

**Optical Sensors**  
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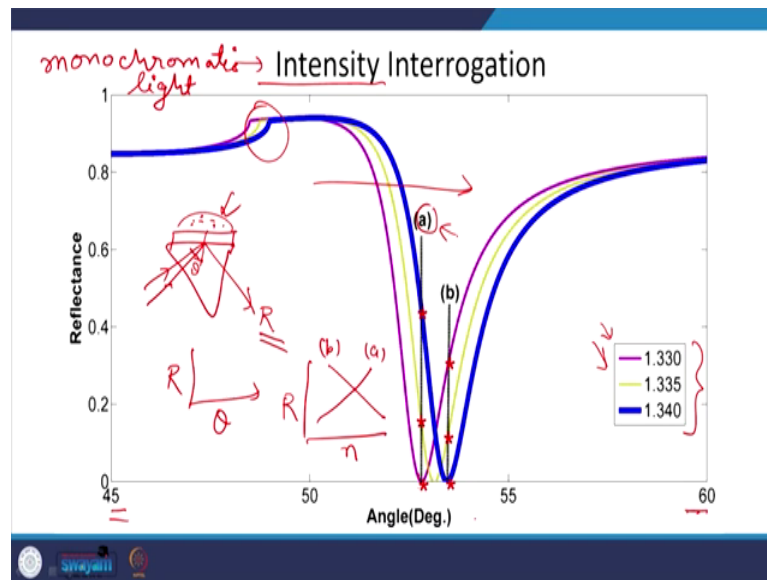
**Lecture – 12**  
**Plasmons - III**  
**Surface Plasmon: Intensity & Phase Interrogation, Simulations Method,**  
**Optimization**

Welcome to the 12th lecture of Optical Sensors course. In the last lecture we discussed the dispersion relation of surface plasmons and we also discussed the issues, which are related to the excitation of the surface plasmons, and there we resolved the issues by discussing the dispersion curve and we said that if you have a high index medium or a grating, and you incident light through that, then you can excite surface plasmons at the other interface of metal and dielectric.

Then, we discussed certain configurations to study surface plasmons (Refer Time: 01:04) and there we studied the angular interrogation, where we were measuring the change in angle that was  $\theta$  or we did the change in wavelength - that was spectral interrogation.

So, that is what we discussed in the last turn. We move ahead and now, today, we discuss what are intensity and phase interrogation techniques. We will also try to study simulation methods - how to simulate a structure for surface plasmon resonance. There we will be using n-layer model. Also, we will discuss the optimization of thickness of this thin film, which were using in the Kretschmann configuration.

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In intensity modulation, you know - I mean, when we plotted the SPR curve - for change in angle with respect to change in reflectance, we saw this kind of dip in reflectance at some angle which was called the angle of resonance.

So, if you increase the refractive index of the medium surrounding the metal, say here it is like 1.330 to 1.340, we see that there is a red shift - here in the angle. So, it is showing right shift. This kind of curve can be achieved, if you have angular interrogation - like you are changing the angle shift from 45 degree to 60 and you are continuously measuring reflectance. So, it is like this. You are changing the angle here - this was the prism and here is the analyte medium and you are measuring the angle - this angle theta and here is R.

So, you are changing the angle theta, say from theta to theta 1 like this and then you are measuring reflectance. When you start increasing the angle, you see that - here - there is a kink. If you remember the total internal reflection, then you can see that this is the angle where total internal reflection is occurring. After that, light becomes total internally reflected. So, it comes back. Before that it was partial reflection. When you are in TIR region and if you continuously keep on increasing the angle, you see that you get a dip in the reflectance and that was SPR. That we are already discussed in angular interrogation technique.

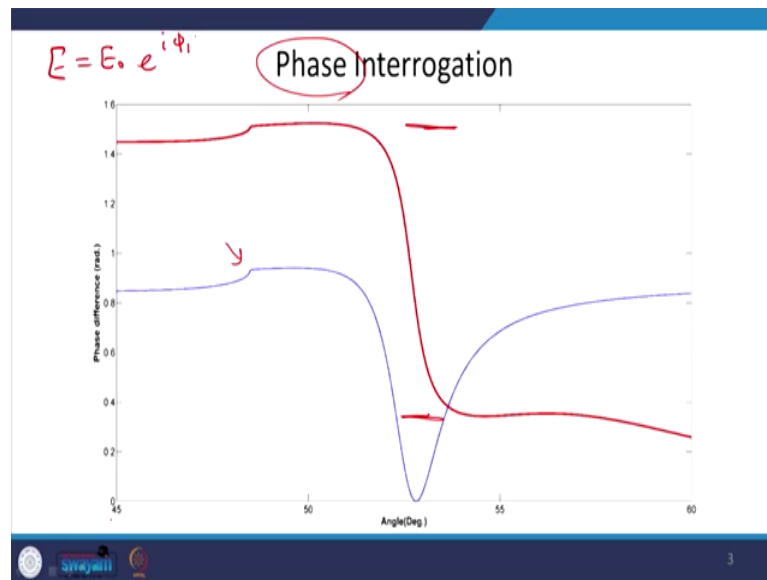
Now, suppose I do not want to change the angle and I still want to measure the SPR. So, what we do is called intensity interrogation. We are still using a laser. So, we are still using a monochromatic light, but we are not changing the angle. I already discussed that if you want to change the angle then it is angular interrogation, if you are not changing the angle then we did spectra interrogation - We were using a polychromatic light and then we were observing the SPR in terms of resonance wavelength. What do you see here - this curve (a) for example - this line, you see that if you just draw a line it intersects these 3 curves at 3 points, ok.

Now, you have another line (b) which is intersecting these at these points - the SPR curve. Let us try to understand what it means? It means that, I have this angle I am just measuring the reflectance. I am measuring the reflected power; I am not changing the angle I am only changing the reflective index of the analyte. So, what will happen that you see that if you have a laser positioned at some angle at (a), then it will happen that when you are increasing the reflective index basically you see an increase in the reflected power.

So, if you plot for  $n$  value and reflected power, you see that with an increase in  $n$  you have increased intensity, while if you position your curve here, you see that it is decreasing. So, it can either be this way or this way. So, it is case (b), this is case (a). So, we see that we can position our laser at such an angle, that you can either measure the increase in reflected intensity or decrease in reflected intensity, while you are changing the reflective index of the analyte.

So, this is how you do an intensity modulation. But, you have to be very precise at which angle you have to put the angle of incidence, because if you are at (b) and you are thinking that it is at (a), then it will be a wrong prediction. So, you have to be careful while you are doing this.

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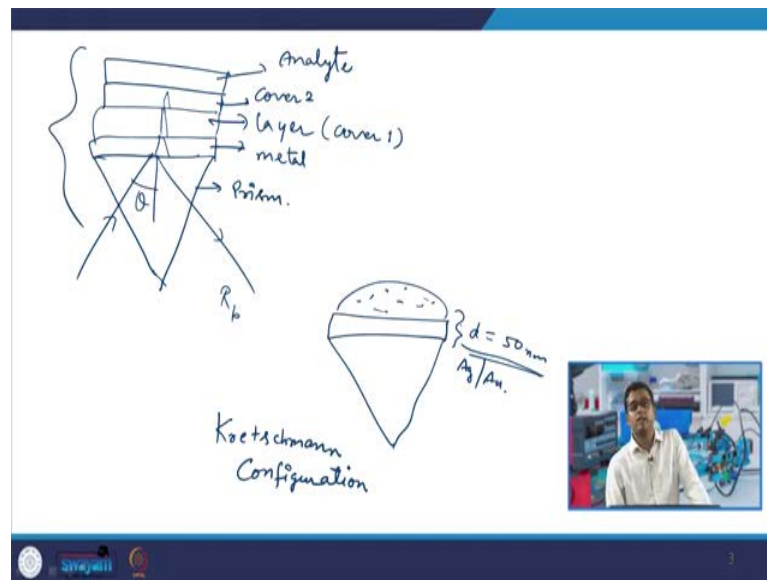


Phase interrogation: if you remember, we were writing an electromagnetic wave as  $E = E_0 e^{i\phi}$  and now when you excite surface plasmons, basically you are changing  $\phi$  - So, it becomes  $\phi + 1$ . And, now what you see is change in intensity that is what will be reflected.

So, suppose this is my SPR curve - this is the blue curve and corresponding to this blue curve, we have this red curve which is the phase curve. So, you see that when there is a resonance the it changes the phase.

So, basically if you are measuring the change in intensity you have this much slope here to here, which is very fast. So, you can either measure the change in phase and this will be very sensitive, because you have this large range and large slope, ok. We will see this when we discuss the sensitivity, you will see that the phase sensitivity much larger than spectral one, angular one and intensity one. But if you want to simulate this kind of films which were discussing -

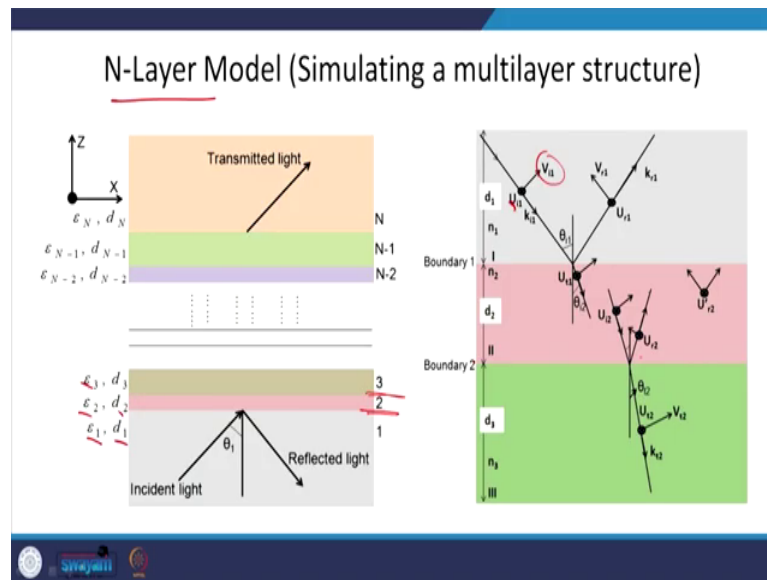
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We were discussing Kretschmann configuration, where you had this prism, then you had this metal and then you had analyte here, this is analyte, you have metal here, it is prism. Suppose, now I add few more layers and then this is layer 1, this is cover say, now say this is cover 2 and then analyte.

If you want to discuss these kinds of configurations, what you do is that you have to solve for electric and magnetic fields, what I showed you in the Fresnel reflection. If, you want to measure the reflected power  $R_p$  as a function of theta, what to do? You have to solve the Fresnel equation. Light will be partially reflected, partially transmitted and then again reflected and again transmitted something like this, right!

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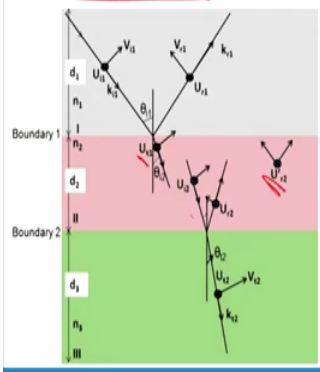


Today we will solve it using matrix method and it is very easy to solve. What we do is - we consider a N layer model - we want to simulate a multi-layer structure. So, suppose we have N layers of refractive indices say epsilon 1, epsilon 2, epsilon 3, so on and the thicknesses of these media d 1, d 2, d 3 and so on. Light is incident in the medium 1, at angle theta 1 and it gets reflected and partially get transmitted after the Nth layer.

So, if you want to simulate it, what you do is that at each of these boundaries, suppose I have taken only 3. So, at each of these boundaries, like this boundary between medium 1 and 2, the interface of medium 2 and 3, you solve for the electric and magnetic field components. So, suppose U is my electric field component, V is my magnetic field component, now I want to solve for it. At this interface, it has to satisfy the boundary condition.

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Relation between U & V

$$\underline{V} = \sqrt{\frac{\epsilon_1}{\mu_1}} n_1 \hat{k} \times \underline{U}$$


Apply boundary conditions for Electric (U) and magnetic (V=B/μ) field

Interface 1

$$\underline{U}_1 = \underline{U}_{i1} + \underline{U}_{r1} = \underline{U}_{t1} + \underline{U}_{r2}$$

$$V_1 = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{i1} - U_{r1}) n_1 \cos \theta_{i1} = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{t1} - U_{r2}) n_2 \cos \theta_{i2}$$

Interface 2 ←

$$U_2 = U_{i2} + U_{r2} = U_{t2}$$

$$V_2 = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{i2} - U_{r2}) n_2 \cos \theta_{i2} = \sqrt{\frac{\epsilon_1}{\mu_1}} U_{t2} n_3 \cos \theta_{i2}$$

So, what we do is that we apply the boundary conditions at all the interfaces. So, if you want to see that electric field we are denoting here U and magnetic field V : for the ease of simulation we do that, because in simulation notations we are keeping U and V as indices. And, the relation between U and V is this; you can always get it from Maxwell equations, I told you in the first or second class. At this interface 1 you have U 1 is equal to U i 1 plus U r 1, that is in the first medium and that should be equal to the electric field in the second medium, that is t 1 dash and U 2 r dash.

It is like this - it is partially getting transmitted, that is U t 1, and partially reflected from here so, U dash r 2 -something like this. Now, it is again getting here so, at the interface 2 again you have to solve for the boundary conditions.

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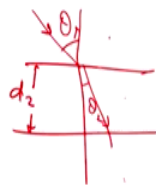
Phase Shift in traversing a film of thickness  $d_2$

$$\beta_2 = \frac{2\pi d_2}{\lambda} (\epsilon_2 - n_1^2 \sin^2 \theta_1)^{1/2}$$

→ Field equations at boundary 2

$$U_{i2} = U_{t1} e^{-i\beta_2} \quad U_{r2} = U_{r2}' e^{+i\beta_2}$$

$$\Rightarrow \underline{U_2} = U_{t1} e^{-i\beta_2} + U_{r2}' e^{+i\beta_2}$$

$$V_2 = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{t1} e^{-i\beta_2} - U_{r2}' e^{+i\beta_2}) n_2 \cos \theta_2$$


So, you apply these boundary conditions and what you see is that the wave which travelled a thickness  $d_2$  - it was like this - if you remember, It was angle  $\theta_1$ , now here say it is  $\theta_2$  and then somehow I do not know what is  $d_3$ ; it is  $d_2$ . So, while travelling  $d_2$ , it will have this much phase difference - this much phase it will acquire while travelling  $d_2$ .

So, if we apply it into the field equation at boundary 2, you will have this much addition of phase to the initial 1. So, if you solve for it -for  $U_2$  you get this relation. You are writing  $U_1$  here and  $U_2$  here and now you are applying the boundary condition. So, what you do is that you to write  $U_2$  in terms of  $U_1$  and see.



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After simplification, we get

$$\begin{aligned} \rightarrow U_1 &= U_2 \cos \beta_2 - V_2 (i \sin \beta_2) / q_2 \\ \rightarrow V_1 &= U_2 q_2 (-i \sin \beta_2) + V_2 (\cos \beta_2) \end{aligned}$$

where

$$q_2 = \frac{(\epsilon_2 - n_1^2 \sin^2 \theta_1)^{1/2}}{\epsilon_2}$$

In matrix form

$$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \cos \beta_2 & (-i \sin \beta_2) / q_2 \\ -i q_2 \sin \beta_2 & \cos \beta_2 \end{bmatrix} \begin{bmatrix} U_2 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = m_2 \begin{bmatrix} U_2 \\ V_2 \end{bmatrix}$$

Handwritten notes on the slide include:

- Diagram of three horizontal lines representing interfaces, labeled 1, 2, and 3 from top to bottom. Arrows indicate incident and reflected waves.
- Matrix equations:  $\begin{bmatrix} U_2 \\ V_2 \end{bmatrix} = m_3 \begin{bmatrix} U_3 \\ V_3 \end{bmatrix}$  and  $\begin{bmatrix} U_3 \\ V_3 \end{bmatrix} = m_4 \begin{bmatrix} U_4 \\ V_4 \end{bmatrix}$ .

Or you write  $U_1$  in terms of  $U_2$ ,  $V_2$  and  $V_1$  in terms of  $U_2$ ,  $V_2$ , where you can simplify it for - you write this index at  $q$ , so, it will become much simple. You see that this is actually a matrix equation. So, electric field at the interface  $U_1$ ,  $V_1$  is multiplication of this matrix which we can write simply  $m_2$  into  $U_2$ ,  $V_2$ .

So, electric and magnetic fields in first layer and second layer are correlated by this relation. Now, suppose if I want to write for the third one, so this was here, at this interface I said that  $U_1$ ,  $V_1$  is connected to  $U_2$ ,  $V_2$  by this relation. This was medium 1, this was 2, now this is 3. So, now, if I want to write here, what it will be -  $U_2$ ,  $V_2$  will be equal to matrix  $m_3$  into  $U_3$ ,  $V_3$ . Similarly, for this one  $U_3$ ,  $V_3$  is equal to  $m_4$   $U_4$ ,  $V_4$ , right! You can write like this.

Now, if I want to write in terms of  $U_2$ ,  $V_2$ , then what it will be? So, it will be like-  $U_2$ ,  $V_2$  will be equal to - in place of  $U_3$ ,  $V_3$  we can write  $m_4$  and this thing. So, it will be  $m_3 m_4$  and this matrix and  $U_4$  and  $V_4$  - something like that right.

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Electric and magnetic fields at the first boundary  $Z = Z_1$  are related to those at the final boundary  $Z = Z_{N-1}$  by

$$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = M \begin{bmatrix} U_{N-1} \\ V_{N-1} \end{bmatrix}$$

$U_{N-1}$  and  $V_{N-1}$  are the corresponding fields at the boundary of  $N-1^{\text{th}}$  and  $N^{\text{th}}$  layers

$$M = \prod_{k=2}^{N-1} m_k = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = m_1 \cdot m_2 \cdot m_3 \cdot \dots \cdot \begin{bmatrix} U_{N-1} \\ V_{N-1} \end{bmatrix}$

- $k$  is the number of layers amongst  $1^{\text{st}}$  and  $N^{\text{th}}$  layer
- There are  $N-1$  interfaces

So, if we keep on doing this thing for  $N$  minus 1 interfaces - you know, if you have  $N$  layers then you will have  $N$  minus 1 interfaces, you will have this relation, where  $M$  is the matrix multiplication for indices  $k$  2 to  $N$  minus 1. So, that is what I was telling you - that you have  $U_1 V_1$  is equal to  $m_1$  into  $m_2$  into  $m_3$  and so on, and then you write  $U_{N-1} V_{N-1}$  - something like this.

So, this is what you do. Now, you do not need to take care of analytical expression at each interface. What you do is that you simply write a matrix for one index and you can keep on changing the index and you can keep on changing the matrix and then you can finally arrive to the last layer.

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Generalized Matrix for any ( $k^{\text{th}}$ ) layer

$$m_k = \begin{bmatrix} \cos \beta_k & (-i \sin \beta_k) / q_k \\ -i q_k \sin \beta_k & \cos \beta_k \end{bmatrix}$$

Where  $q_k = \left( \frac{\mu_k}{\epsilon_k} \right)^{1/2} \cos \theta_k = \frac{(\epsilon_k - n_1^2 \sin^2 \theta_1)^{1/2}}{\epsilon_k}$   $q_1 \dots q_N$

and  $\beta_k = \frac{2\pi}{\lambda} n_k \cos \theta_k (z_k - z_{k-1}) = \frac{2\pi d_k}{\lambda} (\epsilon_k - n_1^2 \sin^2 \theta_1)^{1/2}$   $\beta_1 \dots \beta_N$

Amplitude Reflection Coefficient for 3-layer structure

$$r_p = \frac{(M_{11} + M_{12} q_2) q_1 - (M_{21} + M_{22} q_2)}{(M_{11} + M_{12} q_2) q_1 + (M_{21} + M_{22} q_2)} \Rightarrow R_p = |r_p|^2$$

If you read the generalized matrix for any  $k^{\text{th}}$  layer, that is given by this relation, where  $k$  can take values from 1 to  $N$  and you write this  $q_k$  equal to this thing; here  $\beta_k$  is this.

For each layer you can define it, right! You can write  $d$  and  $\epsilon$  values for each layer and you can define  $\beta_k$  and  $q_k$  and that is how you define  $m_k$ . So, the amplitude reflection coefficient for, say, 3 layer structure is given by this relation. If you have  $N$  layer structure, you replace this 3 with  $N$  - this thing. So, you replace it with  $N$  and you will have  $N$  layers. So, whenever you want to do that you can always use that.

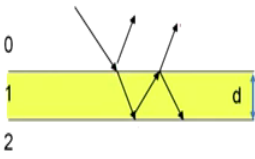
So, generalized matrix for any  $k^{\text{th}}$  layer can be given by this - this expression, where  $\beta_k$  is this one,  $q_k$  is here. So, you just change the index from 1 to  $N$  and you can have  $q_1$  to  $q_N$  values and, here you can have  $\beta_1$  to  $\beta_N$  values. And you substitute it. So, you can get  $m$  matrices from  $m_2$  to  $m_{N-1}$ . If you write the amplitude reflection coefficient say for 3 layer model, then it will have this relation where these are  $q_3$  things, if you want to have a  $N$  then you can have it as  $N$ . So, it will be a  $N$  layer model - just change this.

The reflected power will be given by  $R_p$  is equal to mod of amplitude reflection coefficient square. This is how you solve a  $N$ -layer model. You write a program for this and every time you can change on the integer of  $k$  here, and you run the for loop for this  $q_k$  and  $\beta_k$  and you calculate the  $R_p$ . Now, you change  $\theta$ , this will change and, similarly you will keep on doing this for all the values and then you will have- for an

array of theta you will have an array of R p. So, you can plot it and it gives you the reflection curve.

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**Thin Film Thickness Effect**




Airy formula

$$R = |r_{012}^p|^2 = \left| \frac{E_r^p}{E_0^p} \right|^2 = \left| \frac{r_{01}^p + r_{12}^p \exp(2jk_{z1}d)}{1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d)} \right|^2$$

Where  $r_p = \left( \frac{k_{zi}}{\epsilon_i} - \frac{k_{zj}}{\epsilon_j} \right) / \left( \frac{k_{zi}}{\epsilon_i} + \frac{k_{zj}}{\epsilon_j} \right)$

The dispersion relation (waveguide condition):  $1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d) = 0$



Let us now try to understand what happens to this thin film effect. If you remember, we were discussing Kretschmann configuration. In Kretschmann configuration, we had a prism, on the top of prism - we had a thin layer of metal d is equal to about 50 nanometers of Ag or Au and then we had the analyte on top of it - this is called Kretschmann configuration. We want to see why we need to take d around 50 nanometers, what happens if I take if I take 100 nanometer or so - We need to optimize the thickness.

Let us see what happens to the SPR when we change the film thickness. If we write the dispersion relation in terms of the reflectance from each interface, you can give it through Airy formula which is like this, and you can write it like this. Again, it is the same relation - you know, like r; r is given now in terms of the reflectance from the first interface and the from the second interface, that is why you are writing 0 1, 1 2 and then this is the phase term, which is added after it traverses the film. So, this is a function of d and we want to see what happens.

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### Dispersion Relation

The poles of the reflectivity function give the waveguide modes:

$$1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d) = 0$$

For the symmetric case ( $\epsilon_0 = \epsilon_2$ ) it reduces to:

$$\begin{cases} \epsilon_1 k_{z2} + \epsilon_2 k_{z1} \tanh(k_{z1}d/2i) = 0 \\ \epsilon_1 k_{z2} + \epsilon_2 k_{z1} \coth(k_{z1}d/2i) = 0 \end{cases}$$

For thick metal film (large d) we get:  $\epsilon_1 k_{z2} + \epsilon_2 k_{z1} = 0$

Which together with:  $k_{zi} = \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon_i - k_x^2} \Rightarrow k_x = \frac{\omega}{c} \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$

The poles of the reflectivity give the waveguide modes and if you solve for the symmetric case: let us consider it like this - that this is metal, then here it is also epsilon naught; here also epsilon naught - epsilon 0 may be, then you have this relation and from there we again arrive to the surface plasmon's wave vector case.

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When we started discussing surface plasmons, what we were doing is that we were discussing a semi-infinite model - e were saying that here we have metal and here it is a dielectric.

But, now, we have a thin layer of metal - here only this much, and here also is dielectric. So, it becomes kind of dielectric metal dielectric problem. We call it IMI problem, Insulator - Metal - Insulator. Similarly, you can have MIM also - two metals and a thin layer of dielectric. These kinds of a structures are a matter of interest for lots of people, but what happens, I mean, - how the surface plasmon mode changes with this thickness is something which is of interest to us.

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SPR Shift due to Thin Metal Film

When the metal film becomes thin, the dielectric above (glass prism for example) starts to affect the resonance and causes a shift of the SPR wave-vector by:

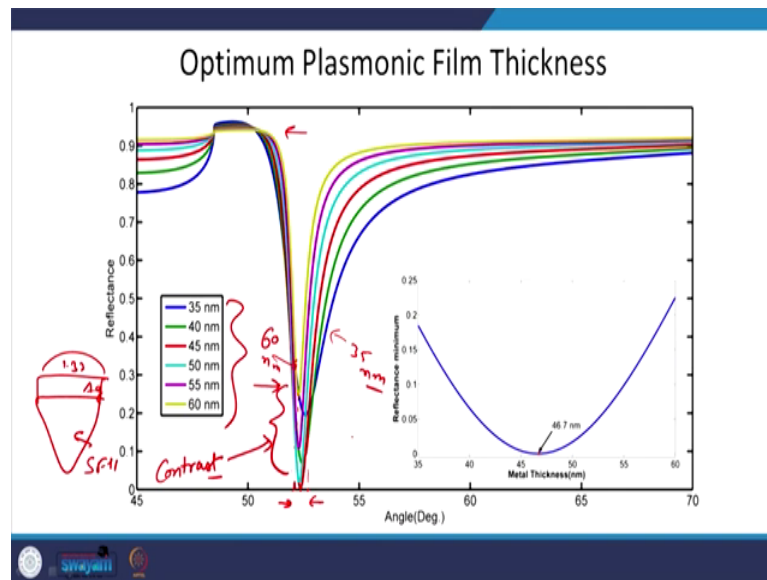
$$\rightarrow \text{Re}\{\Delta k_x\} = \text{Re}\left\{2r_{01}^p \exp(2jk_{z1}d) \left(\frac{\omega}{c}\right) \left(\frac{\epsilon_1\epsilon_2}{(\epsilon_1 + \epsilon_2)}\right)^{3/2} \left(\frac{1}{(\epsilon_1 - \epsilon_2)}\right)\right\}$$

Depending on the thickness, the light reflected and the light released by the plasmons interfere destructively. This will happen at an optimal thickness  $d$  at which they will be totally out of phase. The typical thickness is approximately 50nm.

So, let us see what happens. SPR shifts due to this thin metal film. So, what happens that when the metal becomes thin, the dielectric above it starts to affect the resonance and causes a shift to the SPR wave vector. This is the small change in the real part of the wave vector with respect to change in the thickness of the film. So, if you make it thin, say from 50 to 40 nm, there will be a small change in the wave vector and that wave vector is because of the change in the prism layer, ok.

So, depending on the thickness the light reflected, and light released by the plasmons interfere destructively. And, this will happen at an optimal thickness only, where the light which is coming from the film and light going towards the film will be completely out of phase. And, this thickness is approximately 50 nanometer.

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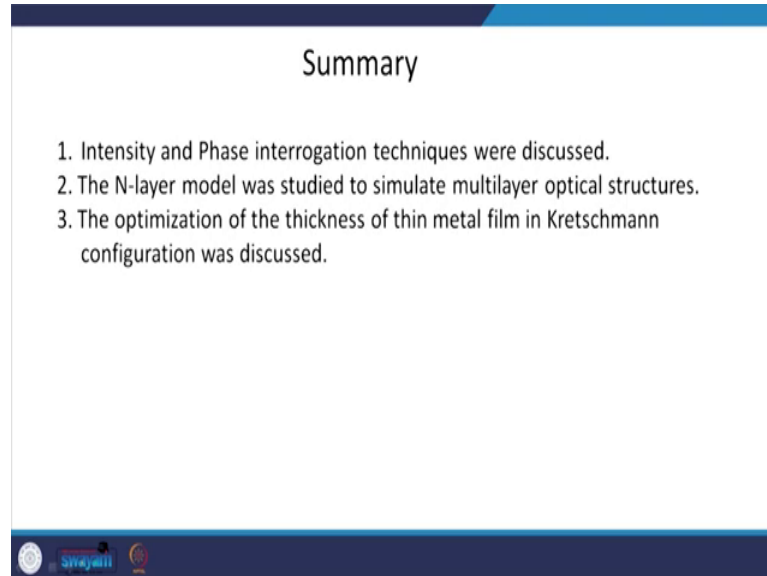
So, here I show you SPR reflection curves for different thicknesses of silver. Here this is a silver configuration on SF 11 prism. You have SF 11 prism - this one and then you have Ag and on top of it you have water, 1.33. What you see here, that when you start increasing the thickness from 35 nanometer to 60 nanometer, this blue curve – here shows a resonance here. And, then it keeps on going down, down, down and then for 60 you see this one. This one is for 60 nanometer, this was for 35.

So, what you see here? Two things – first, there is a slight change in the resonance angle. You see that the blue curve - its resonance is here, for the red one it is here, for this one it is here - somewhere here. So, there is a small shift in the resonance angle, that is what we were discussing that when you lead to change in phase, basically you will lead to change in the resonance angle. That is why you see a small shift. Also, you see this contrast thing - contrast means the maximum minus minimum.

So, you see, this is up to here and in for the red one it is completely dark. So, when you have the optimum thickness - you can see it from here also actually that from the inset, that at about this thickness the reflectance minimum is equal to 0. What it means? It means that there is a particular thickness on that one only you will have 0 reflectance. And, that is because the waves which are going towards the film and the ones which are released by the plasmons - only when they interfere destructively, you will have this

condition. So, it means that the difference in intensity also occurs, when you change the thickness - it is not just the phase, but also the change in intensities which does matter.

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To summarize, we discussed intensity and phase interrogation techniques today and we said that in addition to angular and spectra interrogation, we can use these 2 techniques also. Then we discussed N -layer model and learnt how to simulate multilayer optical structures. From here on, we will be using it for modeling SPR configurations. And then we discussed what should be the optimum thickness of a metal film in Kretschmann configuration and why it is important.

You can see that the tolerances are very small - I mean like 15 nanometers plus or minus, you have a huge change in the reflectance minimum contrast. So, it is very important to be near the resonant structure.

Thank you.