

Optical Sensors
Prof. Sachin Kumar Srivastava
Department of Physics
Indian Institute of Technology, Roorkee

Lecture – 11
Plasmons - II
Surface Plasmon Excitation of, Penetration depth, propagation length

Welcome to the 11th lecture of Optical Sensors course. In the last turn, we discussed what are plasmons and what are surface plasmons, what kind of waves these are and we saw that when it comes to bulk plasmons, they are basically longitudinal oscillations. However, these surface plasmons are decaying waves which are traveling at the interface of a metal and dielectric. And we solved for the dispersion relation for this kind of wave, and we drew two important conclusions. One was that this kind of wave can only be excited at the interface of a metal and dielectric; I mean a material interface which has dielectric functions of opposite signs. And the second thing was that it can only be excited by a p polarized wave.

In the present lecture, we will discuss it further and we will see how do we excite surface plasmons. Also since it is a decaying wave, we want to know what is the penetration depth and what is the propagation length of this wave.

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Propagation Length

Wave-vector of a surface plasmon wave

Please note:

$$k_x = k_o \sqrt{\frac{\epsilon_s \epsilon_m}{\epsilon_s + \epsilon_m}}$$

Real

$$k'_x = k_o \sqrt{\frac{\epsilon_s \epsilon_{m1}}{\epsilon_s + \epsilon_{m1}}}$$

Imag.

$$k''_x = k_o \left(\sqrt{\frac{\epsilon_s \epsilon_{m1}}{\epsilon_s + \epsilon_{m1}}} \right)^3 \left(\frac{\epsilon_{m2}}{2 \epsilon_{m1}^2} \right)$$

Please note:

$$\epsilon_m = \epsilon_{m1} + i \epsilon_{m2} \Rightarrow k_x = k'_x + i k''_x$$

Propagation Length

$$L_{sp} = (2k''_x)^{-1}$$

	$\lambda = 543 \text{ nm}$	$\lambda = 633 \text{ nm}$
Gold-Air (μm)	10.17	15.34
Silver-Air (μm)	29.82	43.67

Miniaturized sensors!!

Since the dielectric function of the metal is complex, the wave vector which is k_x - we were solving for the wave - this was z , and this was x . The wave is travelling in x direction. Basically, it has complex nature because k is a function of ϵ_m . So, if ϵ_m is complex, basically k will also be complex. You can put $\epsilon_m = \epsilon_1 + i\epsilon_2$ in this relation and then you can separate out the real and imaginary parts, this is real part of the wave vector and this is imaginary part and what you see is that it is almost the same.

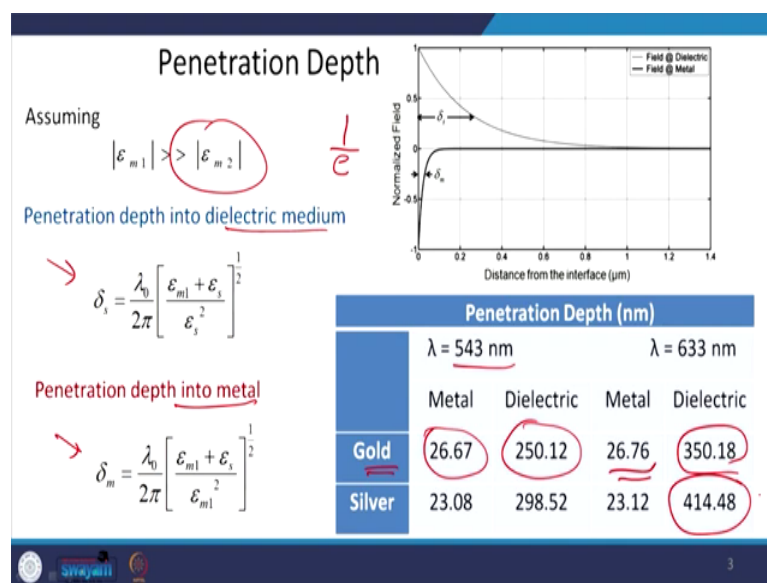
So, the basic nature - this is the one which is responsible for the propagation of the wave; this is the component which is responsible for the wave to die out while it propagates. When the intensity of this wave becomes $1/e^2$ - amplitude to $1/e$ and the intensity becomes $1/e^2$ that length is called propagation length and that is defined like this relation. L_{sp} is inversely equal to $2k_x$ imaginary part.

So, it depends on the imaginary part of the dielectric function like this and for the real part it is inversely proportional. So, you can see that the decay will be faster if the absorption is faster, so the L_{sp} will be smaller. For example, I have given two values for λ is equal to 543 nanometer and 633 nanometer. For gold air interface, the propagation length is about 10 microns while at higher wavelengths it is 15 micron. And similarly, for silver-air interface, this is about 30 micron and 44 micron respectively. So, you can see that for silver the propagation length is larger; that means, the absorption is smaller. If it absorbs less, the surface plasmon wave can propagate to a large distance before it dies out.

If you want to make a waveguide for surface plasmons, you want to prefer a material which has smaller losses, so that the propagation length is larger so it propagates to a large distance. But in this particular case, we will have miniaturized sensors. So, you can see that all you need is about 10 micron size film and it will still work as a sensor because you have a wave which dies out after moving to 10 micron or so.

So, the structures which are involved in these particular kind of configurations are very small in dimensions and if you have 150 micron or so, that is fine. But if you want to make a waveguide of metal and propagate some light which gets coupled into surface plasmons, then it will die out only after 50 micron. So, one has to choose some material which is better in terms of propagation length.

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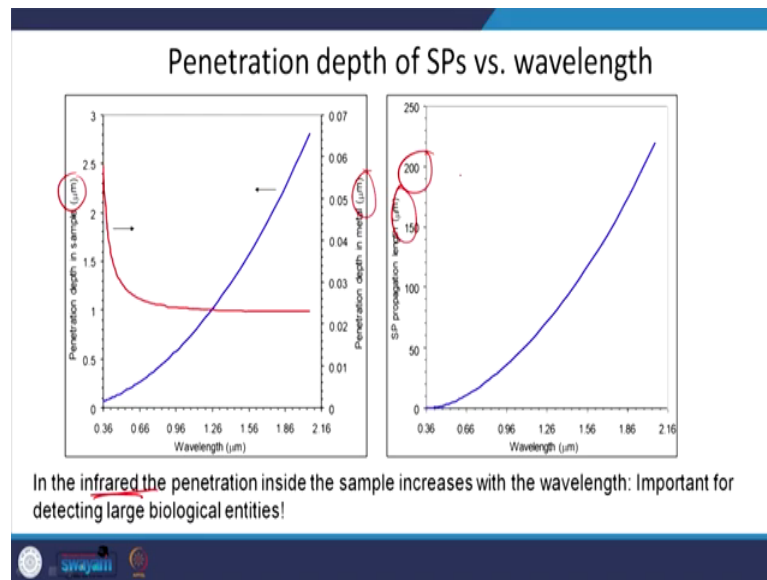
And penetration depth: if we assume that the imaginary part is much smaller then we can define the penetration depth like this. This is the expression for penetration depth into the dielectric medium and this is into the metal and you can see that from here as we told that the field decays both exponentially in the metal and in the dielectric and went up to the value, when it becomes 1 by e of the field at the interface then it is called the penetration depth. So, up to the distance where it becomes 1 by e.

You can see that the penetration depth in the dielectric is larger than that of metal and that is because of the absorptive nature of the metal. Since it is absorptive, the field dies out quickly into the metal region, while for the dielectric, it is larger.

Let us see some numbers - you can see that at 543 nm for gold and air interface, for metal only it is 26 nanometers, while for air it is 250 nanometers. So, you will have a penetration of about 200 to 300 nanometers in depth. If you put an analyte which is larger than this, it will not sense it completely. So, there are special methods you have to choose. This is basically a wave which is traveling at the interface and its field is only up to about 200 nanometers into the dielectric medium, while in the metal, it is even smaller - it is about 26 nanometers or so. So, it does out quickly.

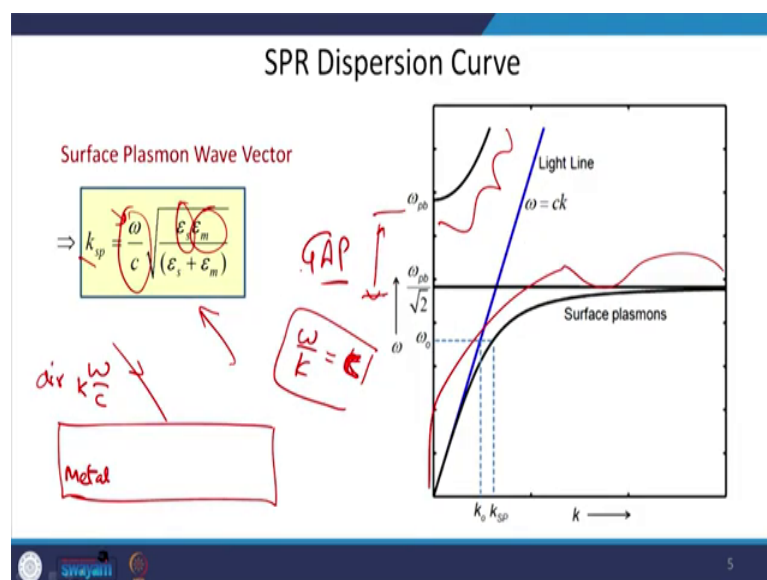
At larger wavelengths, you can have larger penetration depth in both the media - actually for metal it is not much affected, but for the dielectric it is affected a lot due to the same reasons and then you have better penetration.

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So, we see that in the infrared, it becomes even further larger. So, what we have plotted here is the penetration depth in the sample (in microns) and penetration depth in the metal (in microns). You can see it is very small here while it is much larger. And if you move it to infrared wavelengths, it is about 10 to 20 orders of magnitude larger for infrared - you can see these wavelengths here, right.

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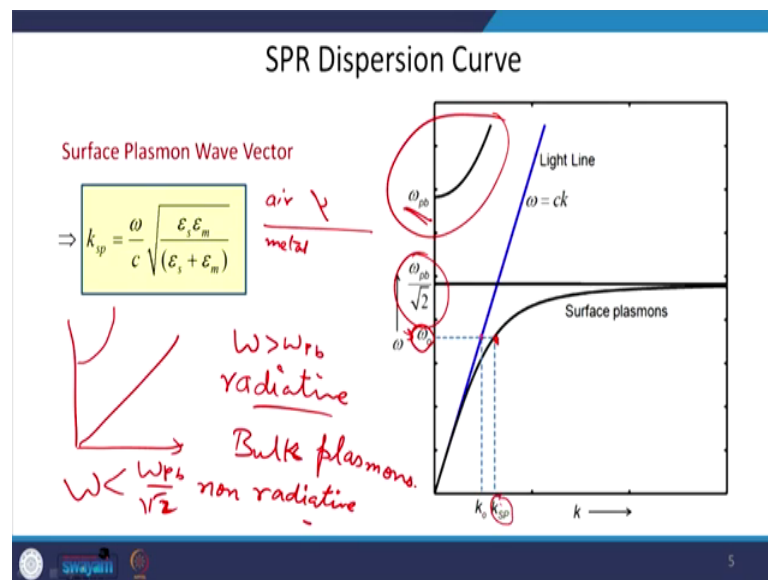


Surface plasmon wave vector: we already derived this expression for surface plasmon wave vector, where epsilon s is the dielectric function of the dielectric. Epsilon m is the dielectric function of the metal and omega by c was a constant or you can write it k 0.

So, if you plot k versus omega curve what you see is that for a light traveling in a dielectric medium; it is a straight line - this is called light line. However, for light travelling in this medium, suppose I have air interface where we have a dielectric, suppose this is air, it is a dielectric and its metal and then we have incident a wave, so it will have k is equal to omega by c - that will be a straight line. So, omega by c - this basically is equal to k. So, the omega versus k curve - let us say this omega versus k curve is c - which is constant, so its slope is constant - so you will have a straight line.

However, for surface plasmons you see two branches, one is here, another one is here. And in between this and this, you have forbidden region, there is a gap - you do not see any branch here. What does it mean? It means that for this region there is no surface plasmon.

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If you remember, when we plotted for the dispersion curve for the bulk plasmons, it was like this - This was the straight line and this was the curve. So, this curve is similar to here. Omega pb is the plasma frequency for bulk metal and for omega values greater than omega pb, all the modes are radiative. So, you have bulk plasmons. However, for values which are smaller than this value - I will come to how to get this value; And for

these values, this is non-radiative. So, omega less than omega pb by root 2, it is non-radiative. So, how do we arrive to this value - omega pb by root 2? Let us see.

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$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$k_{sp} = \frac{\omega}{c} \sqrt{\frac{\epsilon_s \epsilon_m}{\epsilon_s + \epsilon_m}}$$

$$\rightarrow \epsilon_s = 1 \text{ (air)}$$

$$k_{sp} = \frac{\omega}{c} \sqrt{\frac{\epsilon_m}{1 + \epsilon_m}}$$

$$\underline{\epsilon_m} = \text{At resonance}$$

$$\boxed{\epsilon_m = -1}$$

$$\text{Resonance Condition is}$$

$$\epsilon_s + \epsilon_m = 0$$

$$\epsilon_s = -\epsilon_m$$

$$-1 = 1 - \frac{\omega_p^2}{\omega^2}$$

$$-2 = -\frac{\omega_p^2}{\omega^2}$$

$$\omega = \frac{\omega_p}{\sqrt{2}}$$

We have epsilon omega is equal to 1 minus omega p square upon omega square. We had this value. Also, we had k sp is equal to omega by c under root epsilon s epsilon m divided by epsilon s plus epsilon m.

We can write it in terms of omega p. And, so from here if you have epsilon omega, epsilon s is equal to 1 which is for air you will have k sp equal to omega by c under root epsilon m upon 1 plus epsilon m. And here, we want to calculate omega in terms of omega p. So, if you put epsilon m at resonance, at resonance epsilon m is equal to minus 1. The resonance condition is epsilon s plus epsilon m is equal to 0. This term has to be 0 for resonance to happen and epsilon s will be equal to minus epsilon m.

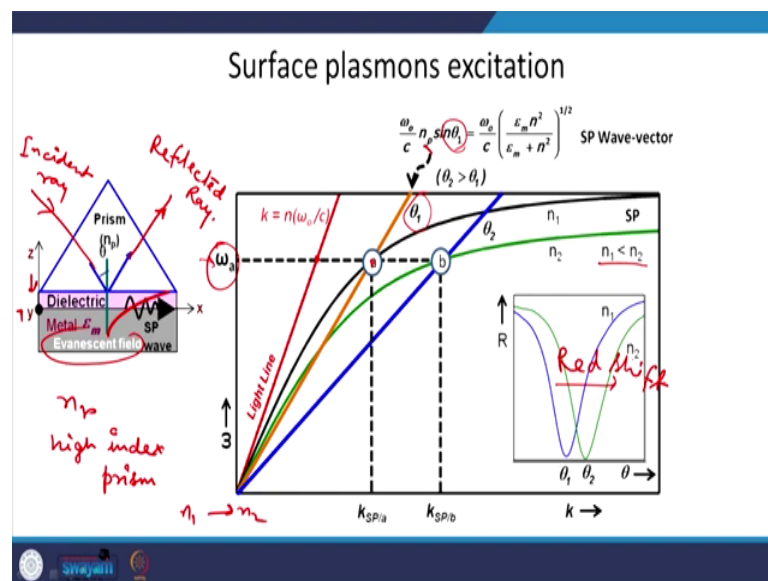
So, here if epsilon s is equal to 1 then epsilon m is equal to minus 1. So, if you put minus 1, minus 1 is equal to 1 minus omega p square upon omega square. So, it will be minus 2 is equal to omega p square upon omega square. If you take it up here, so minus minus cancels out, you will have omega is equal to omega p by root 2. So, basically if we have omega pb; if you put b here, b for bulk because we will discuss nano particles also we have changed it to omega pb. So, you have omega is equal to omega pb by root 2. This is the asymptotic value. So, you have the asymptotic value of omega which becomes

ω_{pb} by $\sqrt{2}$. For this value only you can have surface plasmon excitations - asymptotic values, ok.

There is something more important and that is that suppose you have this interface of say dielectric - say this is air and this is metal and you shine laser light, you can see the wave vector of laser light is much smaller than the wave vector required for surface plasmon excitation. This is called phase mismatch.

It is like this - you have this interface. I am wearing a gold ring. So, you have air and gold interface. If I shine laser on it like this, can I excite surface plasmons? That is the question. This kind of interface supports this kind of modes - surface plasmon modes, but how do we excite them? Because we have seen now that the wave vector of this light which is impinging on this interface is much smaller than the wave vector required to excite surface plasmons. This is an issue.

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We know already that this kind of interface: a metal-dielectric interface supports an electromagnetic wave which is surface plasmon wave, which is traveling along the interface. We already saw the wave vector for surface plasmons is given by this black line, while that for the light line it is straight, this is the red one. And when you want to shine laser on this interface, there is a mismatch between the phase between these two and because of this mismatch you cannot excite surface plasmon.

So, somehow you want to increase the wave vector from here to here to match this condition. And to do that what we do is that we make it very thin and bring out a prism of high index there. The n_p is refractive index of the high index prism. You bring a high index prism, which is kept very close to the metal and you fill it with your dielectric, and you shine from one of the ends and you measure the light which is getting reflected from here. This is the incident ray, this is reflected and what you see is that if you change the angle, there should be some condition when you have the resonance. What happens actually - let us see.

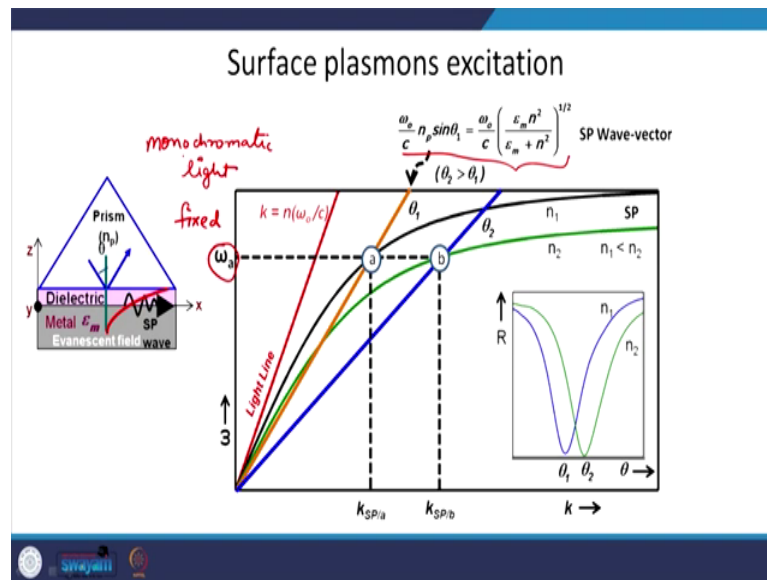
When you pass it through the prism, basically you will have total internal reflection as I told you earlier and you will have evanescent field here, this red one. This evanescent field will interact with the metal dielectric interface here and if it has the wave vector which is equal to that of the surface plasmons, you get a resonance. So, what happens actually, when it passes through the prism material as a function of angle- because there is a multiplication of n_p , you increase the wave vector and at that certain value of θ for that particular frequency, you get a resonance.

What will you see in the reflected light? So, if you measure the reflected power as a function of θ at that particular value when this matches - when this intersects - this light line when it intersects this surface plasmon wave vector curve, you get a resonance dip for that particular refractive index of the dielectric medium.

Now, what happens if I change the refractive index of the dielectric medium from n_1 to n_2 , then the nature of the dispersion curve for surface plasmons will change. So, now, it is this green curve and we have considered that n_2 is greater than n_1 . So, now, for this same incident light we need to match the phase again and to do that we need to keep changing the angle.

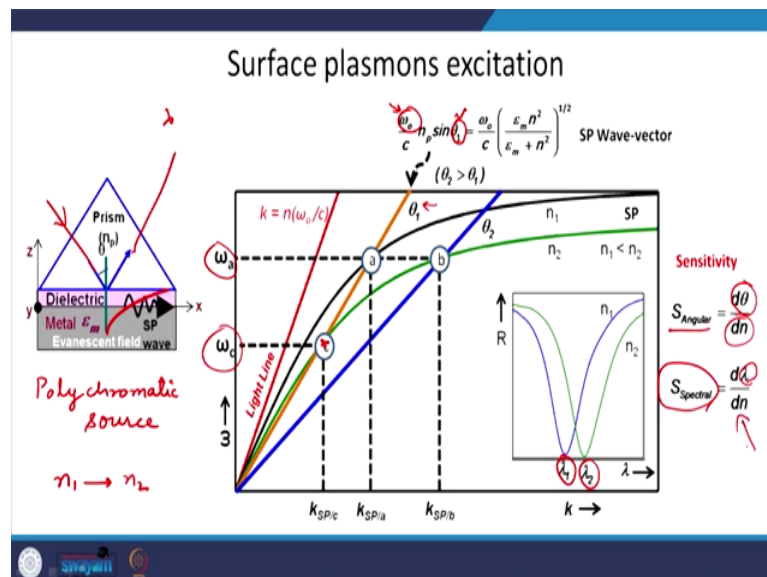
So, let us say it moves from a to b and then you have a resonance again at θ_2 . If you increase the refractive index from n_1 to n_2 , you have to increase the angle of incidence from θ_1 to θ_2 to match the resonance condition. So, there is a shift and it is a red shift. So, there is a red shift in the resonance angle and this is the resonance condition which is satisfied for now θ_2 , ok.

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Now, what happens actually if I do not use a monochromatic light? So, you know, the omega was fixed, and this was monochromatic light. So, it was like this laser. Now, I do not want to use a laser. I have a polychromatic source. Now, let us see what happens with a polychromatic source.

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So, you shine at an angle theta and now you do not want to change the angle, you change the wavelength lambda. If I change the refractive index from n 1 to n 2, what will

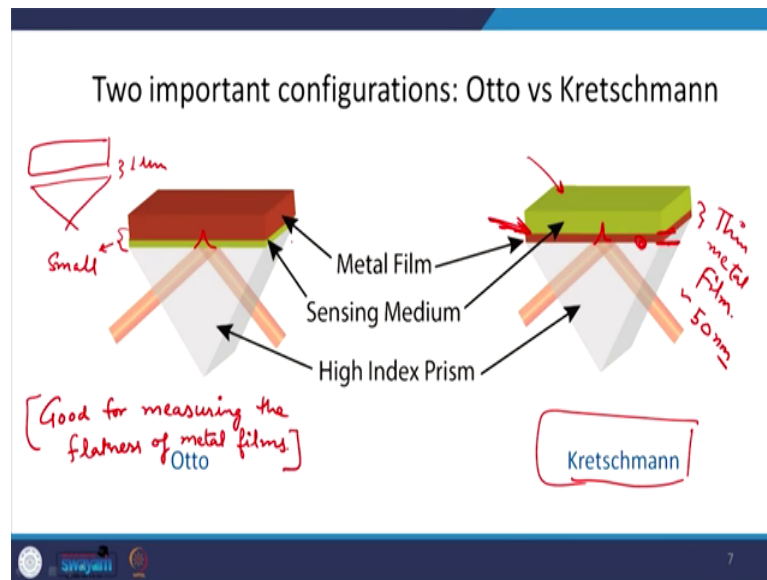
happen? There are two parameters, one is θ , one is ω . I fixed ω in the first case and θ was varying. Now, I fix θ and ω is varying.

I have a polychromatic source. What will happen at particular angle of θ ? We will have a resonance for this particular value of ω and we will have a dip. Now, I move from n_1 to n_2 , we will have a different dispersion curve which is the green one. We had a black one, now we move to green one. But this intersects it here. So, what I can do is that I can move from a to c rather moving a to b - we do not need to change the angle then it is on the fixed angle, but we change the wavelength. So, we change the spectrum, you do not change the angle and then what you measure is that is change in spectrum. That is called spectral interrogation. Again it shows a red shift, right.

So, there are two ways to do sensing, one is called angular interrogation, where you measure change in θ with respect to change in the refractive index and another one is the spectral interrogation where you measure change in λ with change in refractive index and that is how you define the sensitivity. If a beautiful lady is sitting there asks me if I can excite surface plasmons on my earring which is made of gold, the answer is - no. You cannot excite surface plasmons by simply shining light on it. For that, you have to have some arrangement to match the phase of surface plasmons with the light which is incident on the interface.

Here, I showed you that that can be matched using a prism which has high refractive index and you measure the change in angle and from there you can say how much change in refractive index occurred - that is how you use it for sensing. There are two important configurations, one is called Otto configuration and another one is Kretschmann configuration.

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The previous one which I talked about was Otto configuration where, what we did is that - we had this metal and dielectric interface and we brought the prism very close to it. And how close? About 1 micron or so.

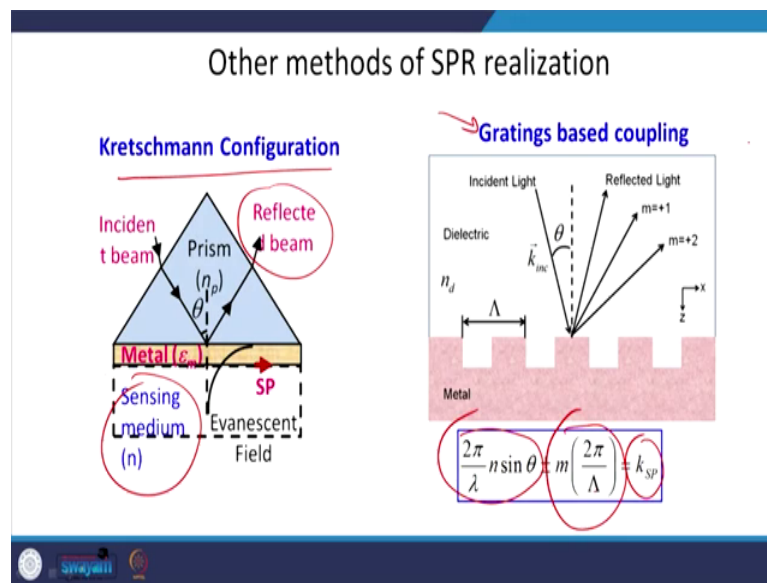
If you remember, when we calculated the penetration depth for the evanescent field, it was only about hundreds of nanometers. So, in this particular case if you want the evanescent field to penetrate through here and intersect with this interface, this gap has to be really small - very small. For this to be small, you have to bring the prism very close to the metal surface, that is really very difficult sometimes, you know, to manage such a small gap of only few hundred nanometers or so, ok. That requires lots of skills. But this technique is good for measuring the flatness of metal films. You bring it close and if at all the places the surface plasmon response changes because it will be a function of distance, you can say that the metal film is not flat. So, this technique is, somehow, useful.

However, most of the people use something called Kretschmann configuration. What you do is that rather making a gap, you put the dielectric otherwise and make the film very thin - thin metal film. So, you have very thin metal film which is only 50 nanometers or so. If you have such a thin metal film, the evanescent field still can penetrate through and excite surface plasmons at this interface. So, you have surface plasmons at this interface.

Also, you have to understand that this is also a metal dielectric interface, but light coming from here - from this medium will again have propagation constant which is much smaller than the propagation constant of the surface plasmons for this interface. So, that is why light coming from this medium cannot excite surface plasmons at this interface, but only at this interface, ok.

Now, if you change the refractive index of this medium, you can excite surface plasmons at this - you can have different resonances. So, that is why Kretschmann configuration is very useful because it is very easy to fabricate, you can directly coat a thin layer of metal on the prism surface. This one is difficult because it is very difficult to bring the prism very close - nanometers close to the metal film and also fill the gap with different refractive indexes. This is challenging, but this one is easy.

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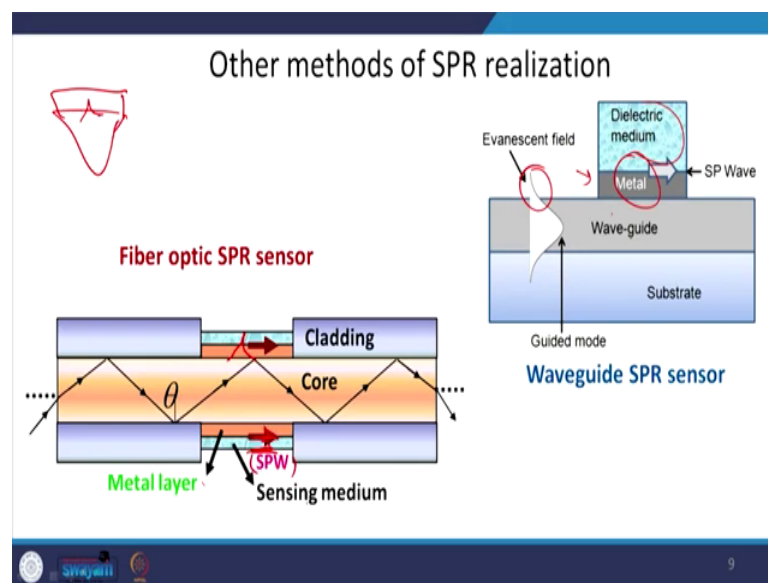
There are other configurations. I already talked about the Kretschmann one. Here it was like - you have incident beam which excites evanescent waves, which eventually can couple to surface plasmon wave and this dielectric medium can be used as a sensing medium - you can change the refractive index here and you measure the power in the reflected beam as a function of angle of incidence.

Other method is grating based coupling, where for light incident at an angle theta, we have different orders of reflected light and these reflected orders will have different wave vectors. If it is in forward direction, this will be a plus sign, if it is a backward direction

this will be in minus sign. This is the wave vector of light incident through this medium - dielectric medium and this is the wave vector, which gets added due to the deflected orders.

And when this becomes equal to that of the surface plasmons supported by this interface of this metal and dielectric, then that order goes missing in the reflected light. So, you do not see that particular order, you see that particular order missing from the reflected light. That is how you excite surface plasmons using a grating.

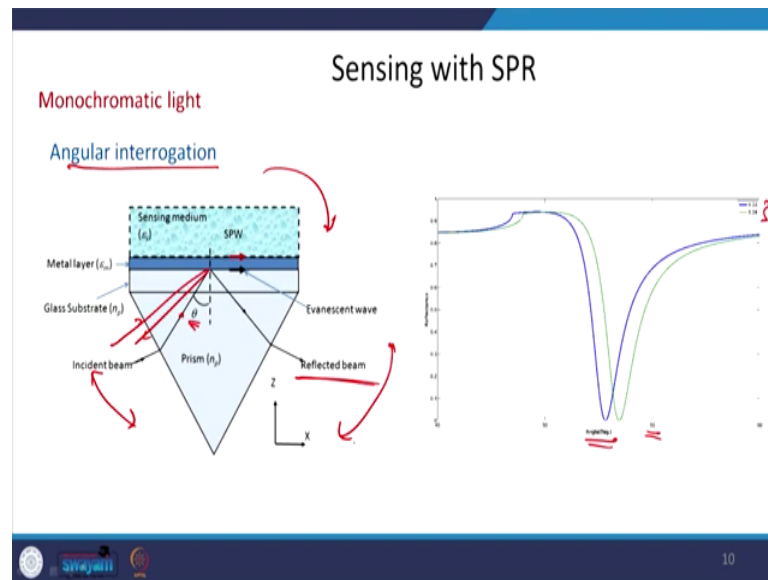
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In a waveguide, again you can have this evanescent field. So, basically there was a prism in Kretschmann configuration and that was having the evanescent field. What you can do is that you can remove a small part of the cladding and do a coating of metal and then this evanescent field from the waveguide metal interface will excite some surface plasmon waves at the dielectric and metal interface.

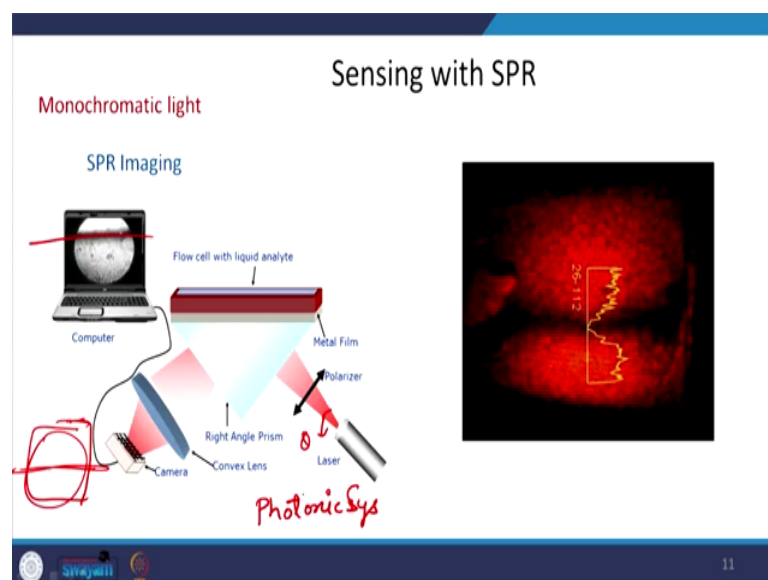
Similarly, what you can do is that you can take a small portion of the cladding out from an optical fiber and do a thin layer of coating of the metal on the cylinder. This is very thin. So, again it will have evanescent field because of total internal reflection. So, it can excite surface plasmon waves at this interface - at the interface of the metal and their sensing medium.

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So, when we are sensing with monochromatic light, we do angular interrogation. As we discussed in the Kretschmann configuration, you change the angle and you measure the reflected power, and what you see is that with a change in refractive index you measure actually how much change in angle occurs. And by using this, you can say what is the change in refractive index.

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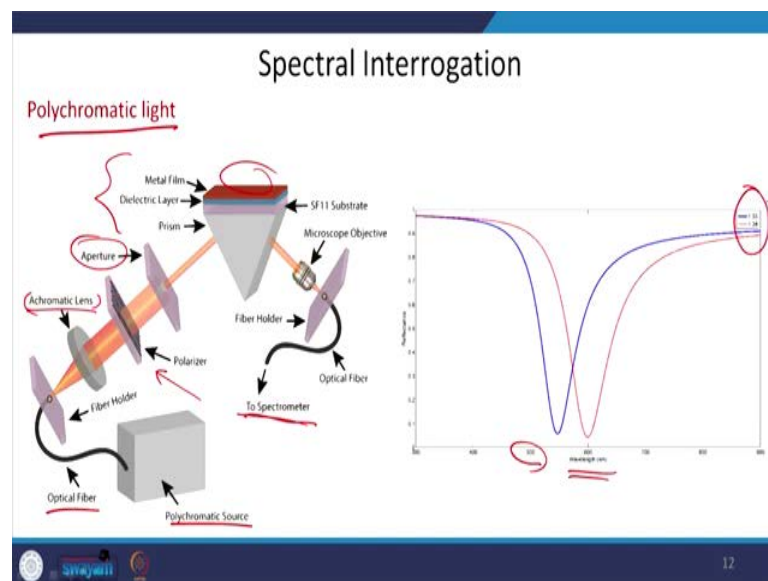
But that is a difficult technique. I mean, what you do is that you have a prism and you keep on changing the angle. You have to rotate the prism, or you rotate the incident

beam, whatever and then you rotate this detector. So, this is a difficult problem. I mean, it needs mechanical components - because you have one directed beam, and if change your angle θ here then it will be deflected by a 2θ and you have to adjust the detector all the time.

To avoid this what you can do is that rather sending a beam which is collimated, you send the diverging beam. So, here you send a beam which is making an angle θ . What happens actually that now it has a range of angles and when you have surface plasmon resonance, you get a dark line here. What you do is that rather using a detector, you use a camera, so you do the imaging. You get the image of it. So, if the beam was all bright, the angle for which when you have surface plasmon resonance, it will go missing here and you will get a dark line there. Later you can do image analysis and you can find out the width of the dip and where the position is. If you can measure the pixels you can know where exactly the dip is.

This is a more advanced technique and most of the companies are using this technique only to make SPR devices. Say for example, PhotonicSys, Photonic Sys company is using this, Biacore is using this. There are many companies which are using this kind of technique.

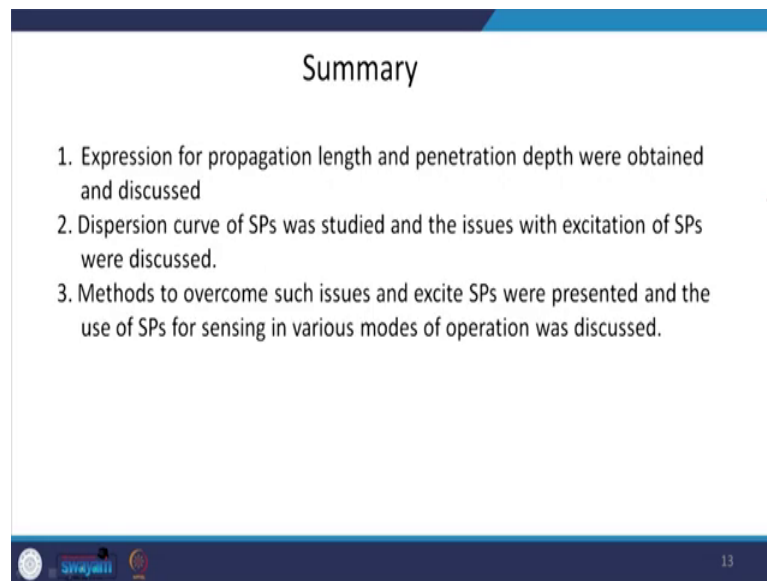
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Other way of doing that was using a poly-chromatic light. What you do is that - you now have a source – where we are using a poly chromatic source with optical fiber and light

coming through it - It can be collimated using an achromatic lens. You put a polarizer which polarizes it as a p polarized light, with an aperture you can control the width of the beam and then it falls on this interface of metal and dielectric. You have analyte here and then you collect the light which goes to the spectrometer. Rather measuring the angle, you measure the wavelength and by measuring the shift in wavelength you can say how much change in refractive index occurred. So, there are these 2-3 methods. There are others also - we will discuss in the next talk.

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Summary

1. Expression for propagation length and penetration depth were obtained and discussed
2. Dispersion curve of SPs was studied and the issues with excitation of SPs were discussed.
3. Methods to overcome such issues and excite SPs were presented and the use of SPs for sensing in various modes of operation was discussed.

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Let us summarize what we studied today. We obtained the expressions for propagation length and penetration depth of the surface plasmons and discussed them. And we also discussed the dispersion curve for surface plasmons and found out what are the issues which need to be resolved to excite surface plasmons and then we discussed various methods for exciting surface plasmas and how we use them for sensing and also we discussed various modes of operation for these kind of sensors.

Thank you.