Introduction to Atmospheric and Space Science
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Lecture - 57
Vacuum Drift and Planetary Ring Current

Hello, dear students in today's class we will try to understand what are called as the Vacuum Drifts.

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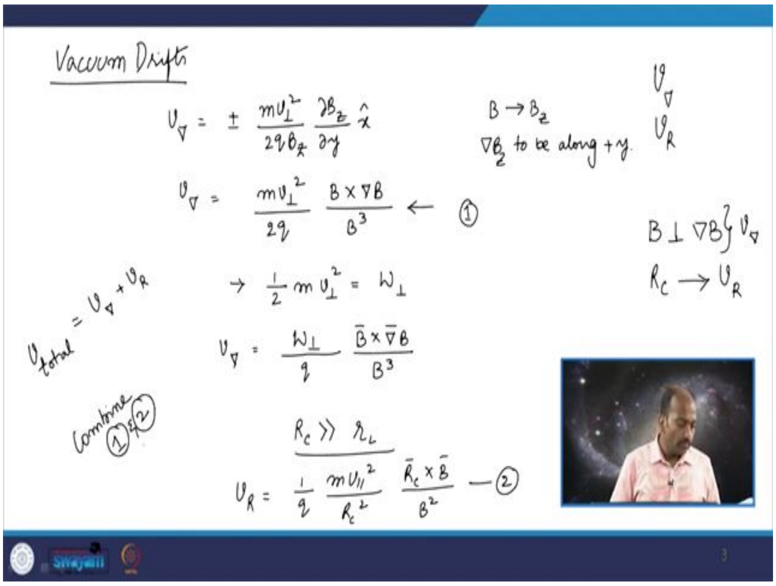

So, in the previous class we have studied the gradient drift and the curvature drift. So, we have also mentioned that the magnetic field imposes a condition that any particle has to always experience both the drifts together; that means, the curvature drift and the gradient drift will always be together.

So, the magnetic field being non uniform kind of imposes that the field line should always be curved. And when charged particle travels across the magnetic field or the magnetic field line due to its mass being circulated around a radius of curvature, it experiences a centrifugal force away from the centre. And this force again was told to give a drift which is called as the curvature drift.

So, we have seen all this aspect. So, we have derived expressions for the curvature drift and the gradient drift. So, the curvature drift was denoted with v R and the gradient drift was denoted with v del. So, the field alignment that we have taken was such that the magnetic field the original magnetic field was perpendicular to the gradient in the magnetic field.

The gradient drift velocity can also be written as

$$v_{\nabla B} = \pm \frac{\mathbf{m} v_{\perp}^2}{2q} \frac{\boldsymbol{R_c} \times \boldsymbol{B}}{\boldsymbol{R_c^2 B^2}}$$

And there was aradius of curvature R C due to which we have calculated the curvature drift. And this particular field alignment the magnetic field being perpendicular to the gradient of the magnetic field has given rise to the gradient drift. So, we can write the gradient drift and the curvature drift in a more convenient form.

So, what we will do in today's class, we will try to see how we can combine these two drifts in a single expression. And we will do this for a situation let us say called as vaccum.

So, we demand that the magnetic fieldis a source free region there are no charges which are present, there are no currents which are present and the magnetic field the nature of the magnetic field is itself is inherent let us say.

So, we have seen that the v del be v del is equals to plus minus mv perpendicular square divided by 2 q B z dou B z by dou y across x cap. So, this is the expression that we have derived we can more conveniently write this as v del is equals to mv perpendicular square divided by two q times B cross del B divided by B cube. Now, if you substitute your magnetic field to be along B z and del B z to be along positive y direction you will still get the same relation.

So, if we say that half mv perpendicular square is the perpendicular kinetic energy we can write v del as W perpendicular divided by q times B cross del B by B cube, B cross, this is the magnitude of the magnetic field. Now, similarly we can also write the curvature. So, if you take the curvature R C.

So, in order for the curvature effects to be reasonable or in order to highlight the effects of curvature because there is one more radius that is involved here which is the r L. In order to make the effects of gyration or particles movement across the radius of curvature we will say that the radius of curvature is much larger than the radius of gyration.

So, in that case we can say that the curvature drift can probably be written here itself. So, that we can understand the full picture. So, the curvature drift v R was derived to be 1 by q mv parallel square divided by R C square times R C cross B divided by B square.

So let us call this equation as equation 1 and lets call this equation as 2. Now, so, in the last class we have seen that any particle will always experience these two drifts together; that means, the total drift. So, v total is written as v due to the gradient and v due to the curvature. So, in this class we will try to understand how we can combine these two drifts for a special case of magnetic field we will assume a particular type of magnetic field and its variations along different directions and its components along different directions.

Then in that particular case we will try to combined how we can combined?. So, we will try to this class is going to be how do we combine 1 and 2. So, we will say that.

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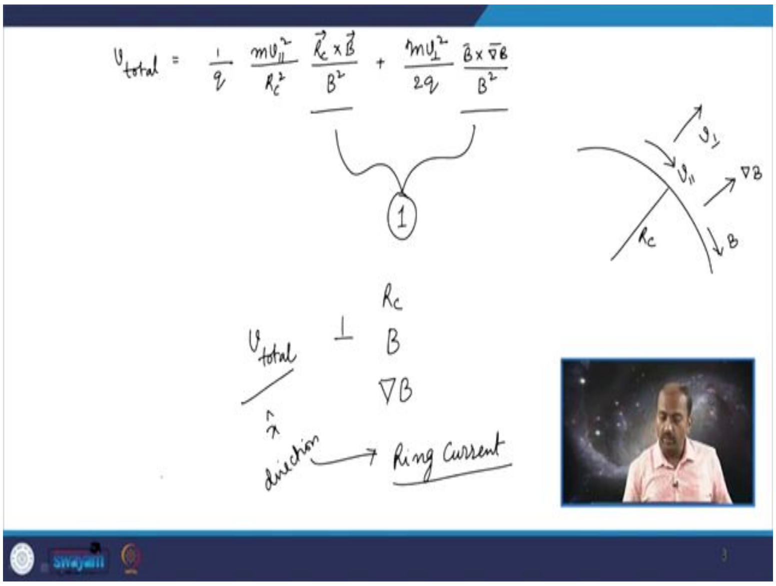

Total drift velocity

$$V_{Total} = \boldsymbol{v}_{\nabla \boldsymbol{B}} + \boldsymbol{v}_{\boldsymbol{R}} = \frac{m}{q}\left(\boldsymbol{v}_{\parallel}^{2} + \frac{v_{\perp}^{2}}{2}\right)\frac{\boldsymbol{R_c}\times\boldsymbol{B}}{\boldsymbol{R_c^2 B^2}}$$

So, for now our objective is v total is equals to 1 by q times mv parallel square divided by R C square times R C cross B by B square plus mv perpendicular square divided by 2 q times B cross del B divided by B square. So, the idea is if you have a magnetic field the velocity component that is parallel to the line of force is v parallel and the velocity component perpendicular to it is v perpendicular.

So, we have assumed that the gradient was in this direction and the magnetic field was in this direction and their radius of curvature was assumed to be this see this is the configuration that we have assumed. So, for now what we will do is we will take this in for combining this say these two curl terms or these two cross products. So, we will try to get instead of these two crossed products we will try to get one cross product.

So, now one very important inference here is that v total is the total drift velocity is perpendicular to R C, is perpendicular to the magnetic field and is also perpendicular to the gradient of the magnetic field these three parameters. So, we have seen what will be the consequence of this particular alignment in which the total drift velocity that is attributed to a particle will be perpendicular to the radius of curvature the magnetic field and the gradient of the magnetic field.

So, this is why we said that the v total will probably be in x cap direction to facilitate the formation of the planetary ring current the equatorial or the toroidal ring current that flows around the planet is called as the planetary ring current. Now, so let us say we transform this into cylindrical coordinates. So, just to be sure. So, I hope all of you have had some introductory class where the different coordinate systems were explained and how this different coordinate systems are different or how these coordinate systems can be used for different situations.

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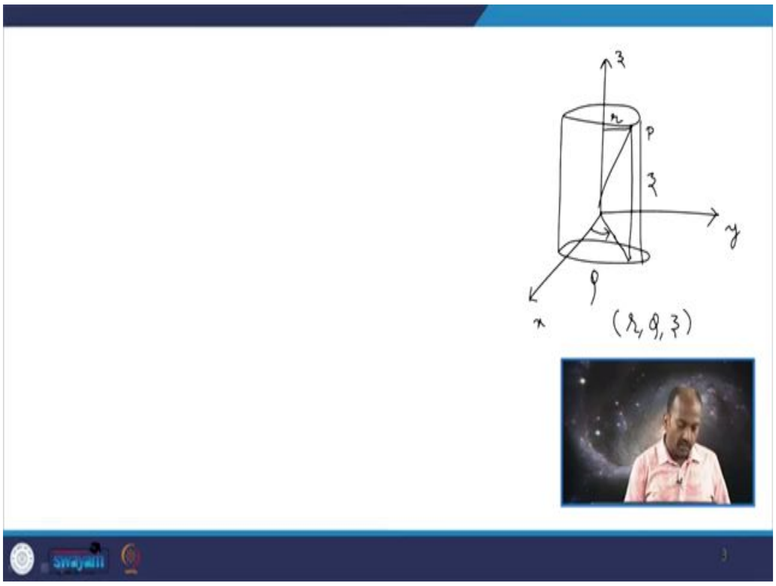

So, if the cylindrical coordinate system is very simple. So, if you take this ,if you take any point P which is lying on this let us say this point this is the point P. So, now, you want to identify the coordinates of this particular point where the cylindrical coordinates come with r, phi and z. So, your coordinates are the cartesian coordinates are x, y and z. Now, what you do is you draw an imaginary plane let us say like this and imaginary plane like this. So, the idea is, you assume the point that you want to represent with coordinates to be lying on this cylinder and the angle that it subtends if you draw a plane like this the angles it subtends.

So, let us say if you want to reach to this point P the in the anti-clockwise direction the amount of angular rotation that you have to do so as to reach this plane is referred to as phi and the point P' s distance from perpendicular distance from the x, y plane is referred to as z. And r is simply the the point P' s intercept on to the z axis is r. So, this is the basic idea of the cylindrical coordinate system.

So, what we will do for today we will assume the particular geometry of the coordinates of the magnetic fields such that.

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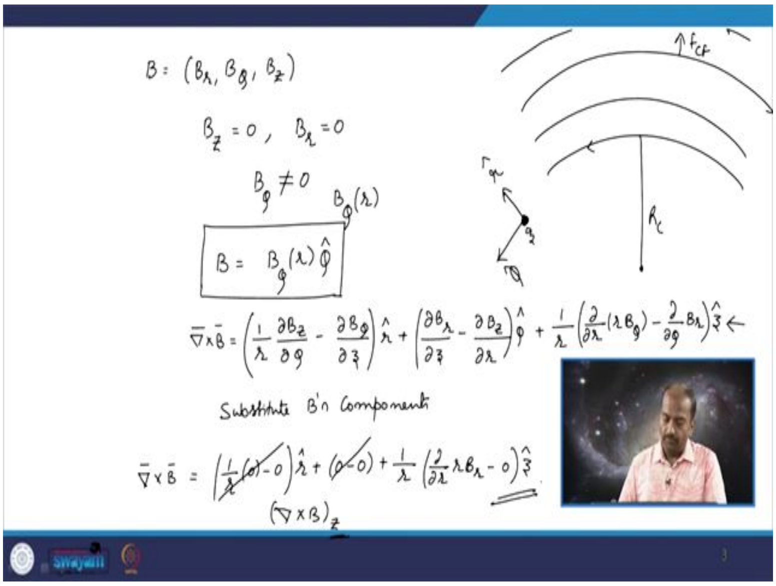

Let us say the magnetic field lines are curved like this. Now, the radius of curvature is this this is R C; now the coordinate system is assumed to be this. So, this is the phi direction the phi cap. So, the magnetic field is directed in the phi cap direction and the centrifugal force is going like this and this direction is the r cap.

So, we take the field such that it decreases with r. So, if you go further of a path the magnetic field lines will be placed further away. So, the idea is now you have the center of this curvature and the radius of curvature at this field line is R C.

Now, the original magnetic field line is pointed in the phi cap direction and the gradient I am saying that the magnetic field lines are placed further apart as you go across r; that means, as you move away from the radius the magnetic field lines are moving further apart that means the magnetic field strength is decreasing as you go away.

So, the gradient is oppositely directed.. So, the magnetic field can now be written in the cylindrical coordinates or the components are B r, B phi and B z. Now, B z there is no component of magnetic field across z direction B z is 0, B r is also 0, but what does it mean B r being 0 means there is no magnetic field in the direction of r, but the magnetic field B phi is not 0 the magnetic field is indeed directed across the phi direction but this magnetic field B phi will vary as a function of r.

So, magnetic field which is directed in the phi direction will have different values at different values of r, but the magnetic field is always directed across the phi direction. So, that is the basic idea. So, the magnetic field can now be written as B is B phi as a function of r and which is directed in the phi cap direction. Now with this type of magnetic field. Now, let us say this is the type of magnetic field that we have taken now is there a gradient of course, there is a gradient across the r direction is there a magnetic field? there is a magnetic field across along the phi direction.

Now, with this type of magnetic field we will try to assume we will try to combine the these two cross products. Now, where is the z component missing I mean so, the z component is out of this page. So, z component is this out of the page which is the z component. So, again the phi component magnetic field is directed in this direction along phi cap the strength of the magnetic field is decreasing as you increase the value of r and there is no magnetic field component across the z direction.

So, simply we have transformed this into the cylindrical coordinates. So, we will try to see how we can combine the both the types of magnetic field. Now, let us say we know from the Maxwell equations the del cross B the curl of the magnetic field in cylindrical coordinates can be written as 1 by r dou B z by dou phi minus dou B phi divided by dou z along r cap plus dou B r by dou z minus dou B z by dou r across phi cap plus 1 by r dou by dou r of r B phi minus dou by dou phi of B r across z cap..

Now, if you substitute B the magnetic fields components we can write the curl of the magnetic field as del cross B why is the curl important here. The curl is important here to find out the gradient and the curvature drifts as 1 by r times 0 minus 0 along r cap plus 0 minus 0 plus 1 by r dou by dou r of r B r minus 0 across the z cap.

So, we just have substituted all the different components of B.

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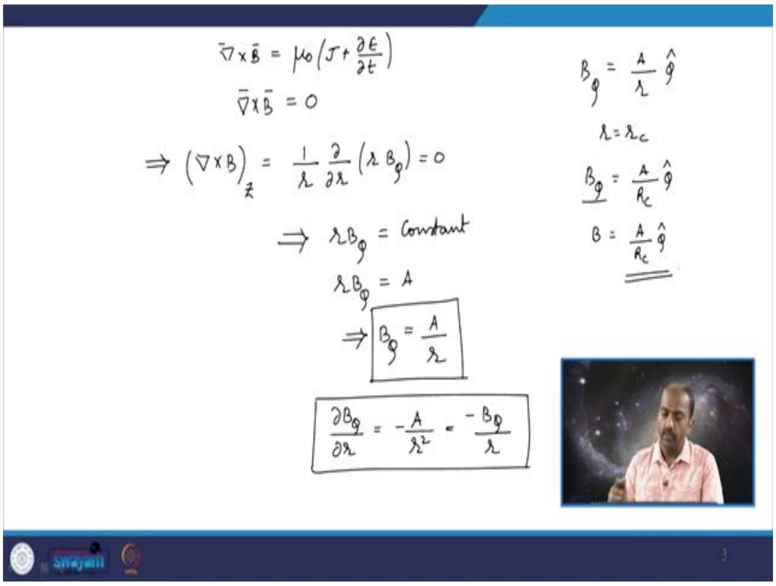

So, from the Maxwell equations we know that del cross B is equals to mu naught times J plus dou E by dou t J plus dou d E by dou t.

So in vacuum we say that del cross B is equals to 0; that means, the products that we have calculated del cross B. So; that means see from here what we say is that the del cross B since this is 0 has only z component.

So, del cross B has only the z component. So, del cross B the z component of del cross B was calculated to be 1 by r dou by dou R of r B phi which is actually equal to 0; that means, r B phi is just a constant so let us say this constant we call it as capital A.

So, the magnetic field B phi is A by r what does it mean? The it means that the magnetic field is changing with respect to the distance as 1 by r, but does it also mean that the magnetic field has a component across the r direction, no it does not mean it simply means the magnetic field is directed in the phi direction, but it varies as a function of r this is the basic difference.

So, now let us say that dou B phi by dou r now simply becomes minus A by r square which is also equal to minus B phi by r, now B phi is A by r in phi direction. So, if we use r is equivalent to R C we say B phi is equals to A by R C across phi direction. So, this is the magnetic field you call it as B phi or B we simply call it as B which is equals to A by R C in the phi direction.

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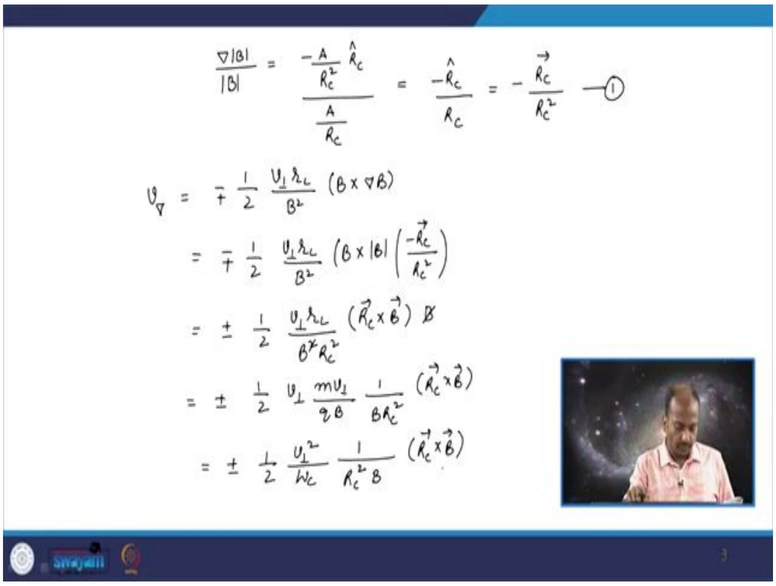

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Now, we will take a ratio let us say the ratio ofthe gradient of magnetic fields magnitude divided by the magnitude of the magnetic field is equals to minus A by R C square directed along R C divided by A by R C which is equals to minus R C cap divided by R C which is equals to minus R C over R C square.

Now, coming back to the gradient drift velocity term which is equals to minus plus half v perpendicular r L divided by B square times B cross del B. Now, what have we done so far? Let us look back before we continue. So, what we have done is we have taken a certain type of geometry for the magnetic field and its gradient.

So, if this magnetic field is true, the curl of the magnetic field should look like this, let us say and having substituted the magnetic fields components which are for example, along the z direction it is 0 and along the R direction is also 0 it has only one component which is the phi component which is varying as the distance varies.

So, we have done that we have realized that the del cross B will only have a non-zero z component. Now, for the vaccum let us say in the absence of any source any charges or any currents we say that the del cross B will be 0; that means, the z component of the curl that we have derived should also be 0.

So, in that case the magnetic field B phi or you call it as the B itself because B does not have any other components B phi should simply be A by r. So, if B phi is A by r dou B phi by dou r simply becomes minus A by r square we are calculating or we are evaluating at a distance of r is equals to R C.

So, we are substituted r is equals to R C in this. So, then we have calculated the ratio of the gradient of the magnetic field only the magnitude divided by the magnitude of the magnetic field turns out to be this. So, this is a gradient this is a vector.

Now,we are getting back to the gradient drift velocity formula that we earlier had. Now, let us say this I will simply rearrange this equation for convenience let us say v perpendicular r L by B square times B cross using del B from equation number 1 let us say as B times R C divided by R C square.

So, which is equals to plus minus there is a minus and the cross product can be rewritten v perpendicular r L divided by B square R C square times R C cross B into B.

So, we cancel this B now we will get plus minus half v perpendicular mv perpendicular by q B times 1 by B R C square times R C cross B which is equals to plus minus half v perpendicular square divided by omega c times 1 by R C square B times R C curl B.

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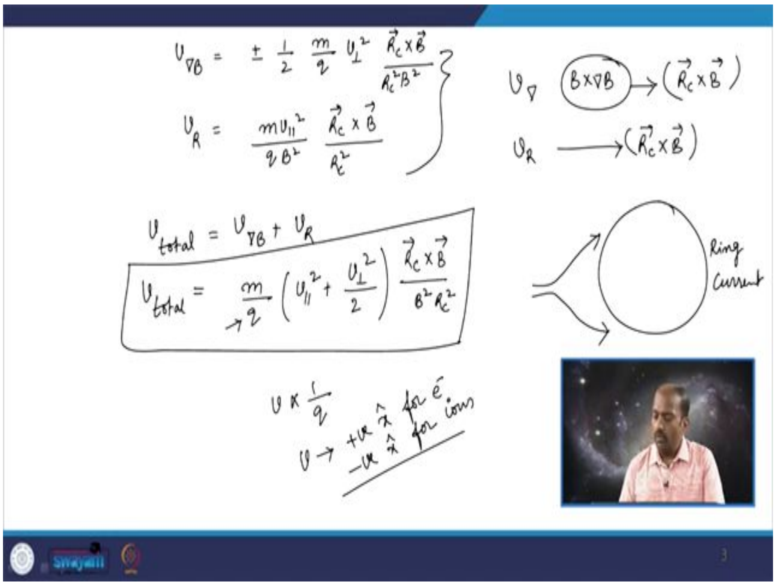

So, we will write it as v del B is equals to plus minus half m by q v perpendicular square R C cross B divided by R C square B square.

So, we know that this is the gradient term the curvature term is already written as a curl of R C and B m v parallel square by q B square into R C again curl of R C times B, R C square B square. So, what have we done? We have done that B cross del B term is transformed into R C cross beta. So, this is further del path and for the curvature part we already had the curl in term so f R C cross B. So, we can be able to combine let us say these two together.

So, v total is now v del B plus v R which can be written as m by q times v parallel square divide plus v perpendicular square divided by 2 the factor 2 coming here into R C cross B divided by B square R C square. So, this is the v total. So, this is the total drift velocity that the particle experiences, when it encounters A inhomogeneous magnetic field whose magnetic field lines are curved. So, like I said in the last class this drift velocity is perpendicular to R C and B and this also seems to be dependent on the charge.

So, v is proportional to 1 by q; that means, the direction of v will be in the positive x cap for electrons and in the negative x cap for the ions. So, then we have seen that for a geometry like this the field the particles will move to the either sides of the earth constituting a current which is called as the ring current.

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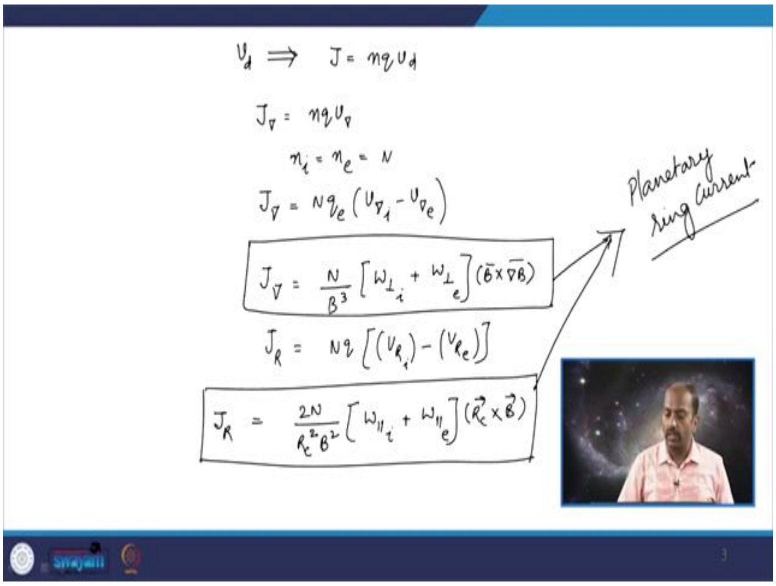

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Planetary ring current is

$$J_{\nabla} = \frac{N}{B^3}(W_{\perp_i} + W_{\perp_e})(\boldsymbol{B} \times \boldsymbol{\nabla B})$$

$$J_R = \frac{2N}{R_c^2 B^3}(W_{\parallel_i} + W_{\parallel_e})(\boldsymbol{R_c} \times \boldsymbol{\nabla B})$$

Where, W is the kinetic energy.

Now, we very well know with any drift velocity v d the current density J can be written as nq v d. Now, the current density due to the gradient drift velocity can be written as J del is equals to let us say nq v del. Let us say n i the number of ions per unit volume is equals to the number of electrons per unit volume which is equals to capital N let us say, we write J del is equals to N qe into v del i minus v del e. And similarly we can simplify it even further by substituting the expressions for the gradient drift velocity we will write n by bq W perpendicular for ions plus W perpendicular for electrons times B cross del B.

So, similarly we can also write the velocity the current density due to the curvature term as n q v R i minus v R e which is equals to 2 N by R C square B square w parallel ion plus w parallel electron times R C cross B. What is this is J R this is the current density due to the curvature drift and this is the current density due to the gradient drift.

So, these two are responsible for what is called as the planetary ring. The planetary ring current becomes a very important phenomena when you want to study the geomagnetic storms or space weather fluctuations in the earth's atmosphere. The planetary ring current is typically of the order of several million amperes and whose strength keeps changing with the amount of plasma that sun throws at the earth.

So, we will have more detailed discussion about the planetary ring current, when we discuss topics about geomagnetic storms or things like that. In this class we have been able to combine the drift velocity due to gradient of the magnetic field and the curvature of the magnetic field. We have got rid of instead of two having two curl terms, we have one curl term and both the velocity components parallel and perpendicular component coming into the picture.

So, this drift velocity is mainly responsible for the creation of what is called as the planetary ring current. So, in the next class, we will discuss something called as the magnetic mirroring and how it enables particles to be trapped within the magnetic field.