Introduction to Atmospheric and Space Science
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Lecture – 58
Magnetic Mirroring

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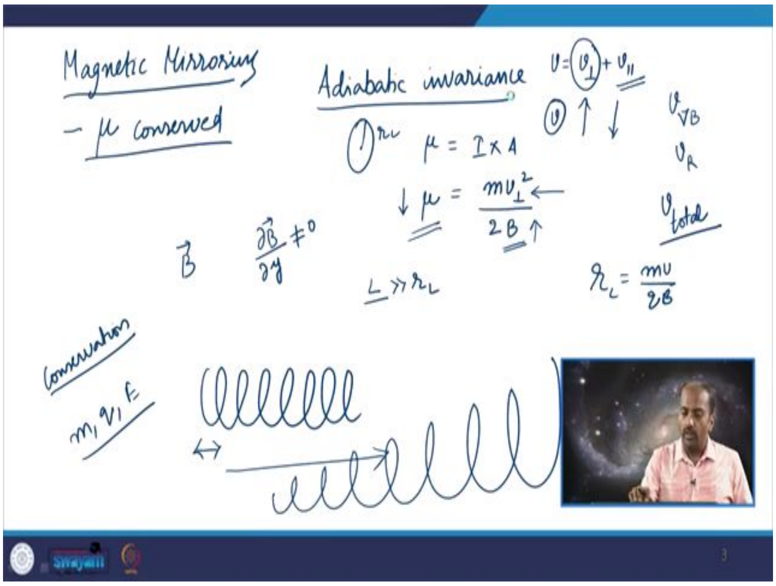

Hello, dear students. In today’s class we will try to understand what is called as the Magnetic Mirroring. So, the idea is , in our earlier classes we have derived expressions for the gradient drift, expressions for the curvature drift and then we learned how to combine these two for what is called as the total drift velocity or the total or the vaccum drifts.

So, in today’s class we will try to understand how a charged particle can be trapped within a magnetic field, what kind of magnetic field can result in trapping charged particles. So, the basic idea is the magnetic field has an ability to keep let us say the magnetic moment as conserved..

So, we have seen that the particle gyrating around a center with a let us say gyration radius r L will constitute a tiny magnetic dipole whose magnetic dipole moment is equal to the current times the area. Now we have seen that, we derived that this magnetic moment is equals to m v square by 2 B.

So, now a very important point to consider is that when you make the magnetic field in homogeneous we have taken the small approximation saying that the magnetic field of course, is changing with respect to space, it is changing with respect to in all our earlier cases the magnetic field was changing across the y direction.

That means the dou B by dou y is not equals to 0 . But what we have done is that the magnetic field can change, but the length scale over which the magnetic field changes is sufficiently large than the radius of gyration, one individual gyration orbit. So, let us say if the particle is grating like this we say that as long as; within this distance let us say within this distance the magnetic field is assumed to be uniform.

So, we are not going to complicate one single orbit of the particle rather when the particle moves across the magnetic field we may expect something like this to happen because of the strength in the magnetic field changing, but we will not expect the particles orbit, one single orbit to be complicated because the magnetic field strength is changing. So, we know that the radius of gyration is m v by q B.

So, in the field alignments thatwe have taken the magnetic field to be low at a particular point, where the radius of gyration will be larger because of the magnetic field, but not due to the particles mass right. now we will try to understand how magnetic field can be trapped by constituting a certain type of magnetic field, which; obviously, changes with respect to space.

So, far we are not included any discussion about magnetic field being non static. So, now, if the magnetic moment needs to be conserved then we will obviously, if you change the magnetic field the magnetic moment let us say if you increase the magnetic field the magnetic moment will decrease, but if you want the same magnetic moment to be retained or if you want this magnetic moment to be conserved even though the magnetic field is changing the only other parameter that you can tweak is the v perpendicular.

So, if you increase the magnetic field the magnetic moment will try to decrease, but at the same time if the perpendicular component of velocity is increasing to the magnetic moment will remain a constant. Now the velocity is always a combination of the perpendicular component plus the parallel component.

Now, if you are changing the magnetic field the v perpendicular is of course, compensating the change in the magnetic field so as to keep the magnetic moment conserved, but v perpendicular changing should also influence the v parallel. So, if the v perpendicular is increasing v parallel should decrease. So, as to keep the v constant . So, this is a special type of conservation I mean this is conservation under a special case let us say.

So, what type of conservation is this? So, generally we are very well aware of the conservation of mass, conservation of charge or conservation of energy. So, while discussing the charge conservation or mass conservation, we do not impose any conditions, we do not impose under this conditions only the mass has to be conserved mass is constant always a respective of any process in the beginning and the end mass is conserved energy is also conserved like that.

So, what it means is that this is a universal conservation; that means, under any given circumstances as far as you are not taking away anything the mass energy charge these quantities are always conserved, but the magnetic moment conservation is not like that. So, in plasma what we do is magnetic moment is conserved only if the magnetic field is not changing much within 1 gyro orbit. So, that is a basic condition that you put on the magnetic moments conservation. So, this type of conservation which is not valid throughout, but rather valid over a certain limit of, let us say variable change is generally called as the Adiabatic invariance,.

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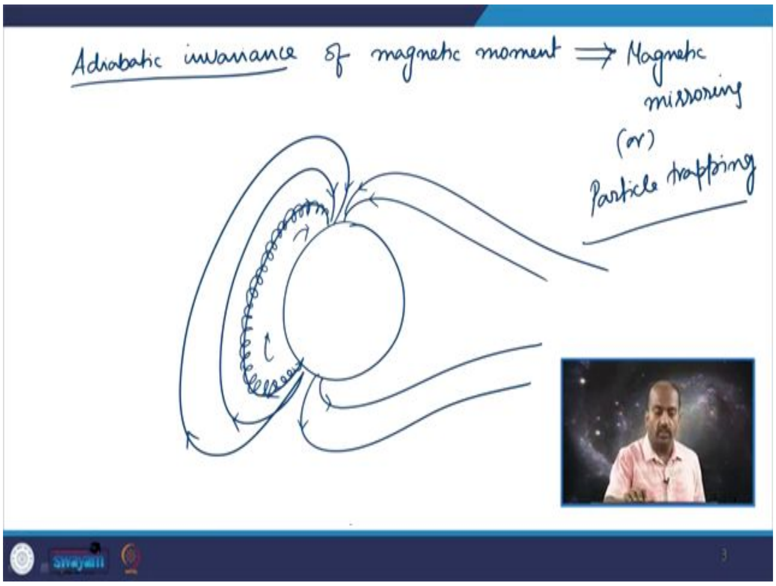

So, let us see what else, we learn about adiabatic invariance. So, we will try to use the variation or the conservation of magnetic field ,magnetic adiabatic invariance. So, essentially the adiabatic invariance of the magnetic moment is what allows leads to the magnetic mirroring or particle trapping.

Now, why are we discussing the magnetic mirroring or magnetic particle trapping in atmospheric or space physics is that, we will always see that the if you have the magnetic field line like this let us say for the case of earth. So, this is the side that is facing the sun, but the magnetic field lines are diverging away from the geographic south pole and converging at the geographic north pole.

Now, what we will see is that the magnetic field lines always accompany or let us say facilitate the gyration of charged particles to be travelling from one pole to the other pole; that means, there are charged particles which are moving from one point to its conjugate point. So, this particle is trapped so this will keep gyrating from here to here like this so it will keep gyrating depending on the charge, it will keep gyrating from one point to other point.

So, what is this alignment of magnetic field what is the nature of this magnetic field which actually facilitates this kind of charge particle trapping is of interest? So, that is why we are studying the magnetic mirroring idea and we will try to see in mathematically what kind of magnetic field will allow such a particle to be trapped, so that is the basic idea.

Now, let us say for this problem . We consider a certain type of certain geometry of the magnetic field because eventually we are now speaking of the inhomogeneous magnetic field so there should be a magnetic field which is changing with respect to space and this inhomogeneity should only be at any given point. If the particle is grating the magnetic field should not change much within the particles orbit radius, but over large distances the magnetic field is of course, changing.

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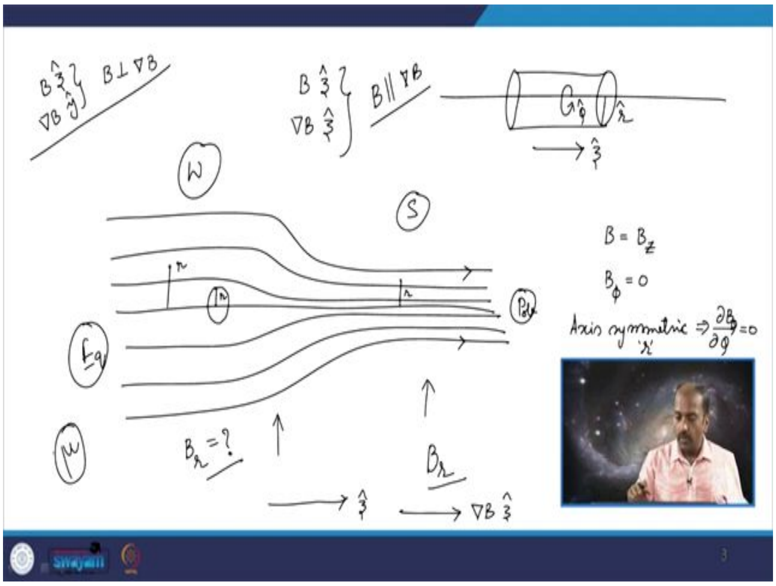

Now let us say. So, the geometry is cylindrical again. So, we take cylindrical geometry like this.

So, the coordinates are; this is the r coordinate and this is the positive z and the anti clockwise direction or is the phi direction. Now, the magnetic field is chosen like this lets say the magnetic field lines are trying to converge. Now what is the direction? So, this is the z direction so the magnetic field is now pointed in this direction the original magnetic field is pointed in the z direction.

Now what is the phi direction? So, this is the r direction so magnetic field has the z component. Now we take that the magnetic fields phi component is 0 and we also assume that the magnetic field is axis symmetric; which means that the magnetic field at for a given value of r any value of phi the magnetic field will be constant.

So, if you take here any point if you take let us say at a at a distance r for any value of phi the magnetic field is same; that means, that the magnetic field is symmetric about the axis. Now the most important thing about this type of field alignment is that the magnetic field is of course, is pointed in the z direction, but we are not saying that the magnetic field has an r component. So, what is the value of B r? What is the possibility of having a B r lets say?

So, let us say the magnetic field is kind of weak here and the magnetic field is let us say is strong here and the magnetic field is weak here. What does it mean? So, there are less number of magnetic field lines per unit area in this region, the magnetic field lines per unit area in this region are less and more number of field lines per unit area here.

So, let us say if you take any distance r the magnetic field strength at this point will be something. If you take the magnetic field at the same distance r the magnetic field strength will be different so; that means, that the magnetic field is changing with respect to r so there is invariably a magnetic fields component across the r direction.

Now, how do we find that component of the magnetic field across the r direction. So, if you observe carefully so the idea is simple. So, in earlier cases we had the just to remind the magnetic field pointed in the z cap direction and the gradient of the magnetic field pointing in the y cap direction so which means that the magnetic field is perpendicular to the gradient.

while we were discussing the gradient and the curvature drifts so. But here in this case the magnetic field is pointed in the z cap direction and the gradient of the magnetic field so here the gradient of the magnetic field. So, what is the direction of gradient of course, this is the direction of gradient. So, the gradient is the maximum change in a particular physical quantity.

The direction of this gradient is; obviously, in the z cap direction itself so magnetic field is changing in is increasing in this direction. So, the magnetic fields gradient is also in the z cap direction. What it means is that now we have a scenario in which B is parallel to del B. So, the magnetic field is parallel to del B so this is a more realistic magnetic field.

So, if you consider the magnetic. So, let us say if this is the pole, this is the equatorwhere the magnetic field lines are placed farther apart and when they approach the pole the magnetic field lines will try to converge. So, this is the equator and this is the lets say this is the pole a more realistic magnetic field.

Now, let us try to understand how this kind of configuration, if you have to conserve the magnetic moment how this kind of configuration we will result in the trapping of a particle? So, what do you mean by trapping, the particle should always travel between two poles of the magnetic field that is what the mirroring is. So, there should be some physical effects due to which the particle should change its trajectory and get reflected from a particular point.

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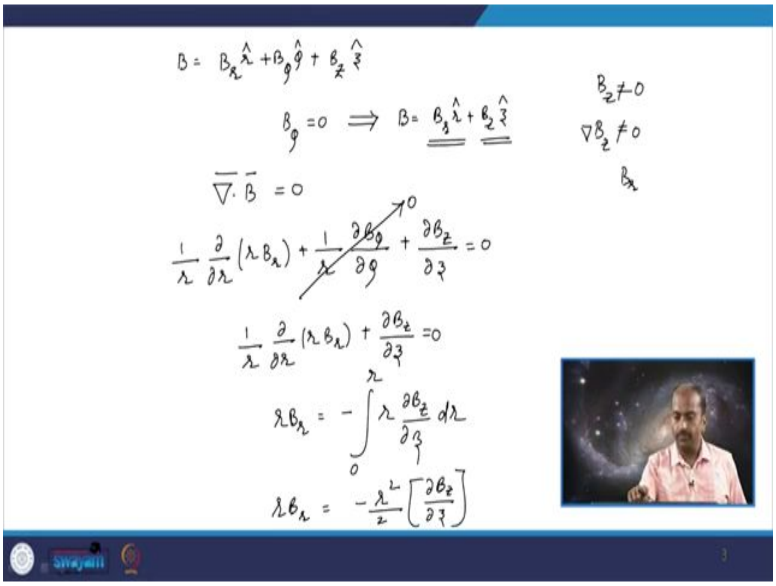

Now, we start from the simple idea we will write the magnetic field as B is B r across r cap plus B phi across phi cap plus B z across z cap. Now B phi is 0 so we will simply write the magnetic field as B r cap plus B z cap.

Now, how do we find B r? We know B z we know that B z is the non 0 component of magnetic field whose gradient is also present. So, B z is of course, non 0 ,del B is also non 0, but how do we found B r? Simple, we will say that the magnetic field the dot product should be 0.

So, how do we write it? We write it as 1 by r dou by dou r of r B r plus 1 by r dou B phi by dou phi plus dou B z by dou z is equals to 0, this is anyway 0 due to the assumption is itself. So, we write 1 by r dou by dou r of r B r plus dou B z by dou z is equals to 0.

So, r B r is minus integral 0 to r r dou B z by dou z into d r, r B r is equals to minus r square by 2 times dou B z by dou z.

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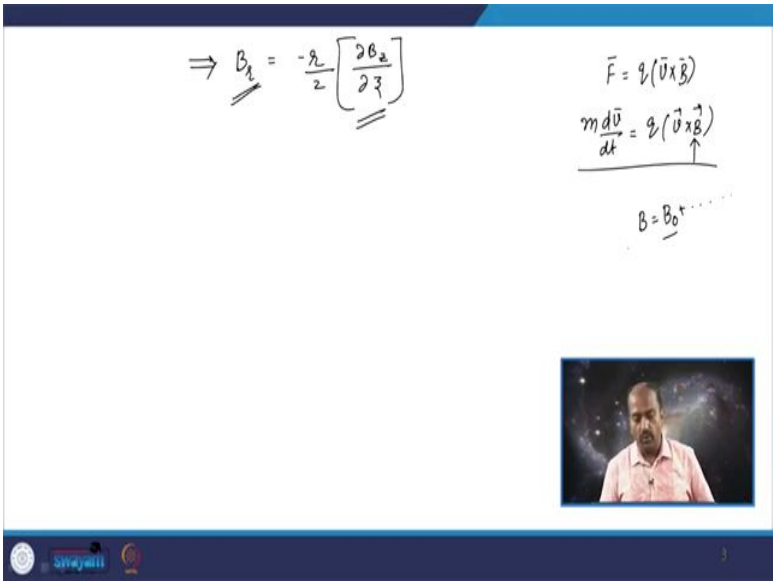

Or, which implies that B r should be minus r by 2 times dou B z by dou z. Now, one very simple thing that we can understand from here is that B r arises only due to the fact that B z changes with respect to z. If there is no change of B z the z component of velocity with z direction there is no possibility that we can get B r.

So, very simple. And how does B r change? B r changes linearly with respect to r this time. So, let us say now what does I mean this is the third time that we are dealing with the magnetic field and charged particle. So, in every case we always say that the magnetic force that is experienced by the particle is always simply q times v cross B.

So, the differential equation is m d v by d t is q times v cross B. So, if you if the magnetic field is changing with respect to space. So, in the first case we solve this equation directly we got the solution as well which indicated that the particles velocity components will add in such a way to give a circular motion.

And the v z component will allow the particle to be travelling parallel to the magnetic field. In the second case we took this magnetic field and we made Taylor expansion such that B 0 is the magnetic field that the centre of the gyration orbit and B 0 remains B 0 within the distance of r L. Then in the third case that this case what we are going to do is, we are still going to do the same thing.

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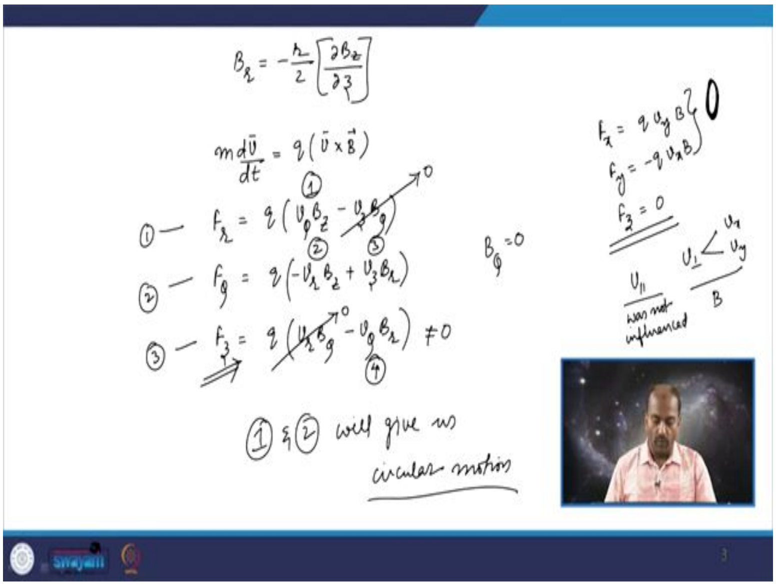

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B r is minus r by 2 dou B z by dou z in continuation. So, m d v by d t is simply q times v cross B. So, in the cylindrical coordinates q times v phi B z minus v z B phi. q times v r B z plus v z B r and f z is equals to q times v r B phi minus v phi v r.

So, these are just the components of the q times v cross b. So, we always had this type of curl products, this only gave us the idea of how the velocity changes with respect to the time in the presence of a certain type of magnetic field. Since B phi is 0 we immediately get rid of this term 0 and this term. So, we have term number 1 the set 2 the set 3 and the set 4.

Now,. So, if you look back these 3 terms let us say term number Let us say take this to be equation 1 this to be equation 2 and this to be equation 3. If you look back term number 1 and 2 of course, they were similar. So, we had F x being given as let us say q times v y B and F y is minus q times v x B and F z was of course 0, when was this? when the magnetic field was not changing with respect to space a homogeneous magnetic field.

Then we said that there is no force that is acting on the particle in the parallel direction. So, v parallel is uninfluenced v perpendicular is what influence. So, v perpendicular was v x and v y these two components of velocity were influenced by the magnetic field and this component v parallel was not influenced by the magnetic field at all. So, this is very simple I mean if the magnetic fields vector is parallel to the velocity then the curl becomes zero. So, there is no point of discussing about it.

But when it is not that so then we combined these two differential equations then we said that the velocity will be such that it is executing a simple harmonic motion around an imaginary guiding centre. But here what you see the magnetic field is along the z direction and the velocity component, since the magnetic field is being across the z direction also there is still a force f z which is non 0, and so F r and F theta. So, terms 1 and 2 will give us the circular motion so it is simply the Larmor precession that we have seen in so many cases. So, terms 1 and 2 will give us the circular motion.

Now, most importantly this is the force that is acting in the parallel direction is something new for us. So, let us see how this non 0 force across the z direction will give us some physical effects.

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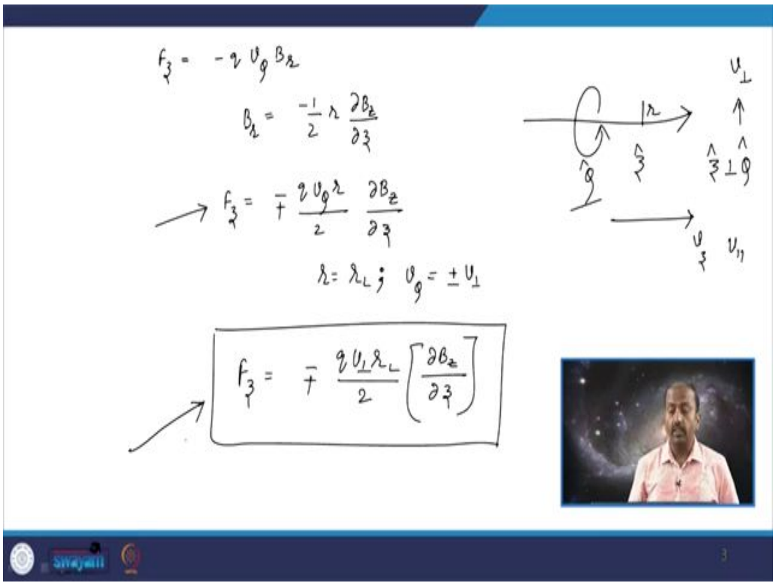

So, let us say F z like we have calculated F z, F z is minus q times v phi B r.

So, we know that B r was calculated to be minus 1 by 2 r dou B z by dou z. If you substitute this and since there is a charged term which can take positive or negative, we can write F z is equals to minus plus q v phi r by 2 times dou B z by dou z. So, we say that r is equals to r L and v phi can be plus minus v perpendicular. Why is it so? Because geometry was like this.

So, this is the z direction and any revolution this is the phi direction and this is the r. So,velocity component which is across z direction is simply the parallel direction and velocity component across the phi direction which is, z is perpendicular to phi so velocity component across phi direction is simply v perpendicular. So, this is the F z so the force is clearly due to the magnetic field nothing else.

So, the velocity component being perpendicular to the magnetic field B z, F z is the force that is resulting from this configuration. So, we can write F z as q v. So, now, F z is minus plus q v perpendicular r L divided by 2 times dou B z by dou z. So, what is the effect of this particular force you know what we have done we have taken a particular type of field configuration in a cylindrical geometry, then we have said that there is a gradient of the magnetic field which is parallel to the magnetic field itself and then if a charged particle starts to experience this type of magnetic field we will try to understand what will be the effect.

So, first result is that the magnetic field imposes a force F z which is non 0 this time and if this force is non 0 this time so what is the effect of this popular force. Now let us substitute this force F z and try to see the differential equation that results.

In case of magnetic mirroring (Particle trapping), μ is conserved.

Force in perpendicular direction is

$$F_z = \pm \frac{qv_\perp r_L}{2}\left[\frac{\partial B_z}{\partial z}\right],$$

And $\frac{d}{dt}\left[\frac{1}{2}mv_\parallel^2\right] = \pm \frac{W_\perp}{B_z}\left[\frac{\partial B_z}{\partial t}\right]$, gives energy conservation.

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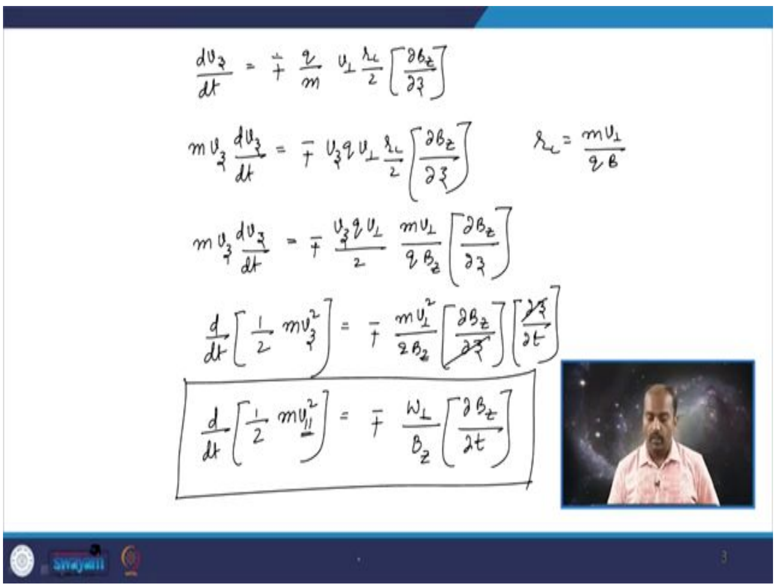

So, F z is d v z by d t is simply minus plus q by m, v perpendicular r L by 2 times dou B z by dou z. Or we will write m v z d v z by d t as minus plus v z, q v perpendicular r L by 2 dou B z by dou z.

We know that r L is m v perpendicular by q B. So, m v z d v z by d t is minus plus v z q v perpendicular by 2 m v perpendicular divided by q B z times dou B z by dou z. Or d by d t of half m v z square is minus plus m v perpendicular square divided by 2 B z into dou B z by dou z into dou z by dou t or d z by d t. It say that d by d t of half v z is parallel, v parallel square is equals to minus plus, m v perpendicular square divided by 2 is W perpendicular divided by B z times dou B z by dou t.

So, this is the consequence is such that. So, we calculate the F z ,F z is nothing, but m d v z by d t and with F z we have made some simplification. So, we have written v z as the parallel component and dou B z by dou t.

So, now see the point is the rate of change of the parallel kinetic energy depends on the perpendicular kinetic energy of course, and the change of the magnetic field with respect to time. So, we will stop here we will try to see how this conservation of energy or leads to the conservation of the magnetic moment which we call as the adiabatic invariant and eventually all these things leading to the trapping of charged particle in a magnetic field. So, we will stop here we will continue this discussion in the next class.