

**Optical Sensors**  
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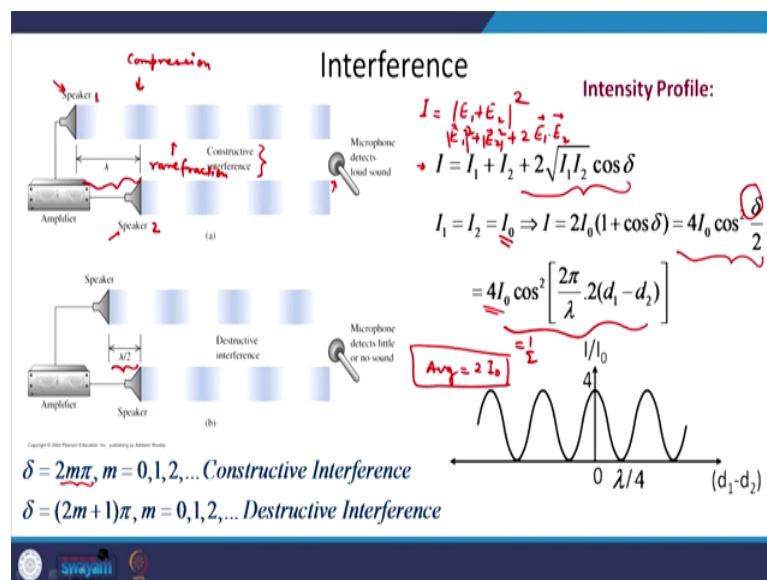
**Lecture – 17**

**Interference based Sensors - Michelson and Mach- Zehnder Interferometers:  
Sensing Applications**

Welcome to 17th lecture of Optical Sensors course. In the last lecture, we discussed optimum configurations for LSPR sensing. And also, we studied fluorescence. Then we described that similar to SERS we can have plasmonic enhancements in fluorescence, in IR absorption and we discussed SEIRA and SEF based sensing. We, then, discussed what is colorimetric sensing and then we concluded with cavity ring down absorption spectroscopy.

Today, we are going to discuss interference-based sensors. In particular we will discuss what is Michelson interferometer and its applications and then, if time permits, we will go to Mach-Zehnder interferometers.

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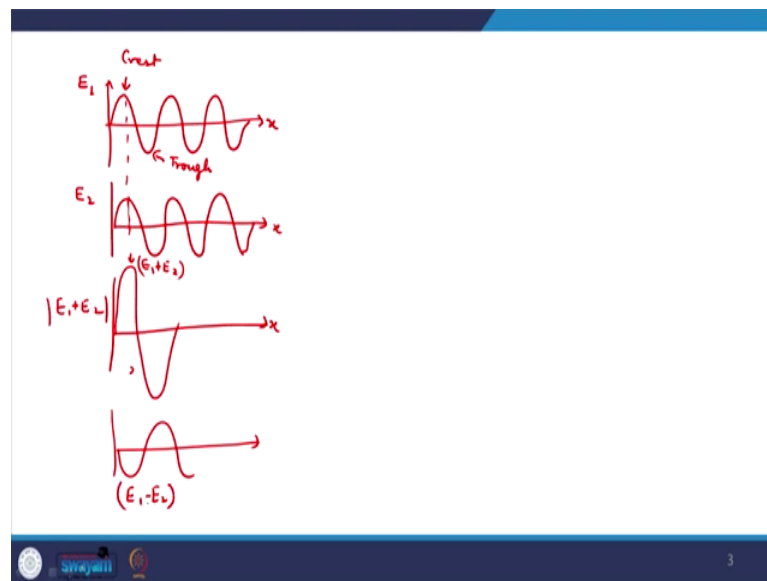


In interference, you know that, if you have two coherent waves and they are superimposed on each other you will get either a maximum or a minimum. Suppose you have an amplifier connected with two speakers and they are producing longitudinal waves like sound. So this travels in the form of compression and rarefaction; these

regions where you have less dense it is rarefaction. So, these longitudinal waves are traveling in form of compression and rarefaction.

And if the waves from the speaker 1 and speaker 2 - let us say, they are super imposed over each other and if they interfere constructively, then this microphone detects a loud sound. However, if they interfere destructively, the microphone detects little or no sound.

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This happens not only for longitudinal waves, but also for transverse wave. Suppose you have a wave, which is traveling in x direction and it has electric field amplitude  $E_1$ , suppose you have other wave which is traveling in x direction and it has amplitude  $E_2$ . And if they are super imposed over each other and say at certain position of x and if their crest - this is now called crest and this is called trough - if the crest lies over crest and trough lies over trough, then you will have the amplitudes added. So, it will be  $E_1$  plus  $E_2$ , here at this point. So, resultant amplitude -  $E_1$  plus  $E_2$  everywhere.

And if they are out of phase - this is mod  $E_1$  and if they are out of phase like this, so it will be  $E_1$  minus  $E_2$ . This is how the electric fields vary and when these electric fields vary, then the intensity can be given by  $I$  is equal to mod  $E_1$  plus  $E_2$  square, which becomes  $E_1$  square plus  $E_2$  square plus  $2 E_1 \cdot E_2$ . These are all, actually, fields so it is like this.

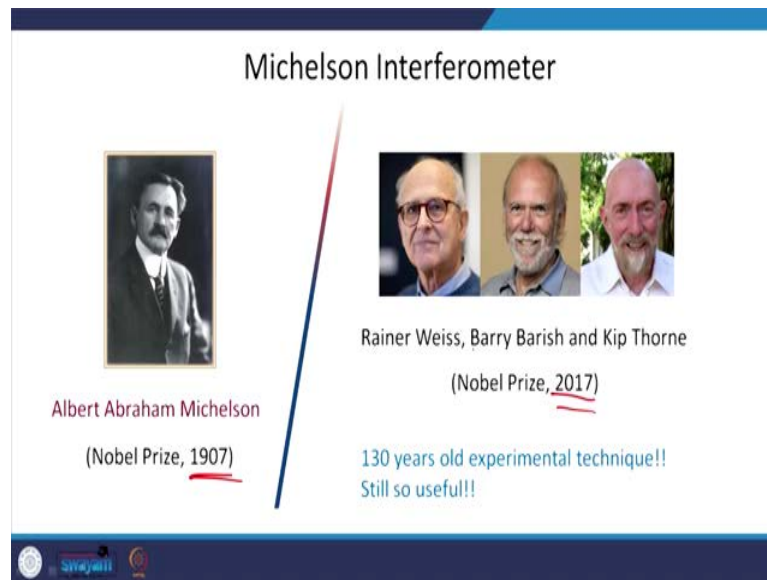
If you write it in terms of intensities, you get  $I$  is equal to  $I_1$  plus  $I_2$  plus this extra term. So, the resultant intensity is not just equal to the intensities of these two waves, but also this extra term which is the interference term. And if the intensities of the two waves which are interfering with each other are same, let us say it is  $I_0$  then  $I$  becomes equal to  $4 I_0 \cos^2 \frac{\Delta}{2}$ .

This  $\Delta$  is the phase term and if the phase difference between these two waves is, say, a multiple of  $2\pi$ , then you will have a constructive interference. If you have  $\Delta$  equal to  $2\pi$  plus one  $\pi$ , then it is destructive interference. So, if you have waves which have phase difference of  $0$  or  $2\pi$  or  $4\pi$ , then you have constructive interference. If the waves have a phase difference of  $\pi$  or  $3\pi$ ,  $5\pi$ , so odd multiples of  $\pi$  - you will have destructive interference.

And this phase basically can be converted in terms of the path difference. And here you can see that if you have a path difference of say  $\lambda$ , then you will have constructive interference. If you have path difference of  $\lambda/2$ , you will have destructive interference. And when you have constructive interference, the maximum intensity can go up to 4 times  $I_0$ . Suppose this term becomes 1, then you have maximum intensity can go  $4 I_0$ .

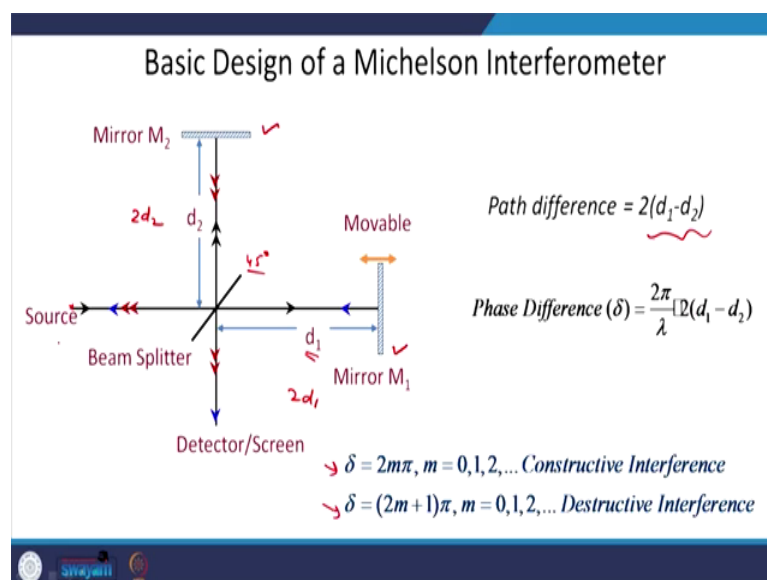
So, the maximum intensity cannot just be the sum of the two intensities, but it can go to 4 times, also the minimum can be 0. But if you write the average intensity, so this  $\cos^2$  - this term, the average value is  $1/2$ . So, average value will be  $2 I_0$ . So, it is still OK, that if they have two waves with intensities  $I_0$  each, then the average intensity will be  $2 I_0$ , but when you see the intensity distribution over a period of space it is  $4 I_0$  at certain places and 0 at certain places.

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Here we will discuss a particular case of Michelson interferometer. Michelson got Nobel Prize for this kind of interferometer which we are going to discuss, and it was in 1907, 130 years after this in 2017 these 3 people got Nobel Prize. So, see this is a 130 years old technique, but it is so powerful, it is so useful. We will discuss why they got this Nobel Prize, but let us see what the basic design of a Michelson interferometer is.

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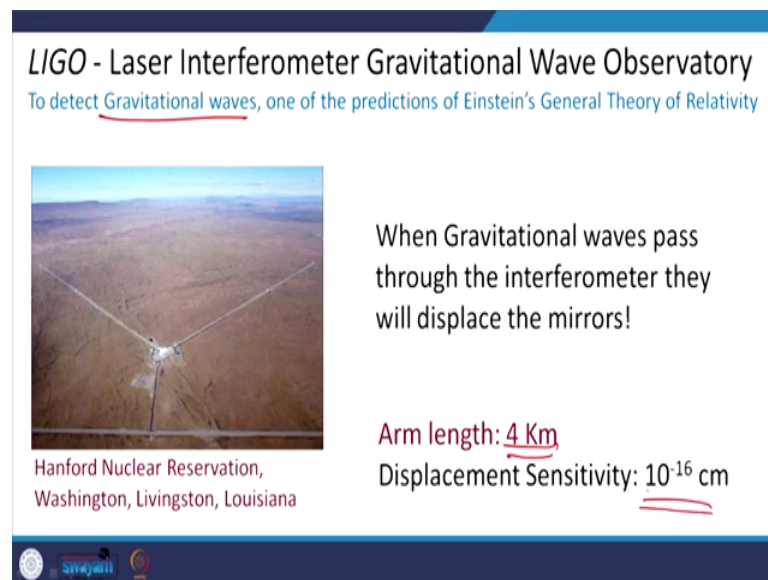


Here you have two mirrors and one of these mirrors can be movable, say mirror M 1 and M 2 and then they are kept at 90 degrees to each other. Then you have a source, and from

the source what happen that you put a beam splitter in such a way that 50 percent light goes here, and 50 percent light goes there. It is kept at 45 degrees to this source. So, half of the light goes up, half of the light goes straight, the light which goes there, say, at mirror M 1 placed at a distance  $d_1$  from this beam splitter, it goes there and it comes back, so it travels  $2d_1$ . Similarly, here it becomes  $2d_2$  - this is the path difference.

So, what happens? If you see it here, you have a path difference of  $2d_1$  minus  $d_2$  here - of the light with got reflected. So, the phase difference is related to it like  $2\pi$  by  $\lambda$  into  $2d_1$  minus  $d_2$ . So, these are the condition. So, we can see always convert it to the condition for path difference.

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After 130 years they got this Nobel Prize because of detection of gravitational waves and that is done using laser interferometer gravitational wave observatory, where you can see that this is a Michelson interferometer and each of the arms is in kilometers. So, it has, again, two arms - one mirror is here, one mirror here and the two arms. And when gravitational wave passes through the interferometer, they will displace the mirror - they are so sensitive. Because of this only, they got the Nobel Prize. And arm length is 4 kilometers. It has displacement sensitivity of 10 to the power minus 16. Can you imagine? I mean - it is so sensitive, that is how it can detect gravitational waves.

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### Bright and Dark Fringes in Michelson interferometer

Path difference  $= 2(d_1 - d_2) = 2d$  (say)

$\underline{2d} = \underline{m\lambda}$  ( $m = 0, 1, 2, \dots$ ): Maxima ✓

$2d = \left(m + \frac{1}{2}\right)\lambda$  ( $m = 0, 1, 2, \dots$ ): Minima

**Order of the fringe:**

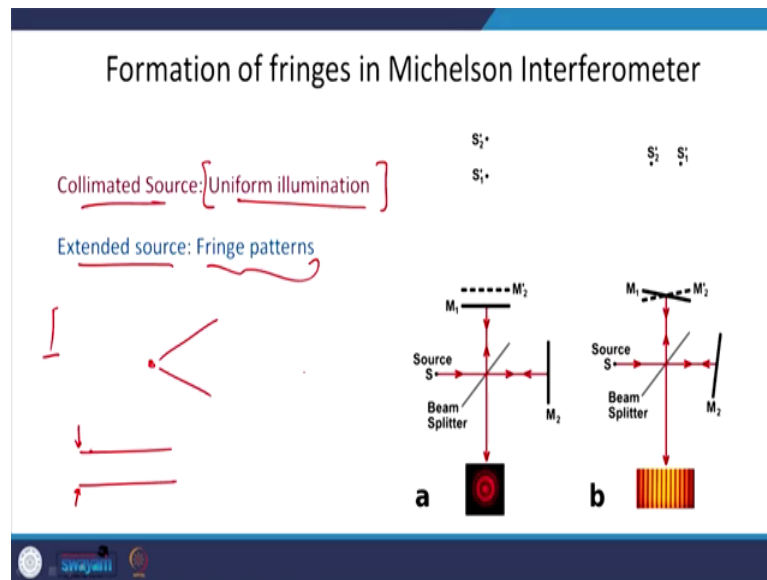
When the central fringe is bright, the order of the fringe is  $\underline{m} = \frac{2d}{\lambda}$

- As  $d$  is increased new fringes appear at the center and the existing fringes move outwards, and finally move out of the field of view.
- For any value of  $d$ , the central fringe has the largest value of  $\underline{m}$ .

So, what happens actually - when you have interference you will have bright and dark fringes, and when we convert phase difference to path difference in the interference condition, we see that when you have this path difference integral multiples of lambda you will have maxima, if you have odd integral multiples, then you will have minima. And the order of the fringe is given by this relation, so  $2d$  by lambda is equal to  $m$ . So, if it is  $m$  is equal to 0 that is 0th order, then first order, then second order, something like that.

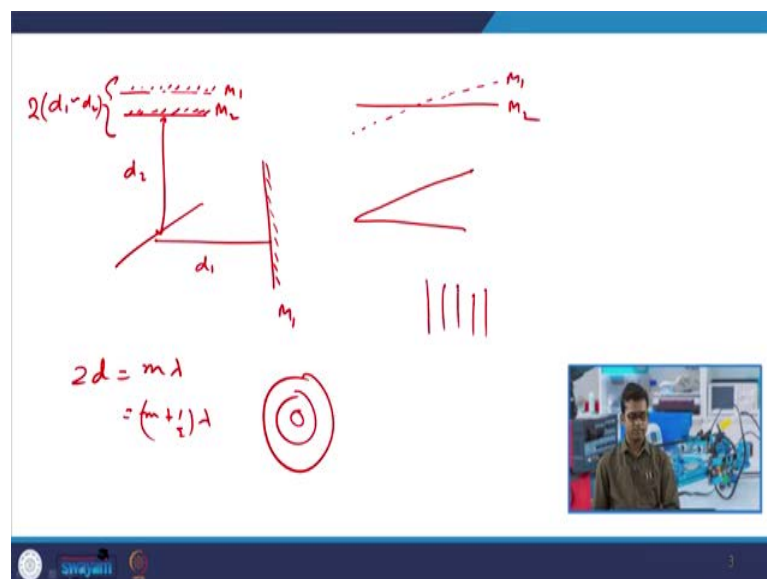
As  $d$  is increased new fringes appear at the center and the existing fringes move outwards and finally, move out of the field of view. So, for any value of  $d$  the center fringe has the largest value of  $m$ , ok.

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Formation of fringes in Michelson interferometer: If you have a collimated source, a source which is having a beam of light like this laser, either you see a maximum or a minimum. So, you see uniform illumination. Either it will be a bright one or dark illumination, you do not see any fringe. But if you have extended source like this or like this point source or extended source - then you see fringe patterns. And if you remember the fringes for a thin film which you might have studied in graduation courses in the 1st year or 2nd year of graduation, what you see is that - you will get circular fringes.

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So, what happens actually? That you can think this setup as this - you have mirror M 1 and M 2, which is say this distance d 1, this is distance d 2, what happens actually that if say image of this mirror can be seen like this, ok. And this is d 1 difference d 2 - actually 2 times d 1 difference d 2. So, it is kind of a film. So, it kinds of a film and if this 2d is m lambda you will have maximum, if it is m plus half lambda you have minimum. So, you are getting circular fringes like this.

But if these are not exactly at 90 degree, there is slight inclination then what will happen? You have this M 2 here and M 1 will be like this - the image of M 1, they are not exactly perpendicular. Then you will have a wedge-shaped film, and this will give you straight fringes – straight line fringes. So, you will get this kind of fringes - depends how you adjust it. There are screws where you adjust the plane of the mirror, so that you get circular or this kind of fringes.

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Applications

1. Measurement of wavelength of light

$$2d \cos \theta = m\lambda$$

$$2d = m_0 \lambda \quad (\theta = 0)$$

Move one of the mirrors to a new position  $d'$  so that the order of the fringe at the centre is changed from  $m_0$  to  $m$ .

$$2d' = m\lambda$$

$$2|d' - d| = |m - m_0| \lambda = n\lambda$$

$$\lambda = \frac{2\Delta d}{\Delta m}$$

Applications of Michelson interferometer: we know  $2d \cos \theta$  is equal to  $m \lambda$ . Now, if you put  $\theta$  is equal to 0 - we assume that there is no tilt, we are getting this thing. Now, one of the mirrors is moved. When I showed you this thing here, one of the mirrors was movable. It changes positions so  $2d$  changes, so the path difference changes. And when the path difference changes what will happen? - That the order of the fringe will change.

So, by measuring the change in order of the fringe that is  $\Delta m$ , and you know how much you moved, you know  $\Delta d$  and  $\Delta m$ , you can always estimate what  $\lambda$  is. This way you can estimate the wavelength of light used. So, this is very useful. I will show you later that how it can be used for sensing.

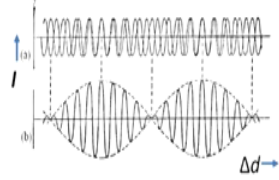
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**2. Measurement of wavelength separation of a doublet  $\lambda_1$  and  $\lambda_2$  ( $=\lambda_1 + \Delta\lambda$ )**

Resultant = Interference due to  $\lambda_1$  + Interference due to  $\lambda_2$

$\lambda_1$  &  $\lambda_2$  Beating: Stable interference pattern is not observed

$$I_{1r} = 2I_1 \left[ 1 + \cos\left(\frac{4\pi}{\lambda_1} \Delta d\right) \right]; \quad I_{2r} = 2I_2 \left[ 1 + \cos\left(\frac{4\pi}{\lambda_2} \Delta d\right) \right]$$

$$I = 2(I_1 + I_2) + 2I_1 \cos\left(\frac{4\pi}{\lambda_1} \Delta d\right) + 2I_2 \cos\left(\frac{4\pi}{\lambda_2} \Delta d\right)$$



The two  $\lambda$ s beat. We cannot measure it. All we can measure is the Intensity. Also, if  $\lambda_1$  &  $\lambda_2$  are quite different, beating will be too fast to be observed.

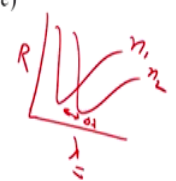
Suppose we are given the beat curve. We can draw inferences about  $\lambda_1$  &  $\lambda_2$  by fitting some value to  $(4\pi\Delta d / \lambda)$ , since  $\Delta d$  is known.

Suppose, you have two wavelengths which are close to each other, then you will have these two relations for these two wavelengths. So, you will have a beating pattern, which will have these two terms. So, we cannot measure it – it is problem. All we can measure is the intensity, ok.

So, if  $\lambda_1$  and  $\lambda_2$  are very different, the beating will be very fast, and it will be too fast to be observed. But what inferences we can draw is that - we can know what is  $\lambda_1$  and  $\lambda_2$ . How? If you fit some value to this, then we know what  $\Delta d$  is. So, what we do is that you have this curve for small  $\Delta d$ , for the same one you have - for  $\lambda_2$ , you will have different intensities and then you can have this beating.

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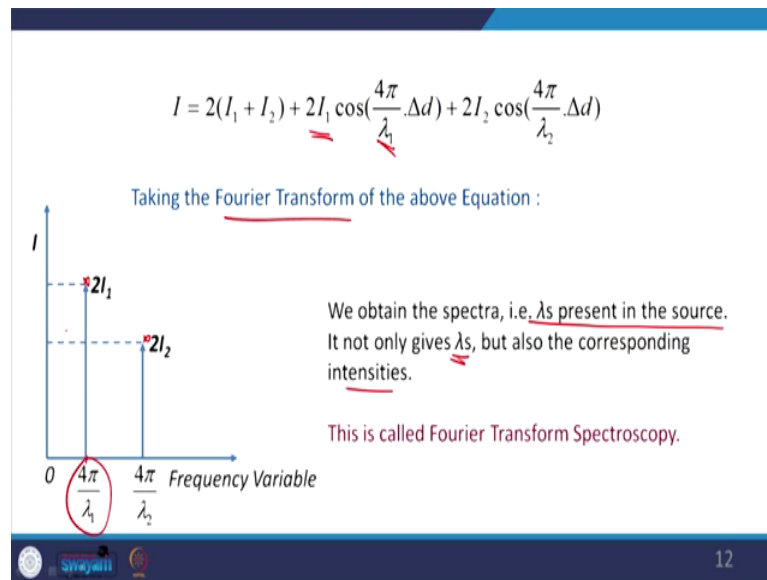
$$\begin{aligned} \rightarrow 2\Delta d &= m\lambda_1 \text{ (For Constructive Interference)} \\ 2\Delta d &= (m+1/2)\lambda_2 \text{ (For Destructive Interference)} \\ \Rightarrow 2\Delta d \left[ \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right] &= \frac{1}{2} \\ \Rightarrow 2\Delta d \left[ \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right] &= \frac{1}{2} \\ \Rightarrow (\lambda_1 - \lambda_2) &= \frac{\lambda_1 \lambda_2}{4\Delta d} \\ \text{For very small wavelength difference, } \lambda_1 &= \lambda_2 = \lambda \\ \Rightarrow \Delta\lambda &= \frac{\lambda^2}{4\Delta d} \end{aligned}$$


Let us say that for  $\lambda_1$  it becomes constructive and we choose  $\Delta d$  in such a way that for  $\lambda_2$  it becomes destructive. So, if you solve for it what you get is that  $\Delta\lambda$  is equal to  $\lambda^2$  by  $4\Delta d$ . For very small wavelength say if the difference is very small then we get this relation.

Suppose you have two lines - spectral lines which are very close say by  $\Delta\lambda$ , you can use Michelson interferometer to detect the shift in  $\Delta\lambda$ . Now, see how it can be used for sensing. So, I told you in the beginning that - when we were discussing say plasmon resonance sensor we were plotting here  $\lambda$  and here you were plotting the intensity reflected light and then you were getting this kind of curve if you remember this was for  $n_1$ , this was for  $n_2$ .

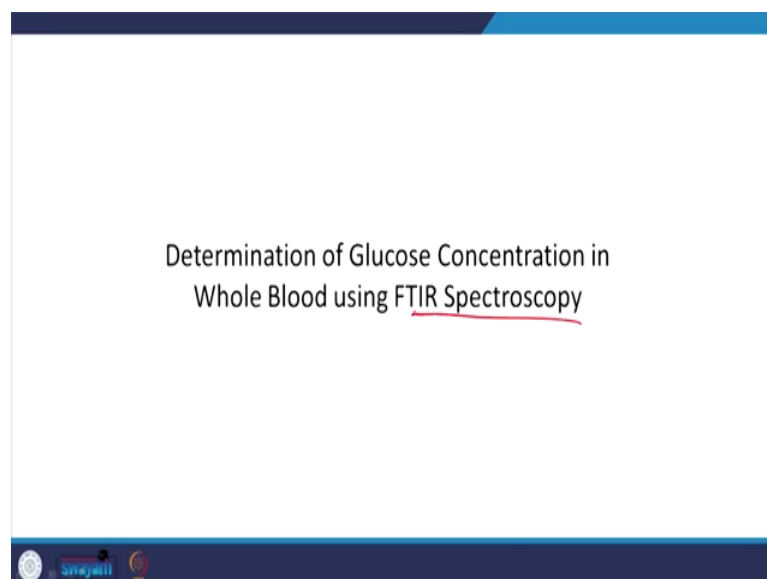
Now, I want to measure  $\lambda$ . I told you how to measure  $\lambda$ . So, if you have Michelson interferometer, you can measure the wavelength of light, ok. So, you can measure which  $\lambda$  is coming. Also, you can measure this  $\Delta\lambda$ . If they are very close, you can choose a  $\Delta d$  in such a way - you have to tune  $\Delta d$  in such a way that you know what  $\Delta\lambda$  is, but this happens only for very close. So, this can be used for high resolution sensors, ok.

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I come, again, back to the same thing that if you have two beating frequencies, what you can do is that you simply take a Fourier transform of above equation. So, for every  $4\pi$  by  $\lambda_1$  you will have intensities like this. So, from this you can know what the  $\lambda$ s are present in the source and it will not just give you  $\lambda$ s, but also the corresponding intensities - it is very distinct. This is called Fourier transform spectroscopy. It is very useful technique, let us see how.

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We will show you something called Fourier transform IR spectroscopy. Let us say that our source is in IR and we want to determine the glucose concentration in the whole blood using this technique. So, FTIR technique - what is this? You have an IR source and this IR source is having multiple wavelengths. It is not just one wavelength - it is a broadband source and you have a sample here in (Refer Time: 17:46) one of the arms. What would happen?

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**What is FTIR Spectroscopy?**

The reference mirror is scanned and the interferogram is recorded:

$$\underline{I(\omega, t)} = \int_0^{\infty} S(\omega) T(\omega) \left\{ R_1 + R_2 + 2\sqrt{R_1 R_2} \cos(\varphi_0 + \omega t) \right\} d\omega$$

This is the cosine Fourier transform of the source spectral power  $S(\omega)$  multiplied by the transmission of the sample  $T(\omega)$

Hence FT of  $I(\omega, t)$  and normalization to the source gives  $\underline{T(\omega)}$

Or the absorption:  $A(\omega) = 1 - T(\omega)$

You can move the reference mirror which was this mirror, and you can have I as a function of delta d on that will come in terms of phi naught plus omega t into d omega. So, if you solve for the Fourier transform, you can have Fourier transform of I omega t and normalization to this gives T omega, that is the transmission with respect to omega or in terms of absorption. Because you know intensities, you can always convert it to transmission or reflected or absorption power.

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Background

- Whole blood:
  - \*45% cellular components
  - \*55% water and dissolved solids *Serum*
  - \*Clinical level of glucose: 2-30mM (**5mM, 90mg/dL**), (remember: 1mM=18mg/dL) *Normal glucose level*

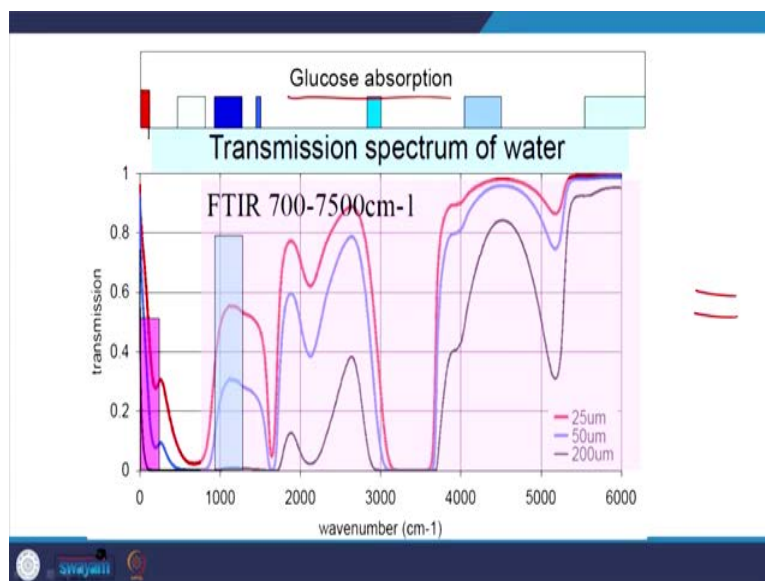
FTIR spectroscopy

- • High water absorption
- Complex blood matrix

Let us go to the background to know what the whole blood is. It has 45 percent of cells, and 55 percent of it is water and dissolved solids - it is like serum. Clinical levels of glucose are like this, normal blood glucose is like this. So, what is the use of FTIR spectroscopy? Because in IR you have high water absorption, so you can have this directly working for the analyte rather, you are having a receptor and also it can be used for complex blood matrix because the blood cells are of few microns, so it is of the same order of magnitude as of the wavelength.

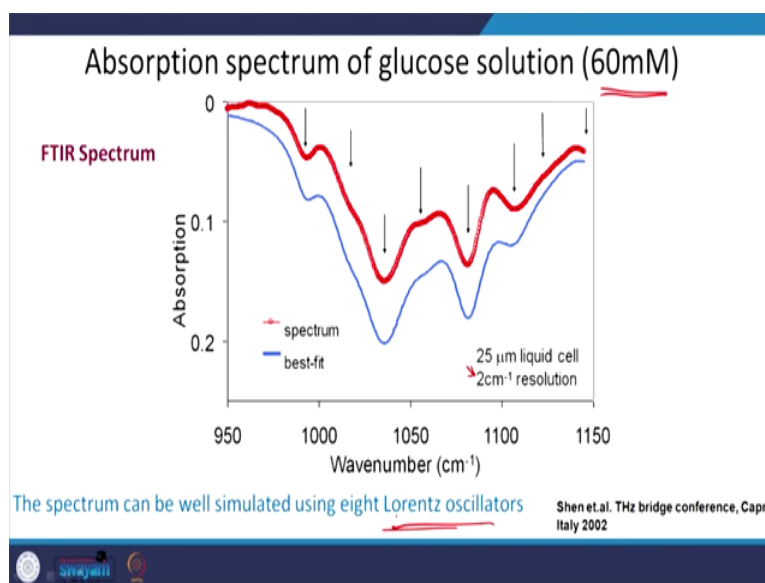
So, basically it is not getting scattered much. It is considered a whole blood matrix as one analyte, does not see much about the composition.

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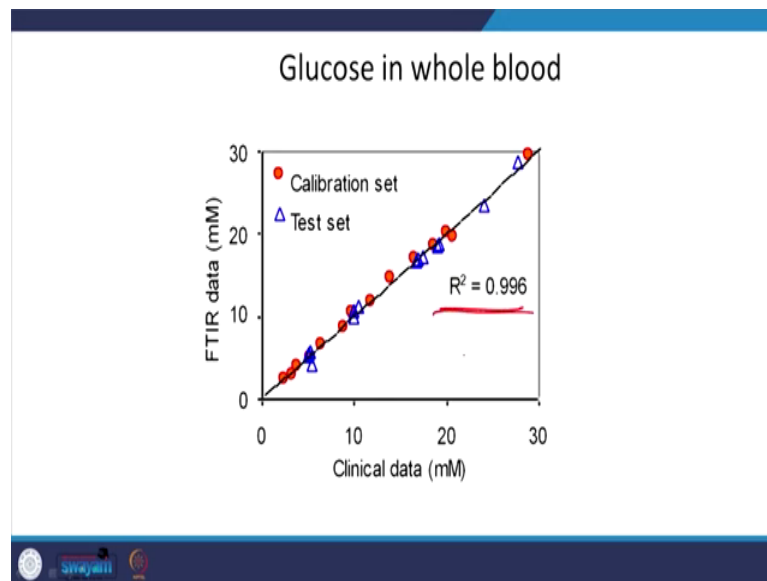
Here I have plotted the transmission spectrum of water for different thicknesses of the say flow cell. You can see that 25-micron, 50-micron, 200-micron flow cells are like the thickness of the water column, and you can see the red, blue and black curves here. And these are the regions - the shaded ones, which are the glucose absorption regions. So, you can see that there is a fair match about 1000 and these are the wavelengths where you can have glucose absorption and from the FTIR spectrum you can see the peaks here to determine what are glucose concentration.

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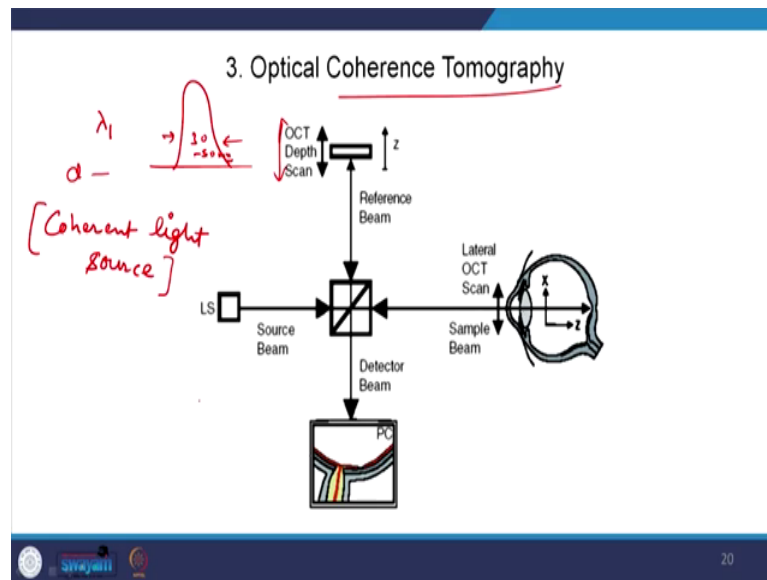
Here I show you the absorption spectrum of glucose, say for 60 milli molar concentration and for the FTIR spectrum you can see that here until a 2-centimeter inverse resolution you can have this kind of curve here. So, it shows you the glucose concentration. Now, if you change the glucose concentration this will start changing - the absorption. This spectrum can be simulated also, if you use Lorentz oscillators. What you do is that you have Lorentz oscillator model of this system and then you incident on it with IR wavelength. So, that is what you can do.

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So, using this technique what was done is that for FTIR data and the clinical data you can see that there is very fair match. So, you can use it for determination of glucose in whole blood.

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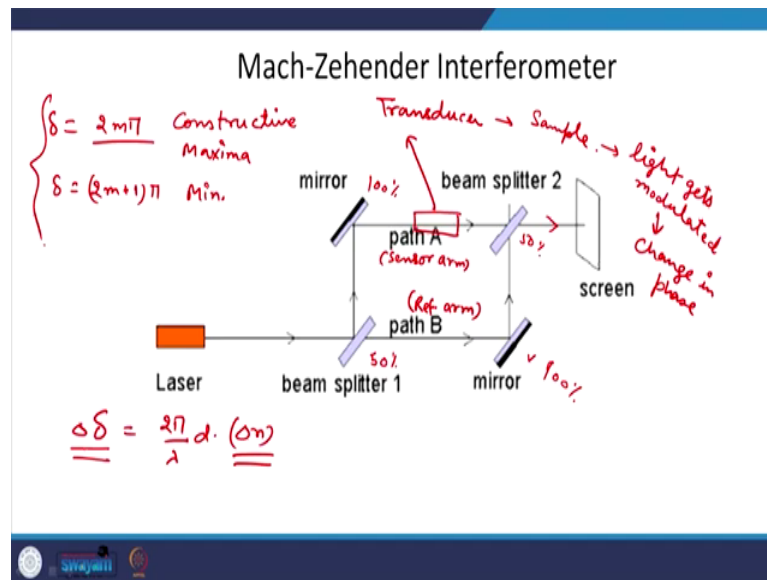


Optical coherence tomography: We know in Michelson interferometer that these two beams interfere and then you get the information about  $\Delta d$ . That happens for say wavelength  $\lambda$  -  $\lambda_1$ . Now, if you take a polychromatic source, not exactly, but say having a small spectral range - narrow spectral range, say 30 to 50 nanometer and for each wavelength, it has a maximum at different  $d$  values. Then what you can do is that you can do imaging using this technique. So, for each  $\lambda$  you can have a depth scan.

This way you can have a scan of few microns inside the thickness of the skin, ok - you can do that! That is what is called tomography - optical coherence tomography. You have to have a quasi coherent beam. It has to be coherent, because you know, that this is the condition for having a sustained interference pattern - that you have to have coherent light sources. But if you have a small spectral width, you know, the coherence starts decreasing.

So, there is a compromise. Where you have this kind of light source which has partially coherent and has a small spectral width, then for each value of  $\lambda$  you can have this maximum or minimum condition - whatever you want to do at different thicknesses of the  $d$ . That is how it is used for imaging of a sample. For different thicknesses it has maximum value and then you can have interference maxima, from there you can conclude how this depth profile looks like.

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Mach-Zehnder interferometer: In this kind of interferometer, you have two mirrors and two beam splitters. Light goes like this. You have a beam splitter which partially reflects and transmits, the light can go from here and here, then again from - this is 100 percent reflecting, this is say 50 percent reflection, and same here 100 percent and here it is 50 percent reflection. So, what happens actually - That these two beams combine here to give you the interference pattern.

So, the basic principle is the same. If you have delta which is  $2m\pi$ , then you will have constructive and then you will have maxima. If delta is  $2m\pi + \pi$ , you have minima. This is the condition, which has to be followed here. So, you will get a maximum, when you have this phase difference and then you can always convert it to the path difference. Now, what we do is that you can always put here a transduction mechanism. So, if you put a transduction mechanism - transducer here and when you put a sample here, the light will get modulated in one of the arms - that is called sensor arm. Suppose here you put - this will become sensor arm, and this will work as reference arm. So, light gets modulated here, this will lead to change in phase and the phase difference here which occurred will lead to change in the intensity profile. So, by measuring the change in intensity profile you can convert it how much change in phase occurred and from there you can conclude that how much.

If you remember that phase was  $-\Delta\phi$  was  $2\pi$  by  $\lambda d$  is fixed into  $\Delta n$ . So, if you change the refractive index by  $\Delta n$ , the phase will change by  $\Delta\phi$ , that is how you measure. So, this kind of phase change can be translated to how much change in refractive index occur. So, that is how you use it for sensing application.

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### Mach-Zehnder interferometer with directional sensitivity

Basic Arrangement of polarisation interferometer

Examples of polarisation states at output port

$\Delta = 0^\circ$     $\Delta = 45^\circ$     $\Delta = 90^\circ$     $\Delta = 135^\circ$     $\Delta = 180^\circ$

Superposition of 2 orthogonally polarized waves yields not only output intensity but polarization ellipse.

Polarization ellipse carries two informations:

Shape and elevation angle.

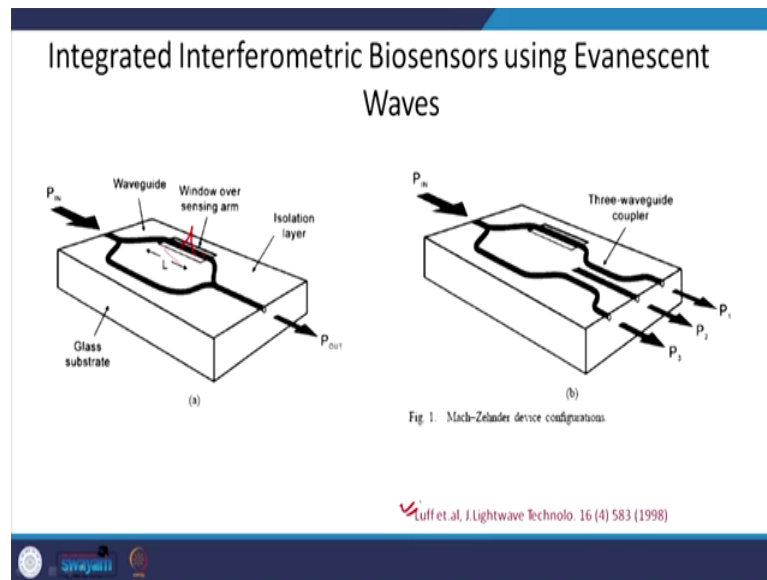
Or:

Phase difference and direction.

Interferometry ITSS 2007

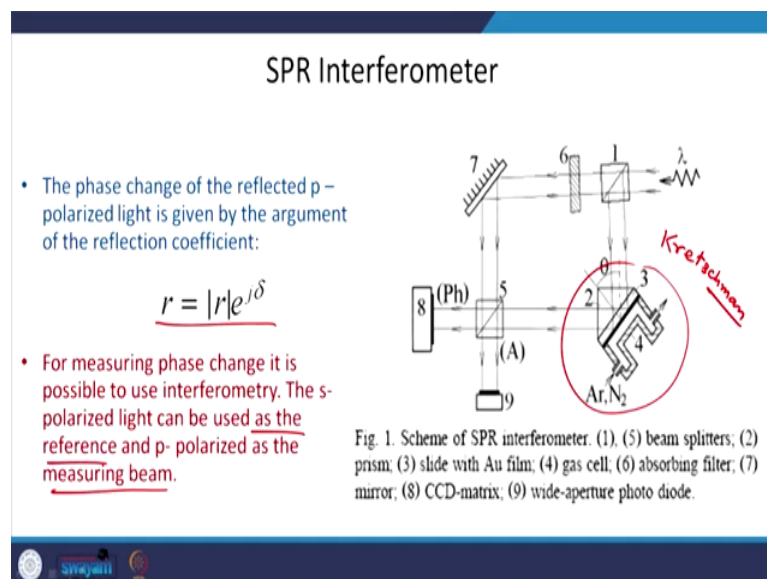
It can also be used with directional sensitivity. Suppose you have now super-position of two orthogonally polarized waves, it will not only change the output intensity, but also the polarization ellipse. Here are two polarizations which are orthogonally polarized and then they are super posed. Polarization ellipse carries two information, one is the shape and other elevation angle - or you can say phase difference and direction. By this way you can have a differential phase in measurements also.

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People have used these kinds of things for integrated bio sensors. I already told you evanescent wave sensing. What they did is that you have a wave guide here on a glass substrate, it has two arms, one of the arms - you put a sample. It is a wave guide, so it will have evanescent field here - evanescent field which interacts with the sample. You can also have 3 wave guide coupler, and if you want to go in more detail you can see this paper. This way you can have evanescent wave sensor using Mach-Zehnder interferometer and this can be integrated on a chip.

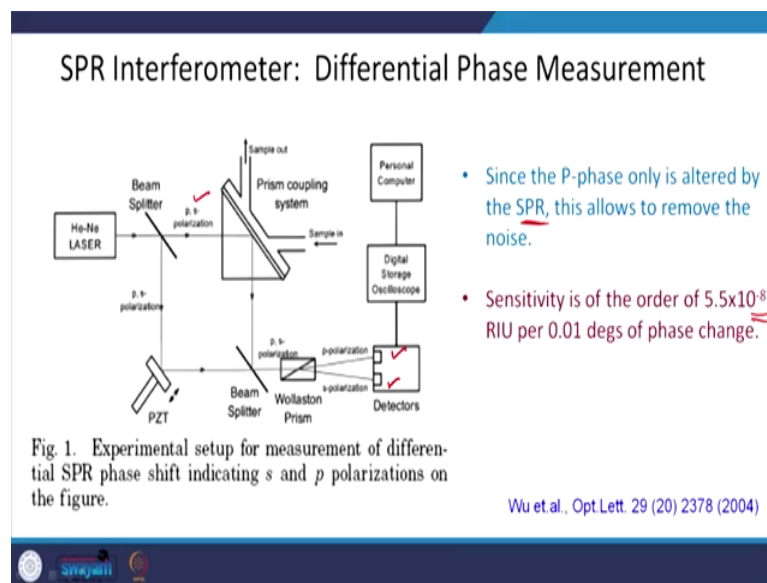
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You can have SPR interferometers. What you do is that in one of the arms you put this prism set up that is Kretschmann configuration, right. So, the phase change of the reflected p-polarized light will be given by this relation and for measuring the phase change it is possible to use interferometry. The s-polarized light can be used as the reference and p-polarized light can be used for the measuring beam because it is the p polarized light which is exciting surface plasmon. That is how you use it for SPR based interferometry sensor.

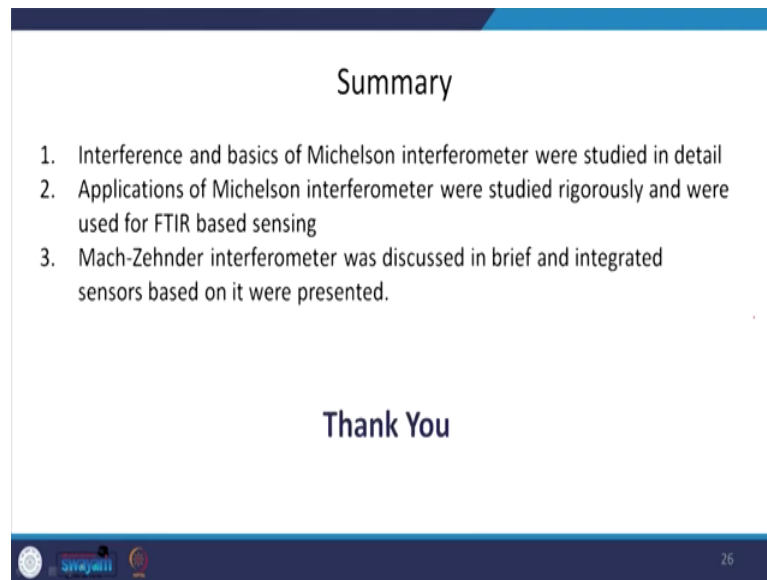
That gives an additional level of sensitivity to it. So, it becomes even more sensitive. You can have differential phase measurement which I already told you that now you have p and s both polarizations which are orthogonal and by putting a Wollaston prism here you can have p and s polarizations and since the p phase is only altered by the SPR, this allows to remove the noise.

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Sensitivity in this particular case is of the order of  $10^{-8}$  RIU per 0.01 degrees of phase change. So, it is very sensitive.

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### Summary

1. Interference and basics of Michelson interferometer were studied in detail
2. Applications of Michelson interferometer were studied rigorously and were used for FTIR based sensing
3. Mach-Zehnder interferometer was discussed in brief and integrated sensors based on it were presented.

Thank You

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Let us summarize this. We discussed today what is the interference and the basics of Michelson interferometer. Then we studied what are the applications of Michelson interferometer and how can it be used for sensing, and we showed one example of Fourier transform IR sensing. So, Fourier transform spectroscopy - we studied what it is, the basics and then why we are going in IR and then how we are using it for detection of glucose in whole blood.

Then we discussed briefly what is Mach-Zehnder interferometer and also, we saw how integrated biosensors on chip, and we also discussed the differential phase measurements which are very important; there you use two orthogonal polarizations to make it more sensitive.

Thank you.