Optical Sensors

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Lecture – 10
Plasmons – I
Surface Plasmon: Dispersion Relation

Welcome to the 10 th lecture of Optical Sensors course. In the last lecture, we discussed the absorption and dispersion properties of materials and we arrived to the Drude model using Lorentz oscillator - damped oscillator model. We arrived to Drude equations for metals and from there we saw for noble metals what are the coefficients. We also saw that the real part of the dielectric function of the metals is negative, while the imaginary part is positive. It implied that the metals are absorptive because of this positive nature of the imaginary part.

In the present lecture we are going to study what are plasmons and from plasmons we will arrive to surface plasmons and their dispersion relations.

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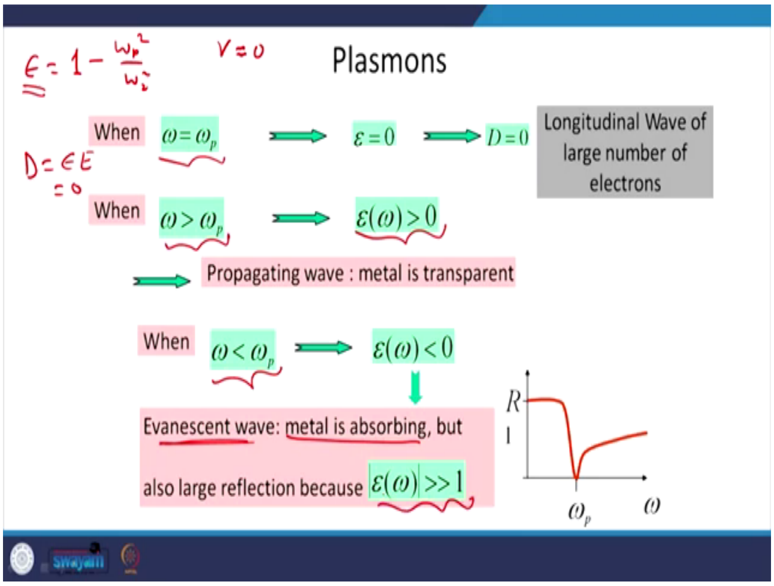

From the last turn, we saw that epsilon was defined as equal to 1 minus omega p squared upon omega square, when we did not consider the damping term - gamma was equal to 0. So, when omega becomes omega p, essentially, epsilon becomes almost equal to 0;

that means, D is equal to epsilon E equal to 0. It means that there are longitudinal waves of large number of electrons.

When omega is greater than omega p, then epsilon omega is greater than 0 and the wave is propagating - metal is transparent. If it is less than omega p, then the metal is absorbing and there is evanescent wave. So, minus sign shows that there would not be any propagating wave and the plus sign shows that there will be a wave which will propagate, ok.

This is the physical significance of the last equation which we derived in the last turn. And, also large reflection because this is much larger than - mod is much larger than 1. So, you saw that it was the order of 100 or so.

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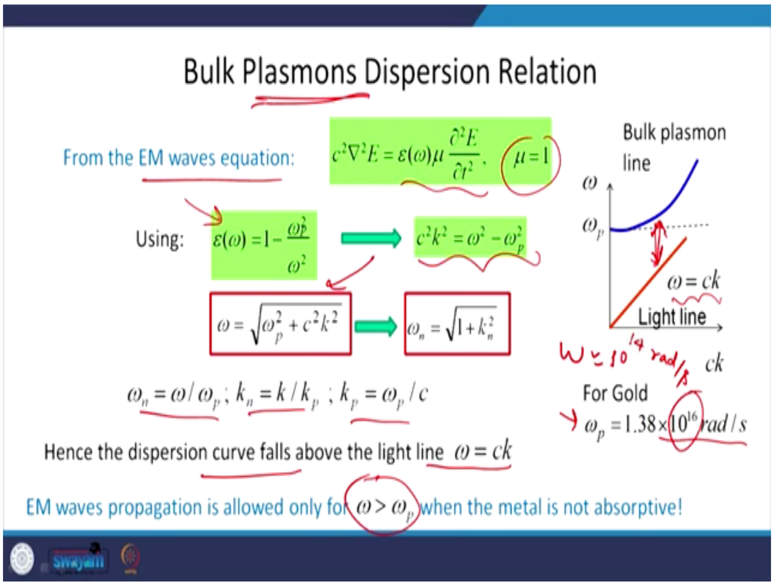

So, if you solve for the electrometric wave equation, what we see is that for mu is equal to 1, you are arrive to this relation using this dielectric function - you can solve it for this value which is c square k square is equal to omega square minus omega p square. From here you can rearrange it to get omega is equal to square root of this thing. So, omega will have basically two values, ok.

You can put it like this and what you get is - this dispersion curve. So, if you have this red line, called light line which is when omega is equal to c into k; but for plasmons, you have this blue curve, which is at a larger omega than to the light line. For example, for a light of any wavelength, the frequency omega is of the order of 10 to the power 14 radian per second. However, the plasmon frequency is this 10 to power 16. So, you can see that it is about 100 orders of magnitude larger. Hence the dispersion curve falls above the light line.

And, EM wave propagation is allowed only for omega greater than omega p. So, if the electromagnetic wave has omega larger than omega p, then the metal is not absorptive. It happens for a very high omega values which is like X-rays, the gamma rays also show that. So, you have to have very high omega values that lambda should be very small so that it does not see the lattice of metal, ok. but for optical frequencies, yes, it is reflective.

What are plasmons? I am, all of the time, speaking about this plasmons thing, what is it? You see, we defined something called plasmon frequency. So, let us try to understand what is plasmon.

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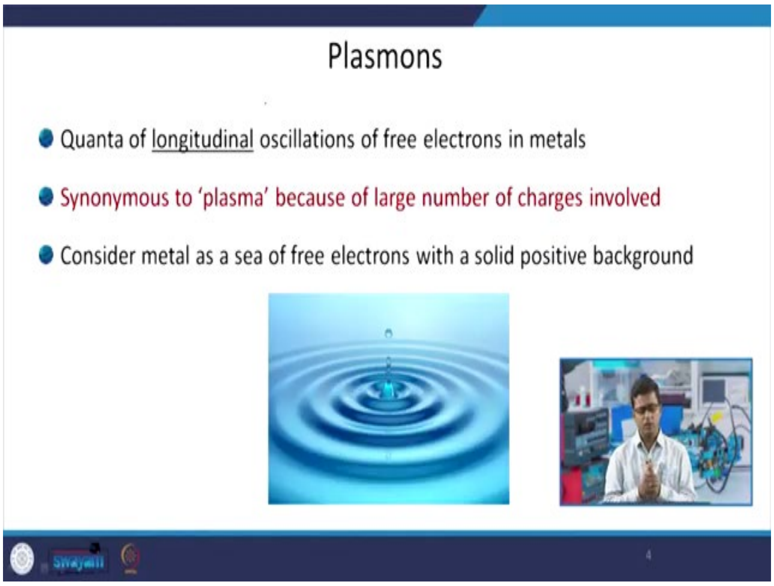

Plasmons are basically quanta of longitudinal oscillations of free electrons in metal and we call it plasmon, because it is synonymous to plasma - because there are large number of charges involved. Say for example, you have a piece of metal, you can consider metal as a sea of free electrons with a positive background.

So, you have a solid positive background, where you have free electrons moving randomly - the oscillation is called plasmon oscillation. Now, try to understand what is happening here. When a wave travels through a metal, how will it look like? I mean, to understand this, I have shown this picture. Suppose I have a pond and this pond is made of free electrons and you drop a pebble in the pond, what happens?

You see the ripples which are moving away from the point of dropping the pebble, right? What happens inside? As the pebble moves inside the water, on the surface you see ripples; inside the water, there will be waves which are cylindrical in nature and they will be travelling away from this point of origin.

So, inside you have waves, which are moving away like this and they are longitudinal - they are cylindrical waves which are moving away. On the surface, you see this kind of oscillations which is kind of transverse wave, but it is not purely transverse because, if you put a small piece of paper on the surface of water, it will start moving this way, right.

It is not just oscillating this way, but it will also oscillate this way. So, on the surface you have a wave, which is partially longitudinal and partially transverse in nature, while in the bulk you have a wave which is purely longitudinal in nature. That is why, if you have a bulk piece of metal the waves in it - that are plasmon waves - will be longitudinal oscillations. And that is why the electromagnetic waves - most of the electromagnetic waves - they cannot excite plasmons in bulk metal.

So, you have to have electron energy losses spectroscopy or something if you want to see. To excite the plasmons in bulk metal you have to have high energy electrons, but not the optical photons. But, on the surface, there happens something - that you can excite something called surface plasmons.

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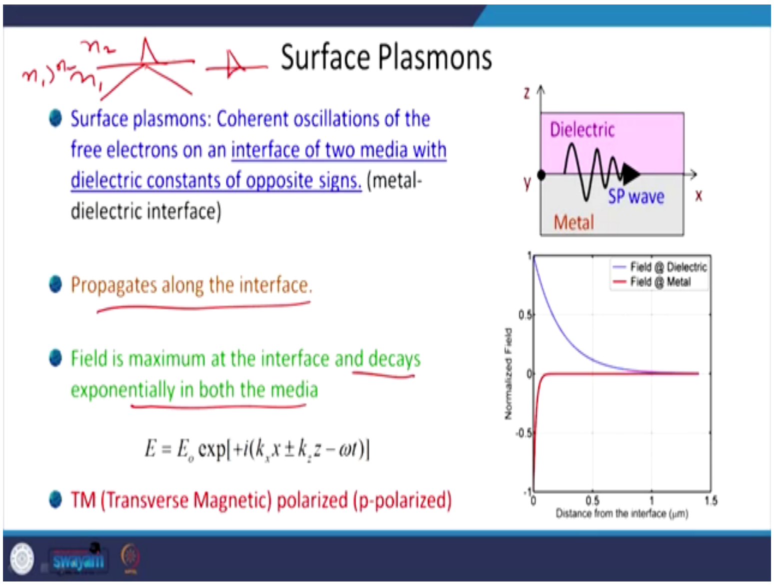

What are surface plasmons? Surface plasmons are coherent collective oscillations of free electrons at the interface of two media of opposite dielectric constant signs - what it means - like metal and dielectric interface. I will come to that when we derive the equation for surface plasmons, but take my word that it has to be interface of a metal and dielectric.

So, what happens actually, this wave - when it travels along the interface, its amplitude decays continuously and it dies out. So, you can see here that we have a wave which is travelling along the interface and when it travels along the interface, it dies out after certain distance. So, surface plasmons are the oscillations and as a result there is a wave which is propagating along the interface and the field associated to this wave is maximum at the interface and it decays exponentially in both the media.

So, this is a wave. If you remember that if you had an evanescent wave, its tail was only in one medium, right! If you remember EM wave was travelling from n 1 to n 2 and n 1 was greater than n 2, so, evanescent wave was travelling here; but this wave - its field penetrates in both the media, larger in the dielectric and smaller in the metal - that we will come to. Also, this wave is transverse magnetic in nature so, it is basically p-polarized.

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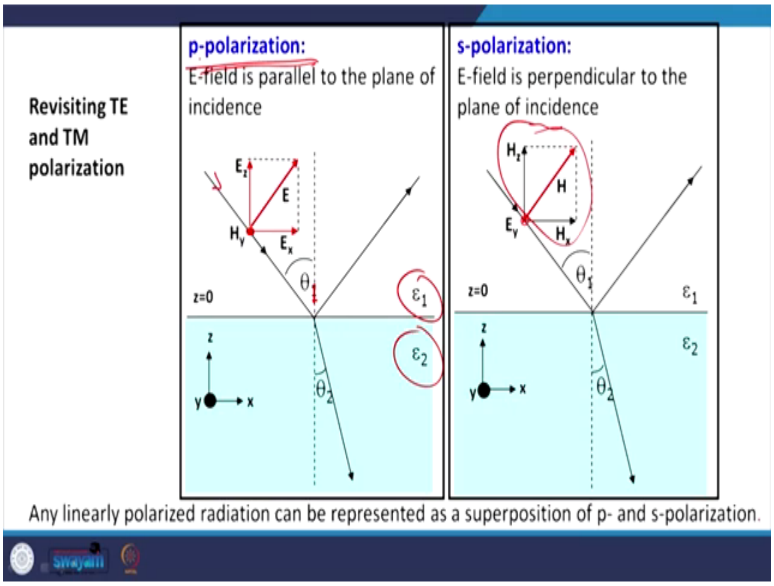

Let us revisit what I taught earlier that, what is the TE and TM polarization for this kind of configuration. So, if you can see this picture, where we have an interface of epsilon 1 and epsilon 2 and then an electromagnetic wave is incident at an angle theta 1, which has electric components in the plane of incidence and magnetic component is perpendicular to this. This is p-polarization right. On the contrary, if you have electric field component perpendicular to the screen then it is s-polarized while the magnetic field components are in the plane of incidents then it is TE polarized. So, you are clear with it. In this case, the electric field is transverse component, here the magnetic field is transverse. We are talking about a wave which is basically p-polarized. So, I will come to that and we will derive it - how to solve this.

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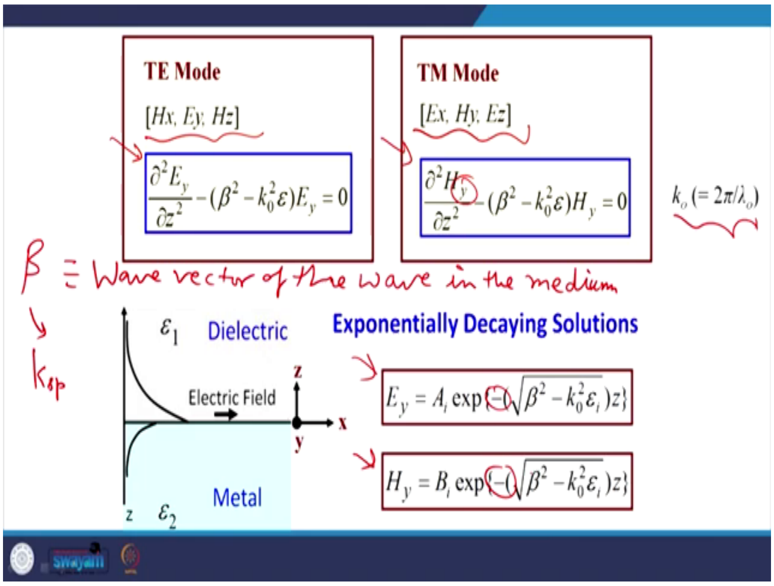

For that kind of configuration, the TE mode is given by this set of components; TM mode is given by this set of components, and you can always write the wave equation for TE wave like this and for TM wave like this. So, this was in terms of the transverse component, where beta is wave vector of the wave in the medium. In our analysis, it will be wave vector of surface plasmons. So, it will be basically ksp - we will come to that. k 0 is the wave vector of light incident, which is in the free medium, ok. Since it is a surface wave, we solve for the exponentially decaying solutions. So, if you solve for the exponentially decaying solutions of this equation, say from TE and TM mode equations, you arrive to this relation, ok.

So, we will have two components and you can see that the plus sign will be exponentially growing - you do not want exponentially growing solutions. So, that is why we have taken only minus ones. So, you have exponentially decaying solutions. Now, we tried to solve these equations for the interface. So, let us consider the TM.

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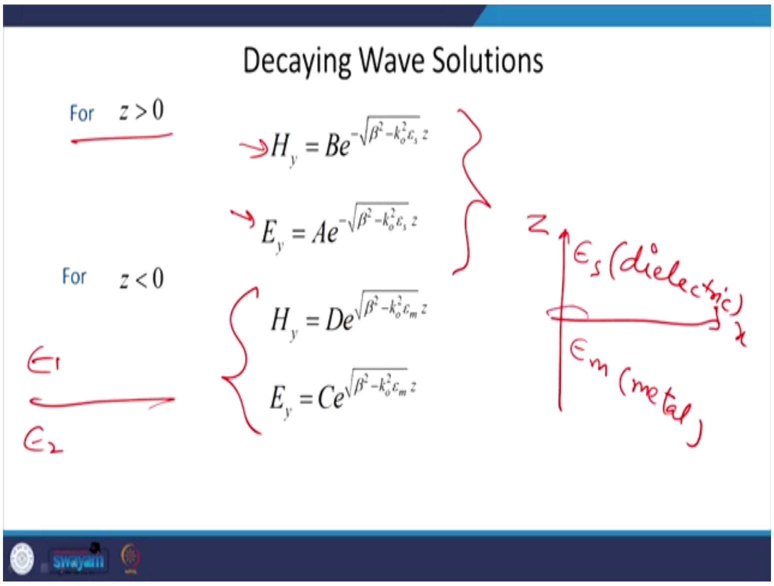

So, for z greater than 0 this is the condition; z smaller than 0 - this is the condition. We had epsilon 1, epsilon 2. Now, here we have epsilon s and epsilon m. So, if you have an interface which has x here and z here, here you have epsilon s, here you have epsilon m - this is the metal, and this is dielectric. You have a metal dielectric interface. You write the E and H components for z greater than 0 and z smaller than 0 and now we apply the boundary conditions at this interface.

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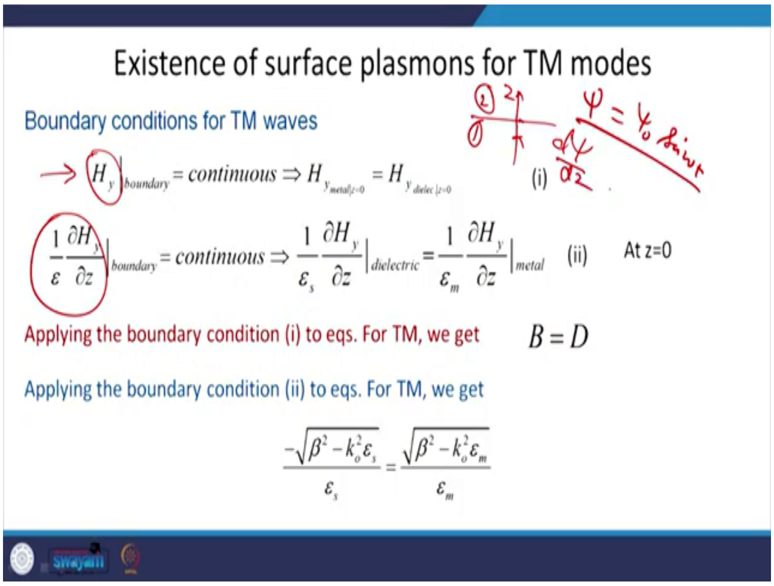

Let us apply the boundary conditions for TM waves. What are the boundary conditions? This is the boundary condition for TM wave. This is the first boundary condition, where the H y at the boundary should be continuous and this at the boundary should be continuous. We have already talked about the boundary conditions. So, from there you can arrive here.

What I am seeing here, I mean the other way of looking at the boundary condition is that you do not worry about the electric and magnetic fields - you have a wave, say, psi equal to psi 0 sine omega t - you have a wave. So, if it is traveling from here to here say from 1 to 2, then it has to be continuous. So, psi should be continuous and d psi by d by d z should become continuous.

So, that is what we are doing here. I mean it is not something you cannot derive from the boundary conditions. And, what you have here is that for the metal at z is equal to 0, it should be equal to the H y for the dielectric at z is equal to 0, right. If you apply that to equation - this H y, you put z equal to 0 this exponent term becomes equal to 0 and what to get is H y equal to D and that is H y is equal to B. So, basically D becomes equal to B right.

From this boundary condition, what you do is that for the dielectric 1 upon epsilon s d H y by d z is equal to this at metal and put z equal to 0. So, what you get? Let us write it.

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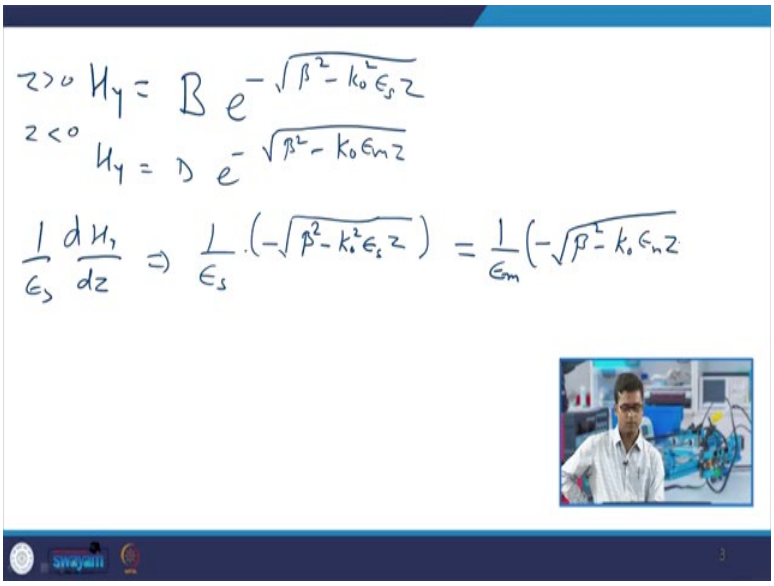

So, we have these equations - H y is equal to B e to power minus square root beta square minus k 0 square epsilon s into z and H y was equal to D e to power minus square root beta square minus k 0 square epsilon m z and we have 1 upon epsilon s d H y by dz; this was z greater than 0 is z less than 0, then it will be 1 upon epsilon s into minus square root beta square minus k 0 square epsilon s z into B. B is equal to D, so, it will be canceling out and that will be equal to 1 by epsilon m minus square root beta square minus k 0 epsilon m into z - this is what we get.

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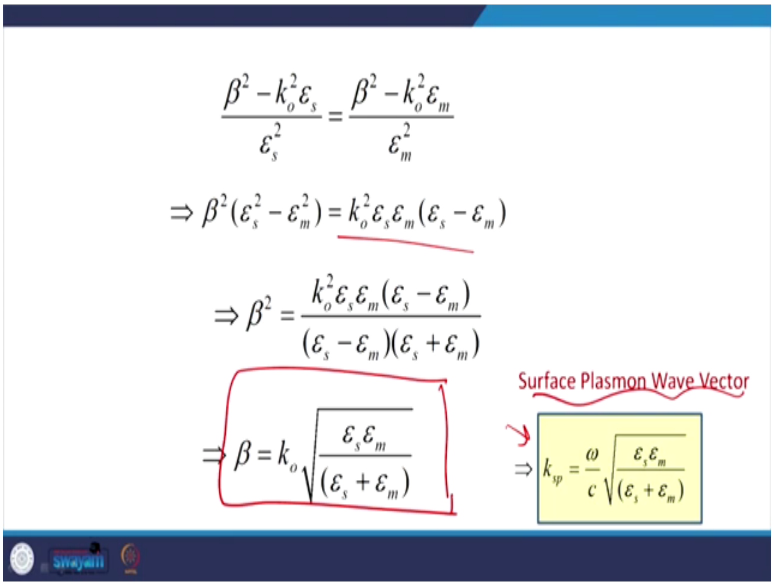

If you simplify it, what you get is from here - you multiply it here and you multiply it here and then you take common to arrive to this. From there you can have beta square is this divided by this and you can basically break it in two parts and you now get beta is equal to this. Something was missing here. So, since this was for z less than equal to 0.

So, it will become a positive here because z is minus, ok. So, now, that is why you get here plus term and the last one here minus and then it is plus, anyway. So, we arrive to surface plasmon wave vector, which is given by this relation. So, now we have a wave which has this propagation constant.

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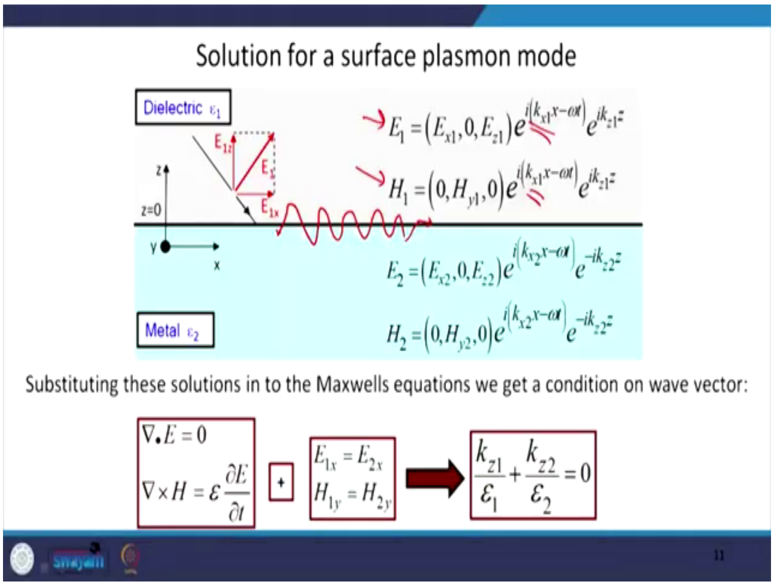

What happened actually that when we solve for the waves, you can see that it is a wave which is traveling along this interface, so you can write the E and H components now in terms of the wave vector and apply the Maxwell's equations to arrive to this condition of the wave vector, ok. What it tells?

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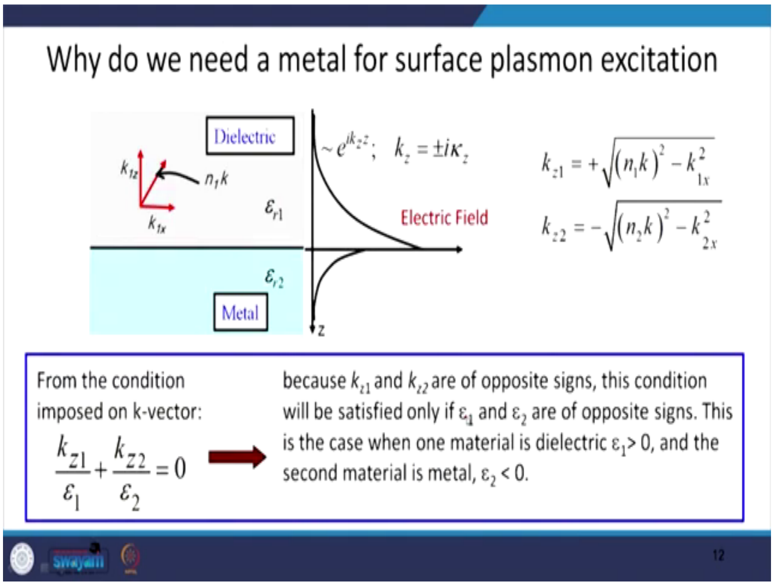

It tells that, from here, that these are of opposite signs - from here. So, if this has to be satisfied these have to be of opposite signs and that can only happen when one material is metal. So, it is an interface of two media and if you want to excite surface plasmons on it, then one of it has to have negative dielectric constant.

And, that is why it one of them has to be metal; metal or similar material which has negative value of the real part of dielectric function. So, from here we have now real part which is negative.

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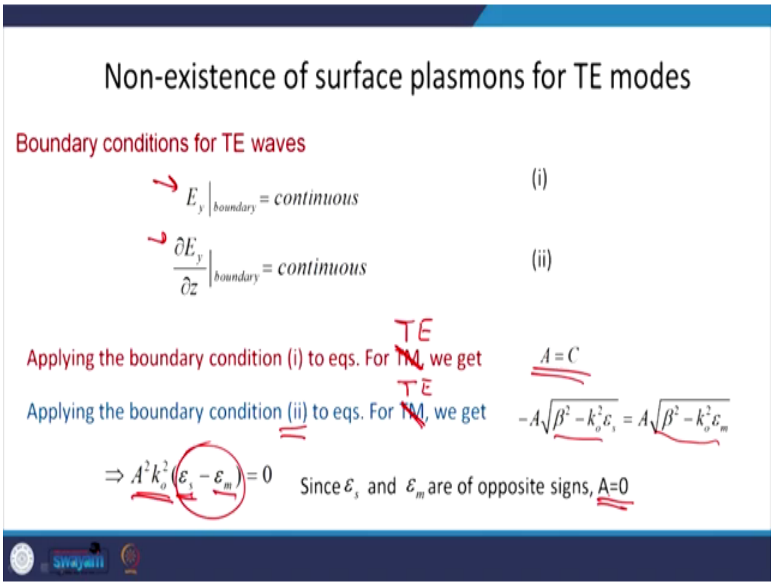

Now, let us see what happens to TE modes. We see that if we apply these boundary conditions, similar to what we did for TM, then for the first boundary condition, we get A is equal to C and for the second boundary condition we arrive for TE, we get this and if you square and sum them - you say that- now we know that these have opposite signs.

So, this cannot be equal to 0. So, what is equal to 0? This term. So, A will be equal to 0. So, practically there would not be any solution for TE wave. Now mathematically we know that for TM, there is a surface wave; for TE there is no wave. So, we want to see what happens actually.

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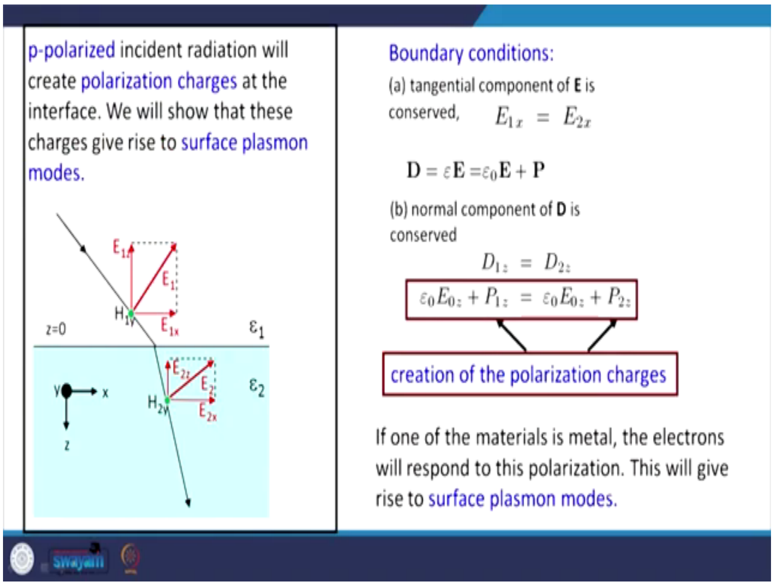

In p-polarized radiation, we create actually polarization charges at the interface, and this gives rise to surface plasmon modes. So, if you again go to the boundary condition for TE or TM, for the TM mode you know that the electric field is in this plane. So, basically if you apply the boundary condition for D you have this P 1 z and P 2 z. What are these P 1 and P 2? These are polarizations.

So, if you have electric field in the plane of variation of the their refrective index at the interface you are creating polarization charges and if one of the materials is metal, the electrons will respond to this polarization and that gives rise to surface plasmon modes. That is something very important here. I mean, if you have an interface, this kind of polarization occurs for any interface - be it a glass and air interface also you always satisfy this kind of boundary condition at the interface. But, if there are electrons which are free, then they respond to this kind of polarization and because of their response to this kind of polarization you have this surface plasmon waves.

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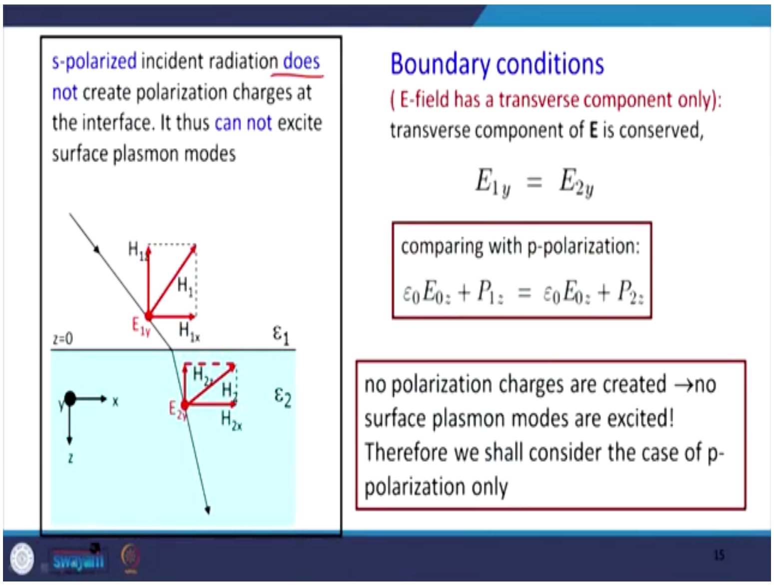

What happens to s-polarized light? Now, incident radiation does not create polarization charges at the interface. Why? You have this plane and the refractive index is varying from here to here in this plane, but the electric field is perpendicular to this interface. So, here what do you satisfy for the electric field is - the tangential component or transverse component of electric field is conserved or continuous.

So, if you compare with the p-polarization, where you had these charges, here you do not see any polarization charges. So, if there are no polarization charges, even if the you have free electrons, you do not have any surface plasmon modes, ok. So, that is why surface plasmons are p-polarized in nature and they can be excited by only using p-polarized light.

So, in the present lecture we discussed the basics of plasmonics; we also discussed what are bulk plasmons and what are surface plasmons. And, then we used the decaying wave solutions and the applied the boundary conditions to arrive the dispersion relation of surface plasmons and from there we also saw that if you want to have surface plasmons one of the materials should have negative value of the dielectric function and that is why we need metals there. From there we also concluded that we need to have only p-polarized light and not s-polarized light for the excitation of surface plasmons. In the next talk we will try to see when this wave travels, as I told that when it travels it dies out. So, what is this distance until when it will die out that we call propagation length.

We also said that the electric filed will be in both the media and it will be decaying exponentially. I want to see what did the extent of this electric field.

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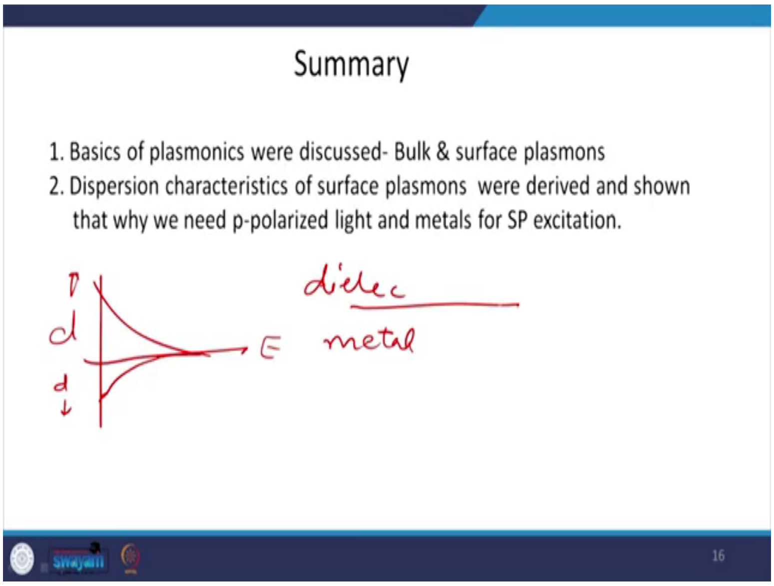

- what is the extent of this electric field in the metal and in the dielectric. So, if you have the electric field here and you want to see – this is the distance from the interface - we want to see how it changes. Also, there are certain issues, I mean, with the excitation of the surface plasmons. If you have a metal and dielectric interface: this is dielectric and metal and we want to excite surface plasmons here, there should be something with the wave vector thing. We will come to that, but there is something very interesting here that till now we have studied that this kind of interface supports this kind of surface modes - surface plasmon modes. In the next talk we will see how to excite these modes, ok.

Thank you.