Introduction to Atmospheric and Space Science
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Lecture – 59
Magnetic Mirroring and Loss Cone

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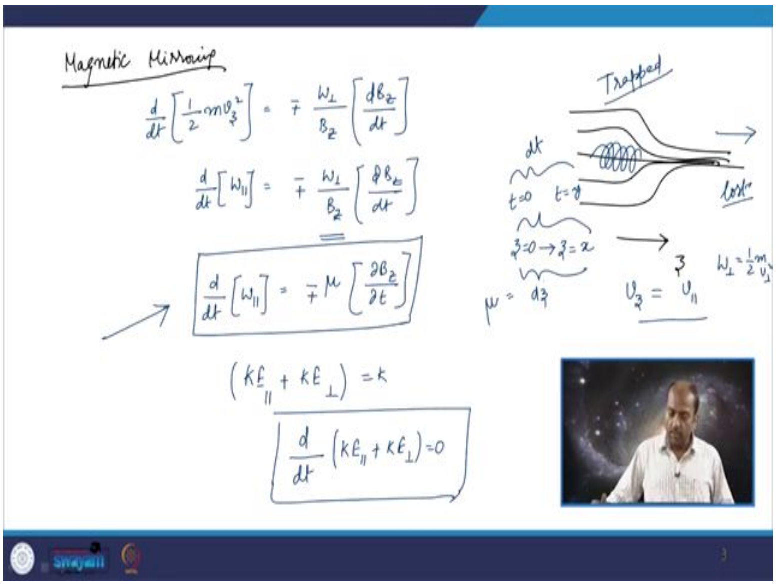

Hello dear students. So, in our earlier class, we have seen a mathematical treatment for the Magnetic Mirroring. So the idea was how a change in the magnetic field with respect to distance will allow the particles to be trapped. So we were discussing how something called as adiabatic invariants of magnetic moment if the magnetic field is changing with respect to time. So we have considered a magnetic field which is converging. That means, the magnetic number of field lines here are more.

And as it goes across this z-direction, the magnetic field lines are converging. So the field strength is increasing with the positive z-axis. So in this particular case, we wanted to understand how this particular field will allow the particles to be trapped. So the particles if they remain within this let us say this magnetic field they are trapped, if the particles escape, the particles can be considered to be lost. So we have to see how the mathematical treatment can help us to understand this magnetic trapping.

So, in our discussion we have stopped at a particular point. So we have seen that the rate of change of let us say half mv z square was derived to be minus plus W perpendicular B Z dB Z by dt. Now this v z is the velocity which can be written as v parallel. And w perpendicular is the perpendicular kinetic energy half m v perpendicular square.

So we can say that the rate at which the parallel kinetic energy changes is W parallel is simply minus plus W perpendicular divided by B Z whole d B Z by d t. So, we also know that w perpendicular by B Z is the magnetic moment. So, we write that d by dt of w parallel is equals to minus plus mu times dou B Z by dou t.

At all times, the total energy must remain conserved. That means, a sum of kinetic energy parallel plus kinetic energy perpendicular should remain a constant or the rate of change of total kinetic energy should be 0. So, we will exploit this energy conservation and we will try to see in order to keep the magnetic moment constant, how the particle gets trapped.

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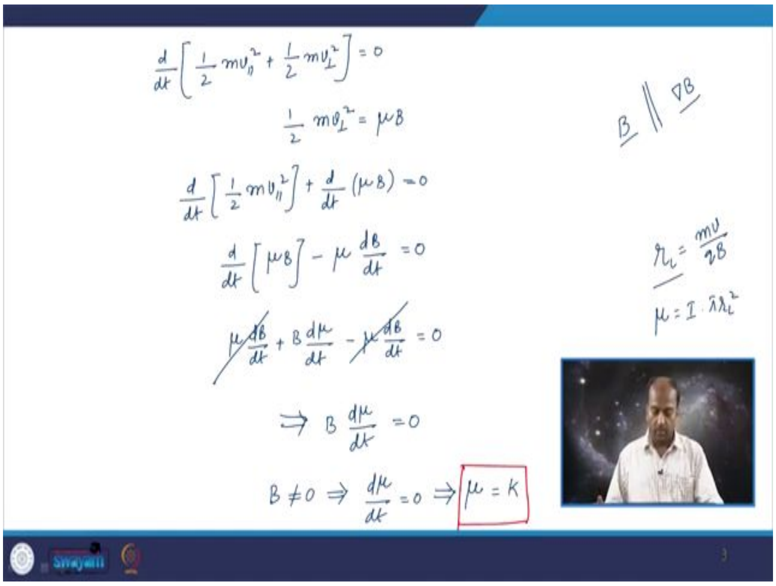

Let us say, we say that d by dt of half m v parallel square plus half m v perpendicular square is equals to 0. Now we know that half m v perpendicular square is equals to mu times B. So, we can substitute d by dt of half m v parallel square plus d by d t of mu B is equal to 0. From this expression, we know that the rate of change of the parallel kinetic energy is mu times dB by d z.

So, if you substitute it. So, we will write that d by dt of mu B minus mu dB by d t is equal to 0. or mu dB by d t plus B d mu by dt minus mu dB over d t is equals to 0. So we get rid off these 2 terms. We say that B is equals to d mu over d t is equals to 0. Now B is of course, not equals to 0 which implies d mu over d t is equals to 0 which implies mu the magnetic moment is a constant.

So, constant of motion, What does it mean? What is the situation that we have taken and how does this magnetic moment being constant helps us to understand the magnetic mirroring? So we have taken a magnetic field which is converging into a point, and if the particle which is charged and also has some mass, travels in the direction of this magnetic field. Here, so the most important thing that we should always remember is, there is a magnetic field and there is also a gradient in the magnetic field.

So in the case of gradient drift we have seen that the magnetic field and the gradient are perpendicular to each other. But here, in the magnetic mirroring case the gradient and the magnetic field are parallel to each other. So this kind of field only allows magnetic particles to be trapped. So if we take a particle to be traveling into the convergence of the magnetic field, we have now realized that in that particular situation, the magnetic moment should remain a constant.

So in our earlier discussions when there was change in a magnetic field, we have seen that even the magnetic field changes, there is a certain parameter which is called as the radius of gyration which also changes so as to keep the net magnetic moment constant. How does the radius of gyration influence the magnetic moment? The magnetic moment is defined as the current times the area. So, if r L is the gyration radius, so the area that it sweeps over 1 orbit will differ when the magnetic field changes by changing the radius of gyration.

For the present case we have been able to realize that if you take a certain type of field, the parallel component of the velocity is changing as the magnetic field is changing. So, this d B by dt is does not signify that the magnetic field is changing with respect to time. But it signifies as the time elapses how the particle travels from one point to another point and how the particle experiences different strengths of magnetic field.

So here, the particle is traveling from let us say 0 to physically speaking, the particle is traveling from lets say for example, for simplicity is traveling from z is equal to 0 to z is equals to x let us say. So and this duration of the space is covered from let us say t is equals to 0 to t is equals to something let us say y. So this elapsed time is dt and the elapsed distance is d z.

So the point is, how the particle experiences different magnetic field strengths as it travels across dz is also signified by saying that how the particle experiences as it travels,. So the point is, if the particle is moving like this it will always make sure that the net magnetic moment will always be a constant,. So, what is the significance of this magnetic moment remaining constant?

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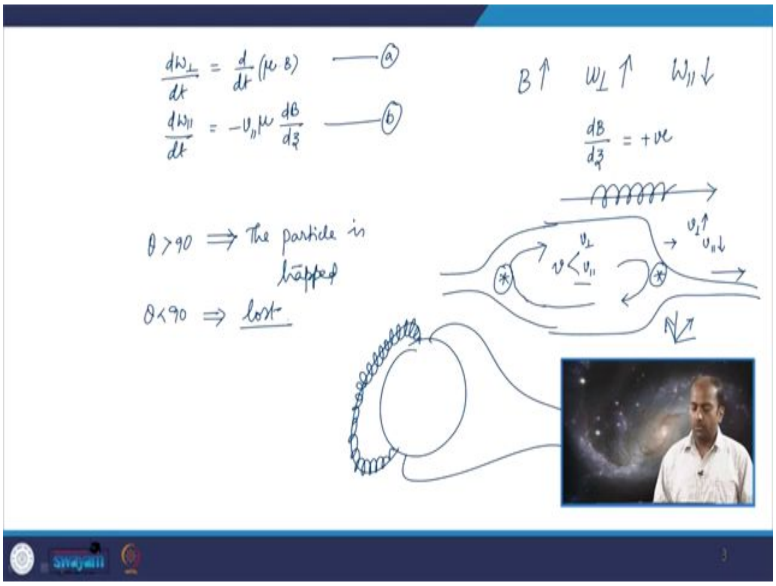

In case of magnetic mirroring, particle's motion is described by

$$\frac{dW_{\perp}}{dt} = \frac{d}{dt}(\boldsymbol{\mu}.\boldsymbol{B})$$

$$\frac{dW_{\parallel}}{dt} = -v_{\parallel}\mu\frac{dB}{dz}$$

Let us say for example. So, our earlier equations of motion dw perpendicular by d t is d by d t of mu time mu dot B and d w parallel by dt is minus v parallel mu dB by d z. So what I have done is, I have not written v parallel as d by dt of x rather, I left it like this. So, the dz by dz terms do not get cancelled here. So here, let us call this equation as equation a and let us call this equation as equation b. Now what do you see here?

So now let us say if the magnetic field is increasing, what happens to the perpendicular component? We are able to see that the perpendicular component of the kinetic energy is also increasing. Now as the magnetic field is converging along the z-axis, so if the magnetic field is changing or increasing as a function of z so, dB by dz will be a positive gradient. So if d B by d z is increasing, if the magnetic field is increasing along the z-axis, we can simply say that that parallel kinetic energy.

Now the parallel kinetic energy is decreasing,. So, if the magnetic field is like this as the particle is traveling in this direction, gyrating of course,. The magnetic field is of course is increasing in this direction as if the particle travels. So it is perpendicular kinetic energy is increasing but if you want the net magnetic moment to remain a constant, the magnetic moment remaining a constant demands the total kinetic energy to remain a constant.

So, this increase in the kinetic energy in the perpendicular direction should always be compensated with the decrease in the parallel kinetic energy. So, at a point lets say if the particle has started from here lets say the particles velocity is v which is which can be in any direction lets say the particles velocity is v. So this can be in any direction. So, this has now v perpendicular and v parallel. So, as it goes to this point v parallel is enormously large just to be able to account the increase in the magnetic field.

So in order to compensate this, v parallel decreases. As a result, at some point in the convergence at some point the v parallel will almost be 0. That means, there is no there is no velocity component which will make the particle to travel along the z-axis or along the magnetic field. So, v parallel becoming 0 the particle will immediately the there is only one component of velocity, the perpendicular component of velocity. The particle will immediately turn back to conserve the energy the total energy.

So, this is idea of increasing the magnetic field making the parallel component of velocity almost 0; allows the particle to return back into the magnetic field which is lower in strength,. Now imagine, if you keep a similar type of convergence on the other side as well. If the particle is traveling in this direction now, and if the particle reaches here and if it kind of experiences a very large magnetic fields change, it will kind of mirror. So these 2 points are called as the magnetic mirrors.

So these at these points, the magnetic field becoming so strong will make the perpendicular component of velocity enormously large and also make the parallel component of velocity almost 0 so that the particle is no longer bound to travel into the positive z-axis. So immediately, things will flip and the particle will immediately return back into the weak field region,. So into the weak field region, it comes back. So if you have another magnetic convergence at the other side, then you can trap these 2 particles,.

So as a consequence so what happens is let us say if you have the earth for our understanding, what happens is if the particles are gyrating along the magnetic field lines as the particles come close to the magnetic poles, magnetic poles are the regions where all the magnetic field lines will converge and as a result, the magnetic field strength at this point is stronger. So here the mirroring effect will happen and the particle will gyrate to the pole and it will again gyrate back towards the other pole.

So this is a phenomena which is actually in place in the magnetosphere of the earth,. So for all this to happen of course, you need to have a very strong gradient in the magnetic field and the particle should be traveling in a direction parallel to the magnetic field. Now let us define some more parameters. Let us say its not necessary that all the particles should be trapped like this. If the particles initial velocity is small, the particle may not get trapped here.

So there are several other conditions over which you can define or you can you can say whether the trapping of the particles will be efficient or will it happen or not. So for example, if the particles velocity at this point of mirroring; if it is greater than 90, you say that the particle is coming back into the weak field region. Of course, if it is not let us say if it is less than 90 you can simply say that the particle is not in the written path rather it is going intothe strong field.

So, that the particle is lost. So for example, particles velocity theta if it is greater than 90 at the mirroring point, we consider the particle to be trapped. Particle is trapped. If theta is less than 90, the particle is simply lost,.

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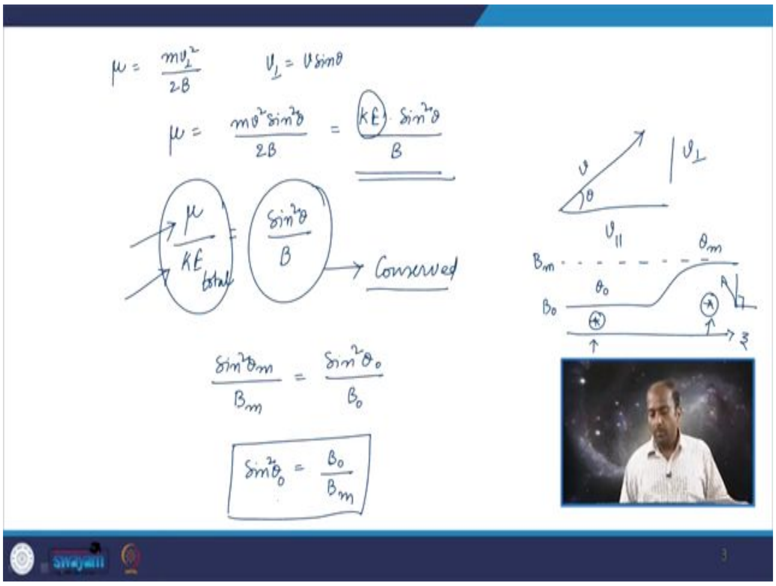

Now let us consider the part random direction velocity v which is in any direction theta. So this is this becomes the v perpendicular component and this becomes the v parallel component,. Let us consider the magnetic field strength is changing like this,. So, along the z-axis so this is the magnetic field in the beginning is assumed to be let us say B naught and later at a point when it becomes very strong, we assume it to be B m.

So and we want the mirroring to happen somewhere in the strong field. So, this the line that you see does not indicate the field line. Rather, it indicates the field strength where it is smaller and where it is larger,. So if it is the case, now we have 2 physical quantities which are conserved one the magnetic moment is conserved, and other the total kinetic energy is conserved,.

Let us say we define the magnetic moment like always as m v perpendicular square by 2 B. So according to this picture v perpendicular is simply v sin theta. So, we will write the magnetic moment as mv square sin square theta divided by 2 B or we will say that we KE times sin square theta by B. Now here, this KE is not the perpendicular kinetic energy. This KE is the total kinetic energy,.

So in that case K is the total kinetic energy. So mu is conserved I mean, so this quantity this entire quantity needs to be conserved. So, sin square theta by B is also conserved,. So here, we take a ratio of let us say mu by kinetic energy. So, always remember this kinetic energy let us say, we write it as total kinetic energy is sin square theta divided by B,

, now the point is in our earlier discussions we have been able to show mathematically that the magnetic moment is conserved,. And kinetic energy is of course, conserved there is no arguing. So the total kinetic energy should always be conserved. So magnetic moment is conserved here, in the sense that its not conserved always its conserved only under certain situations where the change in the magnetic field is very large in comparison to one gyro-radius.

So this limiting condition is what defines the adiabatic invariants. So, it is not an invariance; it is not a overall invariance its a adiabatic invariant; that means, only under certain conditions this invariance is valid,. So the kinetic energy is conserved. So this ratio is of course, comes. So these 2 quantities being conserved the ratio also conserved. That means, sin square theta by B is also conserved. So the particle moves such that sin squared theta by B is also conserved. So we take B 0 and B m.

So we will say that sin square theta m divided by B m is equals to sin square theta naught divided by B naught. What have I done? So sin square theta by B being conserved will simply mean that at 2 points, let us take one point here and let us take one point here. That means, you are taking a point in the weak field region and you are taking a point in the strong field region.

So throughout this path, sin square theta m by B m here and sin square theta naught by B naught is conserved,. Now what should be the limiting condition of theta m? That means, if the particle needs to be trapped. If the particle returns back into the weak field you will say that theta m should at least be greater than 90. If it is less than 90, the particle is not trapped the particle is going away. So that the particle should at least. So, this is your 90 degrees.

So the particle should have an angle at least 90 only above which is the particle consider to be trapped. S, theta m should at least be 90. So, we say that so theta m B becoming 90. So sin square theta m is 1. So, sin square theta naught is now B 0 by B m. So what is sin square theta naught? Theta naught is the initial angle with which the particle starts its journey. So theta naught is defined here and theta m is defined here.

So if the particle becomes trapped, its theta m is at least ninety or greater than 90 of course,. So theta m becoming 90 defines a condition for the initial angle of the particle. So if the particles initial angle is matching this condition only then sin square theta by B remains conserved. If sin square theta by B is conserved, only then the magnetic moment is conserved the magnetic moment conservation is what allows the particles to be trapped.

So all this is a kind of one-to-one conservation rule. So if you say that so we will say that sin square theta naught is equals to B by B naught by B m. B naught is a field strength at the beginning and B m is a field strength at the maximum point.

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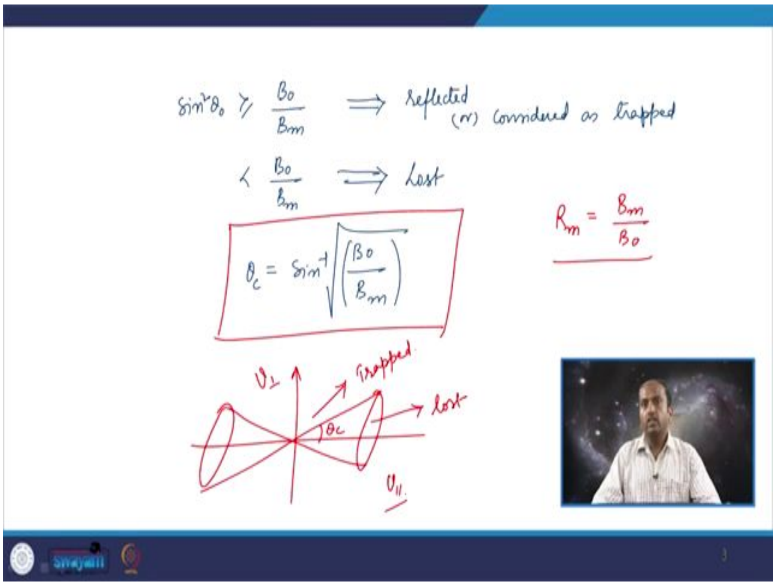

Critical angle for magnetic mirroring

$$\theta_c = sin^{-1}\sqrt{\frac{B_0}{B_m}}$$

So the limiting condition for this is simply sin square theta naught, the initial angle must be greater than or equals to B naught by B m. So only then, the particle will get reflected from a mirror point or the particle can be considered as trapped. And if it is less than B 0 by B m, we will say that the particle is simply lost,.

So, this theta naught is the minimum angle. So if theta naught is the minimum angle, or we can define a critical angle above which the reflection can happen,. So, theta c is equals to sin inverse square root of B 0 by B m. So, theta c is the critical angle above which reflection will happen so this is the magnetic mirroring we also define what is called as the mirror ratio R m as B m by B naught.

Mirror ratio is given by

$$R_m = \frac{B_0}{B_m}$$

So you want the mirror ratio to be x, to be very high only then the magnetic mirroring is a feasible idea,. So we can now define a loss cone. So theta c is the smallest angle above which above which reflection will happen. So this is theta c if the angle is less than theta c the particle is kind of lost, if the angle is greater than theta c the particle is trapped,. So this is the basic idea. So here we define v perpendicular and v parallel.

So, this is the basic idea of magnetic mirroring. How a certain type of magnetic field convergence can result into magnetic mirroring. And so we have defined what is the critical angle above which the magnetic mirroring will happen ?and we have we have drawn what is the loss cone. So just a visual representation of what particles will be lost and which particles will be trapped.

So this module ends here. That means, we have learnt various aspects of the plasma physics. So the idea is these aspects of plasma physics will be helpful for you to appreciate the ionospheric physics,. How the movement of particles: electrons and ions will happen and how the earth's magnetic field will influence the movement of these particles . So I will stop here.