dot B is equal to 0; del cross E is equal to minus del B by del t, and del cross H is equal to J external plus del d by del t. These four relations rely on two constitutive relations which are: D is equal to epsilon naught E plus P which you can write epsilon E, and B is equal to mu H, where I have written out what every term means. So, basically this is displacement vector. So, I assume that you know what this operator – delta is, this is vector operator. Those who have not yet studied, I think they should go and read it first or make a course on that first.

From here you can see that these relations are two from electricity and two from magnetism. And this one was somehow connecting electricity and magnetism, and here there was a missing gap which Maxwell fixed. So, what he said that a changing magnetic field produces electric field, and changing electric field produces magnetic field and this keeps on changing only at a particular speed, it is called the speed of light.

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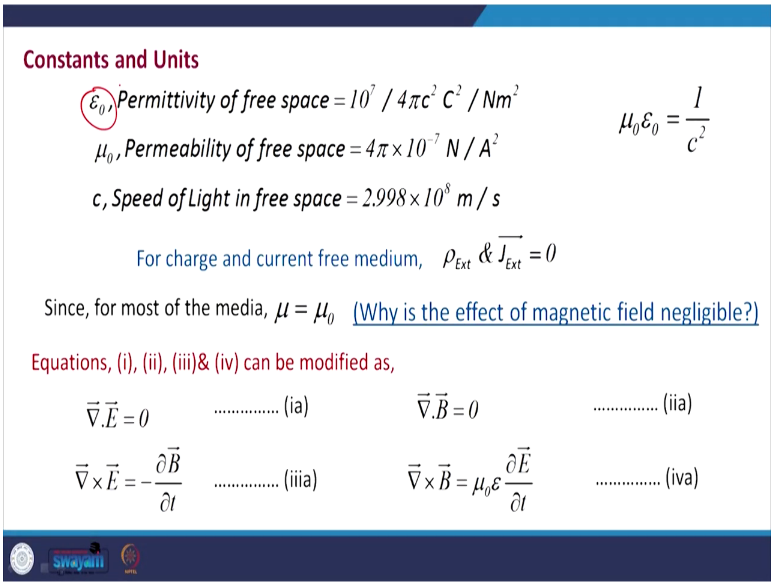

These are the constants: epsilon 0 is the permittivity of the free space, mu 0 is the permeability of the free space, and c is the speed of light. If you multiply these two epsilon not and mu not here, here 4 pi, 4 pi terms cancel out, and 10 to power 7 cancels out, and you are left way 1 by c square.

Now, suppose I take a medium where J external and rho external are 0, then we can rewrite the Maxwell's equations in these forms, where I have assumed that mu is essentially equal to mu not because most of the materials are less responsive to that magnetic fields and more responsible to the electric fields. So, when we talk about optical materials, they are mostly responsive to electric fields.

So, there is the role of epsilon, but mu 0, mu is almost equal to mu 0. We will discuss later why is the effect of magnetic field negligible - we will see. But for now, we move ahead with these equations and try to solve them to find a wave which propagates in the medium or in vacuum.

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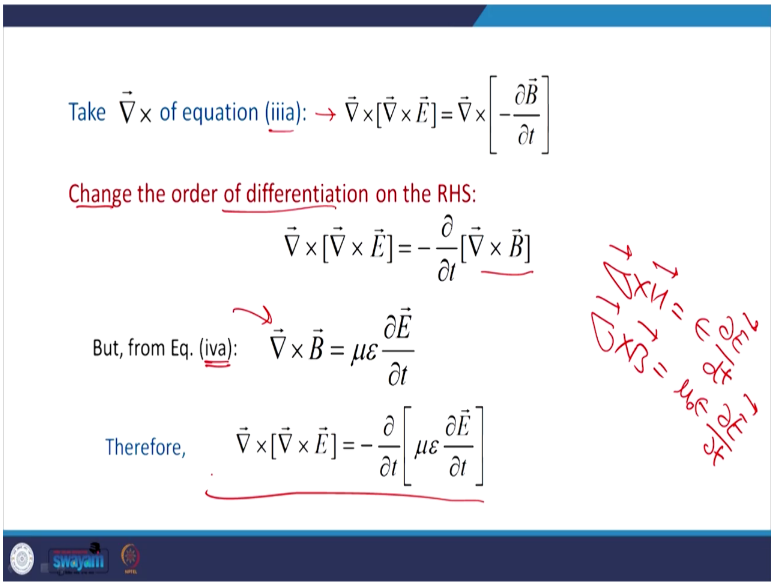

So, take curl of the equation third. We will end up with curl of curl of E is equal to curl of minus del B by del t. You change the order of differentiation. First you differentiate and then take that derivative, you end up with this. From equation 4th a, you can put this directly here ok. So, equation 4th was del cross H was epsilon del E by del t. And when you have del cross B it will become mu not epsilon del E by del t ok.

So, from there we end up with this relation, you can simply substitute for this and we end up here. So, del cross del cross E is equal to minus del by del t mu epsilon del E by del t, so that will essentially become del 2 E by del t square.

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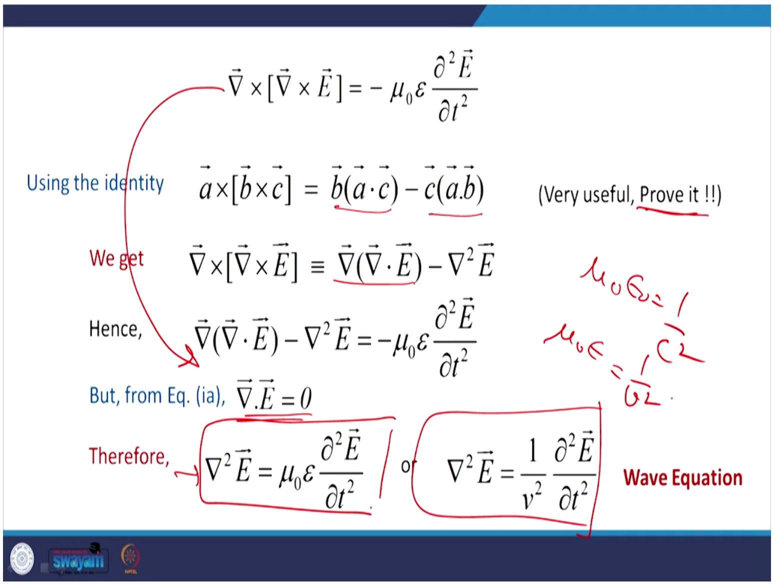

Now, we use an identity that is a cross b cross c is equal to b a c minus c a b back minus cap this is a very useful rule. I suggest that you go home and prove it, that is true ok. So, you simply use vector calculus to prove it. So, if you apply the same identity here, what we get here is del cross del cross E is equal to del of del dot E minus del 2 E ok.

Hence this equation becomes this, but we know that del dot E is equal to 0, that is why we end of with this or is equal to this. So, we know that mu 0 epsilon 0 is equal to 1 by c square that is a speed of light it, it becomes a material medium like mu naught epsilon, it becomes 1 by v square.

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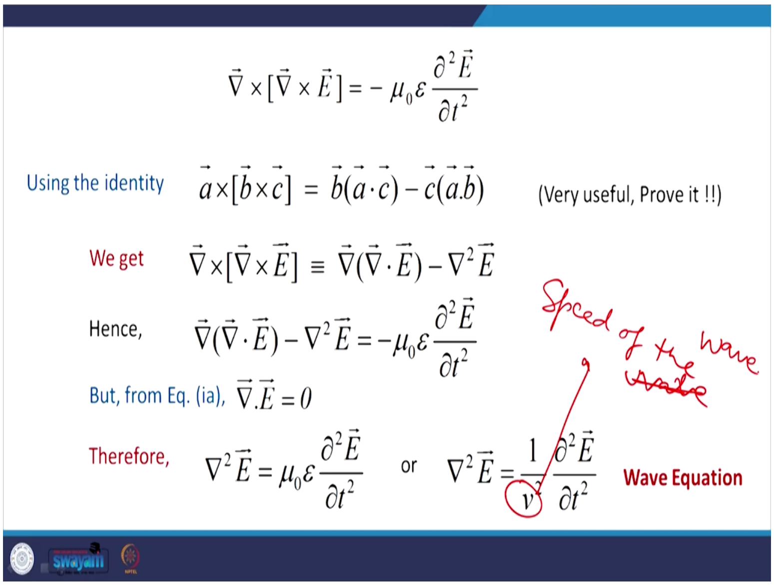

v is the speed of the wave. So, this relation is called a wave equation.

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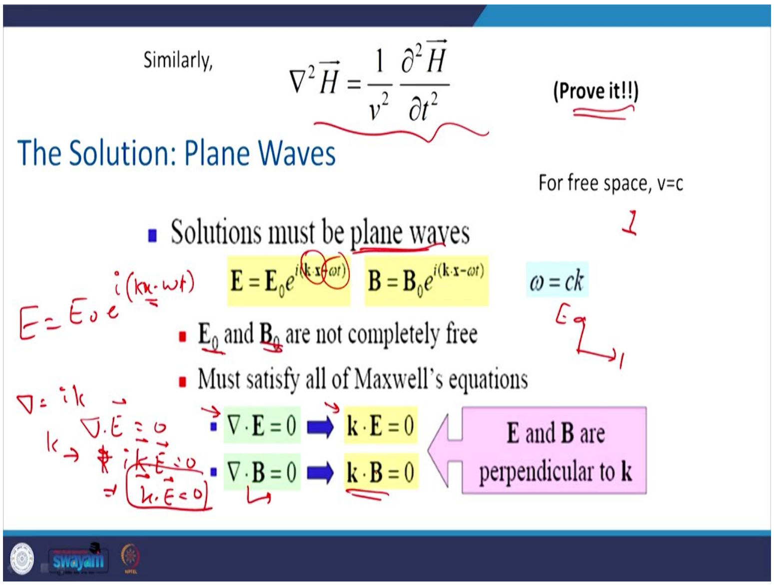

And if you solve for it, similarly for magnetic field, you will end up with this. So, I leave it to you for homework. You go home and you solve similarly for magnetic field. You take the curl of the fourth equation, substitute for the second one and you find out this. So, if you want to have a solution of this, you will have plane wave solutions like E is equal to E 0, e to power i k x minus omega t, or B is equal to this thing, ok.

So, if you have this equation, these are not completely free, I mean now E 0 and B 0, they must satisfy Maxwell's equations. So, let us see what happens. So, if you have E is equal to E 0 e 2 power i k x minus omega t. Why x, let us say we are considering it a one-dimensional case.

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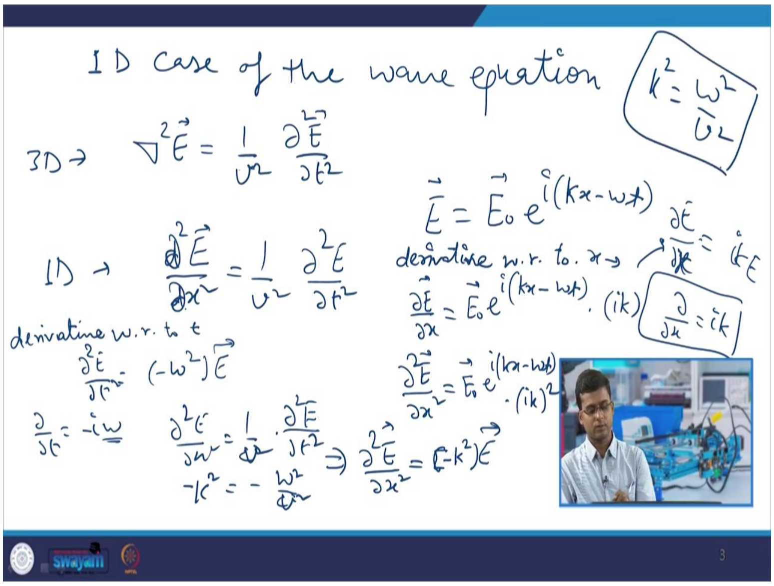

So, if you write a one-dimensional case of the wave equation that will be. So, wave equation was I mean 3D, 3D it was del 2 E is equal to 1 by v square del 2 E by del t square. Now, if you take 1D case it will become d 2 E by d x square del 2 E by d x square is equal to 1 by v square del 2 E by del t square I mean it has to be partial derivative.

Now if you I have E is equal to E 0 e to power i k x minus omega t and I want to solve it, what do I do. Take a derivative of E with respect to x. It will lead you del E by del x equal to E not e to power i k x minus omega t into i k. Del 2 E by del d x square will be will be E 0 e to power i k x minus omega t into i k square. So, I have these two terms. So, you can replace it with E. So, it essentially leads you del 2 E by del x square equal to minus k square i square is i square is equal to minus 1, so minus k square E that is what you get.

Now, let us take derivative with respect to t, so, from here we know that del E by del x is equal to i k E. So, del by del x is equal to i k. From here we know that del E by del t is equal to similarly del 2 E by del t square is equal to minus omega square E. And you can say that del by del t is equal to i omega minus i omega.

So, if you want to solve for the Maxwell's equations, what happens actually that del dot E is equal to 0 means k dot E is equal to 0. Del was equal to i k. So, del dot E is equal to 0 - means i into k dot E is equal to 0; this implies k dot E is equal to 0. Similarly, from here you can say that k dot B is equal to 0. What does it mean? It means that E and k are perpendicular if E is here k is here; similarly, k and B are perpendicular to each other.

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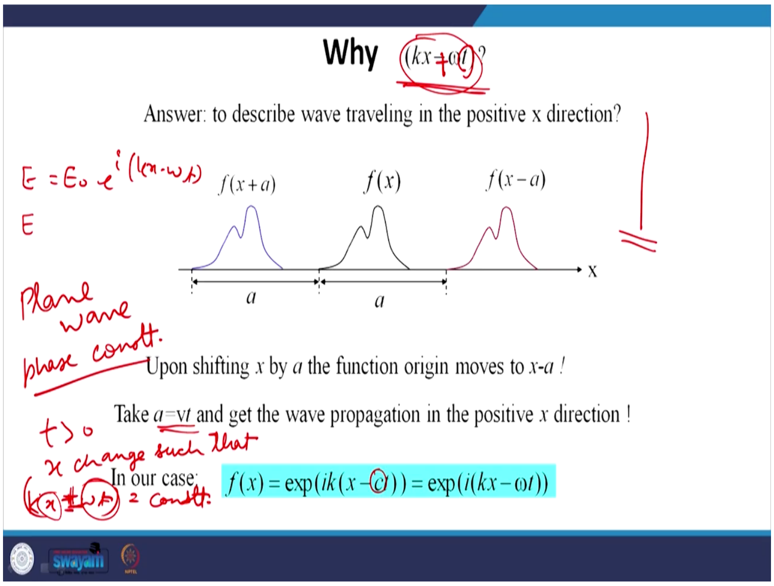

Why we chose k x minus omega t kind of solution, because we want to describe a wave travelling in positive x direction. What it seems? So, if you shift x by a, the origin of the function moves to x minus a, ok. So, it was here, now it became here. If you take a equal to v t; a is the distance, v is the velocity of the wave, and t is the time taken by it to move to a.

So, we get the wave propagation in positive x direction. In our case, we have i k x minus c t, why c because we are considering the wave in free space, that is why we are having c here. Also, you can see that if you have E equal to E 0 e power k x minus omega t, from here you have omega is equal to c k. How do you arrived here?

So, what to do is that you put in this relation from here. So, you get del 2 E by del x square is equal to 1 by c square into del 2 E by del t square. Now, you put it you get minus k square is equal to minus omega square by c v square actually v square. So, k square was equal to omega square by v square.

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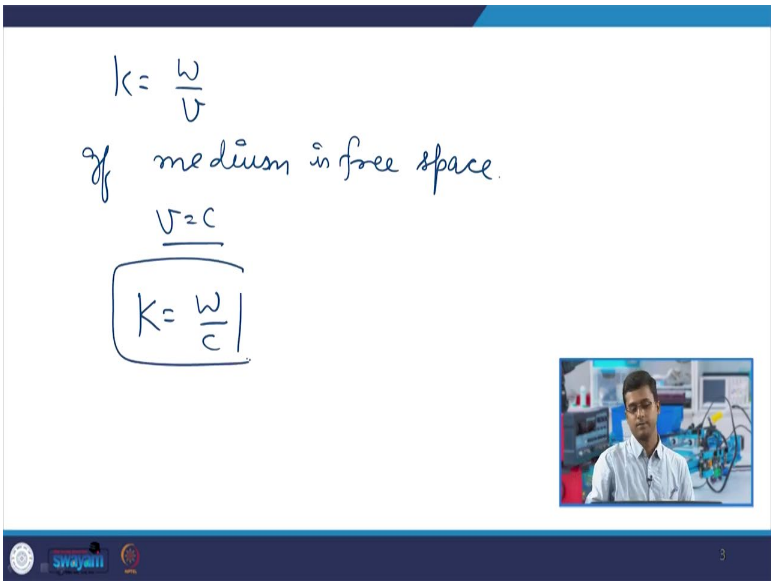

So, k is equal to omega by v. If medium is free space, I mean the wave is travelling in free space, then v is equal to c. So, k is equal to omega by c. So, we arrive here. So, we see from here that it is the wave which is travelling in positive x-direction. If it was positive, say k x plus omega t, then it will be a wave which is traveling in minus x-direction, because when time is increasing because it is a plane wave it has to be fixed. A plane wave means phase has to be constant phase is equal to constant throughout the plane.

Now, if t is increasing then x changes in a way such that k x plus minus omega t equal to constant. So, if t is increasing, then this term is increasing; for if it is a, for if it is plus, then x has to decrease. So, x is going in other direction, so that it nullifies the effect. If this is minus sign, if t is increasing, then x will be increasing, so it is going in the positive x-direction. That is how you say if the wave is going in plus or minus direction.

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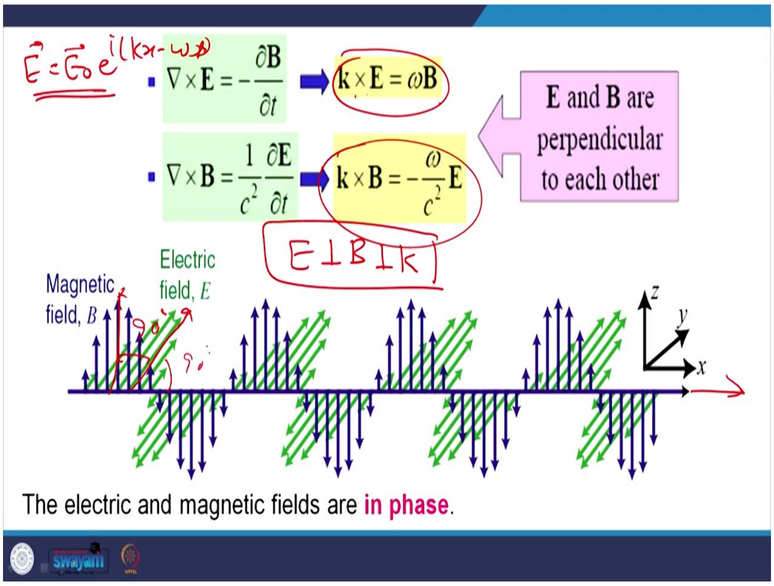

If you put this E is equal to E 0 e to power i k x minus omega t, this equation, you end up with this relation. And the 4th equation you end up with this relation. What these two say? - these two say that E and B are perpendicular to each other. So, from here it seems that E is perpendicular to B and perpendicular to k. So, they are all mutually orthogonal triads.

So, you can see that if the wave is propagating in this direction, the electric field is in this direction magnetic field in this direction, and then all these angles are 90 degree. All of them are 90 degrees like this. Wave is propagating in this direction, electric field in this direction and magnetic field in this direction. So, they are all and the electric and magnetic fields are in phase they are not in out of phase.

So, they are in phase all the time and this is a transverse wave. Why transverse, because they are all perpendicular to one another. Now, from here we have seen that k cross E is equal to omega B and omega is equal to c k, so you end up with this relation mod E is equal to c times B.

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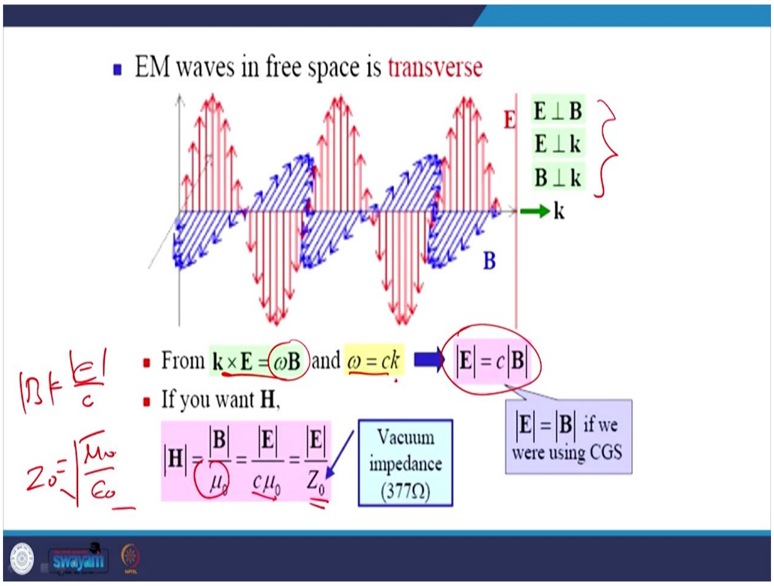

So, if you say B, magnitude of B is equal to magnitude of electric field by c. You solve for it: B is equal to h is equal to B by mu 0, you can write it like c by E by c mu 0, and this is essentially Z 0. Z 0 is defined is equal to under root mu 0 over epsilon 0 and is called the impedance of the vacuum. If you have mu by epsilon then it is the impedance of the medium. Impedance is like the resistance offered by the medium when the wave is propagating through it ok.

Till now we have demonstrated that this is a wave which is transverse in nature, because the electric field oscillations or magnetic field oscillations are perpendicular to the direction of propagation of the wave. Now, I come to the point why the effect of magnetic field is negligible.

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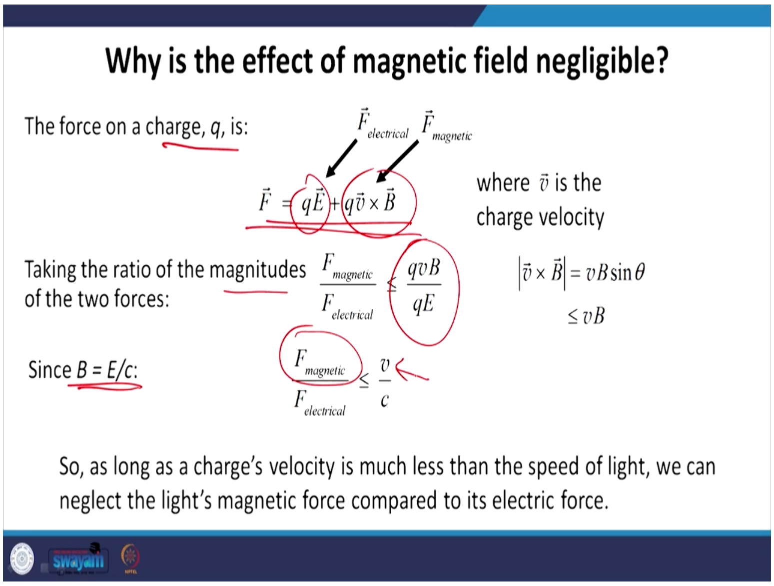

Suppose I have a charged particle, you know this Lorentz force on it due to electrical and magnetic force components, it is q E plus q v cross B. So, if I take the ratio of the magnitude of the electric and magnetic forces, we end up with this thing. And since B is equal to E by c, we see that the contribution of magnetic force to electric force is like v by c.

And, if the velocity of the charge is much smaller than the speed of light, then we can neglect its magnetic force as compared to the electric force, ok. That is why most of th optical materials are responsible to the change in electric field, but not to the magnetic field, all right.

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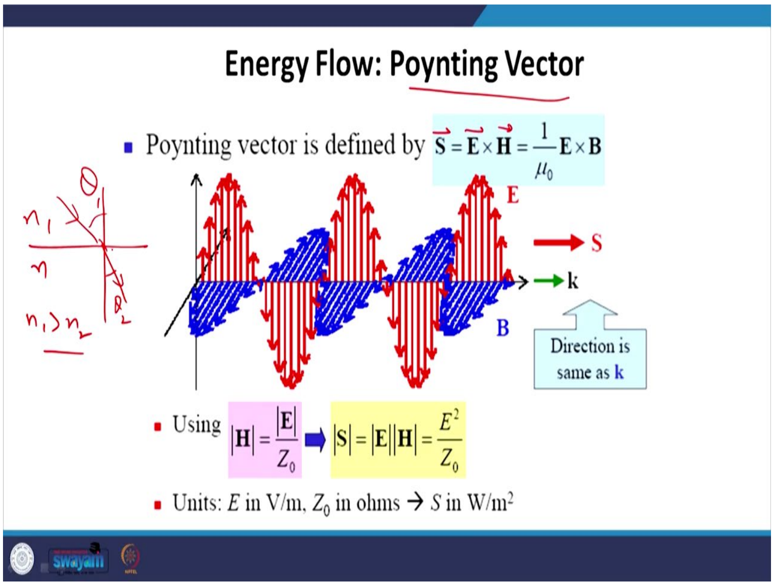

Now, we want to see what the direction of flow of the energy is and that is given by Poynting vector. Poynting vector is defined by S is equal to E cross H. So, the direction of flow of energy in an electromagnetic wave is defined by E cross H. It may be pointing in the direction of the wave vector; may not be even. In case of double reflection you have ordinary and extraordinary rays; and for extraordinary rays, in most of the cases the direction of the propagation vector does not align to the direction of the S vector.

So, there is some specific case, but for most of the cases for example, this case of simple reflection and refraction the wave vector aligns to the Poynting vector. That we will see when we solve the refraction problem. I guess you know that laws of refraction that if a wave comes from n 1 to n 2, and n 1 greater than n 2, then what happens actually is that it goes away from the normal. You know that what happens to the direction of flow of energy that consideration. So, it will be taken. So, we will see that.

So, if the wave is propagating in this direction the in ideal case, it will be just same as the direction of. So, direction of flow of energy will be the same at the direction of propagation of the wave. So, using this relation which we derived earlier, you come to the point that it is E square by Z 0. So, S is in watt per meter square that gives you the Poynting vector.

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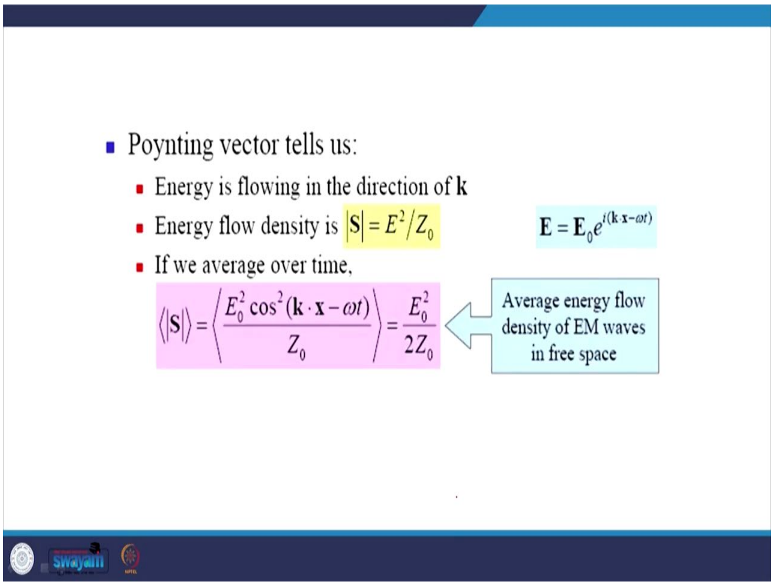

Poynting vector tells us that energy is flowing in the direction of propagation of the wave that is the vector k, and the energy flow density is E square by Z naught. So, if we average over time you get by a factor of half because you have a cos square term, average of which comes out to be 1 by 2. So, average energy flow density of E m waves in free space is E 0 square by 2 Z 0.

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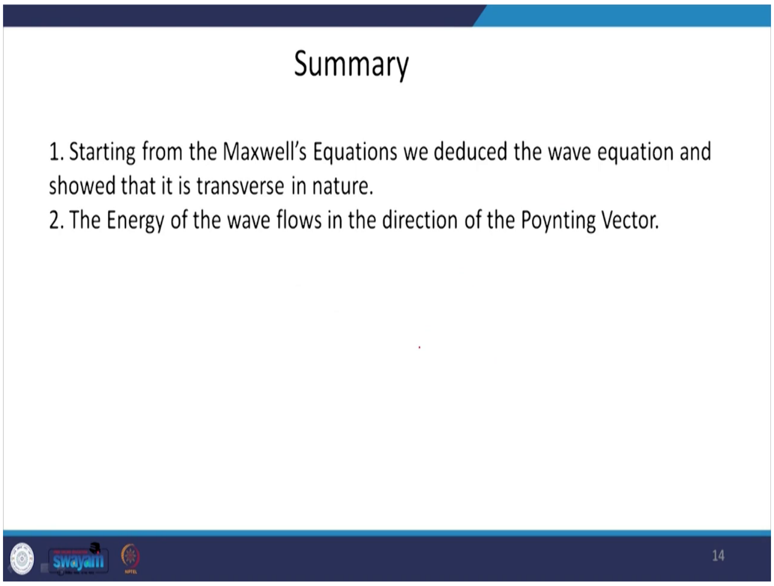

So, to summarize this talk; starting from Maxwell's equations, we deduced the wave equation and showed that it is transverse in nature. Energy of the wave flows in the direction of the Poynting vector.

Thank you.