Introduction to Atmospheric and Space Science
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Lecture – 55
Gradient Magnetic Field

Hello dear students. So, we will continue our discussion on various aspects of plasma physics. So, far we have seen if we consider plasma as a single particle how the plasma will be affected by various different types of magnetic and electric fields. So, the idea was to keep it simple, let us say we considered electric field which is homogeneous and static in nature; that means, the field is not changing with respect to time and the field is also not changing with respect to space.

So, we realized that the particle will be accelerated along the direction of the electric field. When we changed from the electric field to the magnetic field, the magnetic field was taken to be homogeneous that is uniform with respect to space and static. So, if magnetic field is not changing with respect to time, then we realized that the particle will execute a simple harmonic motion or a circular motion to be precise and this particle will keep on moving in a circular motion. If there is no additional component of velocity which is parallel to the magnetic field, the particle will execute a circular motion; if there is a component of velocity which is parallel to the magnetic field the particle will move in a helical path.

Now the most important thing that we should remember from this discussion is that the particles velocity which is parallel will not be influenced by the magnetic field. So, this was a uniform electric field and a uniform magnetic field. Now we combined these 2 uniform magnetic and electric fields and we realize that if this is the case, then a charged particle with a mass and a charge when experiences perpendicular E and B fields, it will experience the guiding center drift which we called as the E cross B drift. We derived relation for the E cross B drift as v E is equals to E cross B by B square.

So, this is the drift velocity that the guiding center experiences due to the electric and magnetic fields perpendicular in nature. So, this was the story about homogeneous magnetic fields and electric fields. Now what will happen if there is an in homogeneity? in homogeneity can be in terms of, let us say space or in time. A simple example could be if the field is changing as the particle is moving, if the magnitude of the field is changing as the particle is moving or if the magnitude of field is changing in time as the particle is moving.

So, it means at each and every point as the particle travels, it experiences different magnitudes of fields. So, this is the inhomogeneous magnetic field. So, for a couple of lectures from now, we will see how the plasma will behave in the presence of non homogeneous or inhomogeneous electric and magnetic fields.

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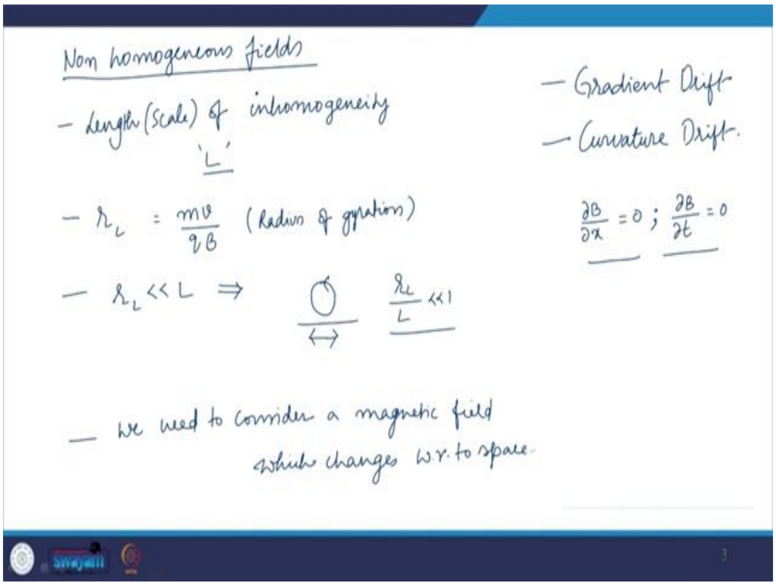

So, our discussion is about non homogeneous fields. So, for example, in today's class, we will try to understand what is called as the gradient drift and the curvature drift. So, the basic idea is as long as the magnetic field or the electric field is homogeneous; that means, it is not changing with respect to space. So, let us say dou B by dou x or dou y or dou y, dou z is 0 or the electric field for that matter and if you keep the magnetic field strength constant with respect to time again. In these cases, the behavior of plasma or the mathematical approach to understand the plasma behavior was simple enough. But the moment you introduce in homogeneity into the picture things will become complicated; that means, the motion of the particle will not be as simple as circular motion rather it will be much more complicated.

So, what we have to do is, we have to understand how we can incorporate the in homogeneity into the plasma motion. Now so, in order to approximate things, we start with introducing what is called as the length or or the scale of in homogeneity. So, what does it mean the scale of in homogeneity is the distance after which the magnetic field becomes in homogeneous.

So, let us say we define the scale of inhomogeneity as let us say capital L, what it means? Physically is that up to the length scale of capital L, the magnetic field can be assumed to be homogeneous and once you cross this limit capital L the magnetic field will become inhomogeneous. Now how do we put it in terms of particles properties, let us how does the scale of in homogeneity matter when you compare some property of the particle?

So, what we can do is, we can define r L which is the radius of gyration which is equal to m v by q B, we have kind of derived this equation the formula for radius of gyration. So, what is this? So, this is the radius of gyration. So, radius of gyration matters only when you have a magnetic field in place. So, if r L is the radius of gyration, for us to understand or for us to make a simple mathematical treatment.

What we can do is, we can say that the radius of gyration will be much less than capital L, what does it mean? It means that the scale over which magnetic field becomes in homogeneous is much larger than the radius of gyration of a particle, it can be an electron or it can be an ion. So,we also know that the particles the ions being massive particle has a larger radius of gyration.

So, it is very convenient for us to say that radius of gyration should be very very small in comparison to the scale of in homogeneity. So, I mean why are we taking the scale of in homogeneity in such a proportion is that. So, let us say if you have a single particle which is gyrating in a magnetic field, in let us say in one particular direction, then we do not want the magnetic field to become inhomogeneous within this distance, within the distance of radius of gyration.

We want at least the magnetic field to be homogeneous at least in the limits of the radius of gyration; that means; as the particle is revolving the particle should not experience different magnitudes of magnetic field at different points of its trajectory. So, things I mean it is not a possibility that is ruled out, but rather we want to make things simple. So, we say that the radius of gyration is at least of the order is that within one radius of gyration itself within one, orbit itself the magnetic field is not changing so much. That is just something we want to keep for the mathematical approach to be simple.

So, what it means is that; that means somewhere r L by L will be much less than 1. Now if what will happen? Let us consider a magnetic field which changes with respect to space coordinates. Magnetic field can also change with respect to the time coordinates but so, our idea is to understand ionosphere as plasma.

So, we are equipping ourselves with enough understanding of plasma physics. So, that we can understand the type of motion that is involved or that is that happens or that is manifested in nano sphere. So, now let us consider a particular type of magnetic field.

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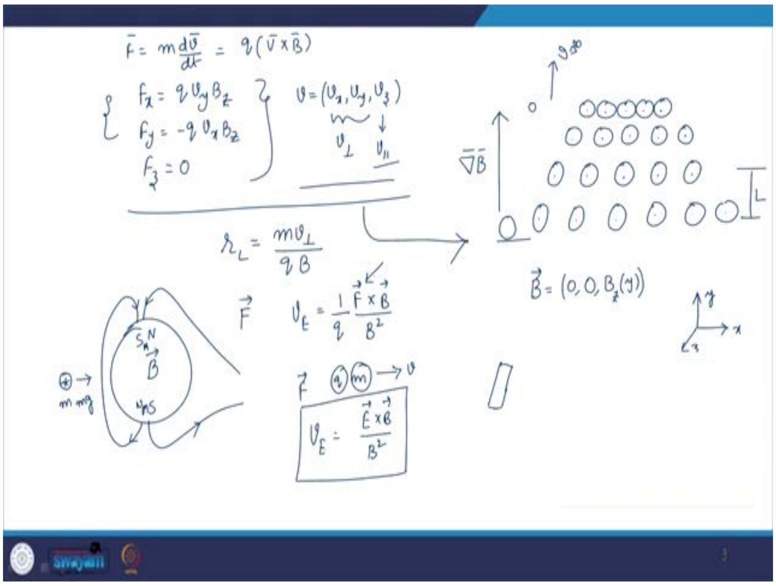

The drift velocity of the guiding centre is

$$v_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2}$$

So, let us say the coordinate axis are in this direction. So, this is your z axis that is coming out of the page and this is your x axis and this is your y axis. So, the magnetic field configuration that I will consider is such that it is pointing out of the page or towards you. So, what happens these are the magnetic field lines that I am drawing. So, magnetic field lines are out of the page. So, the direction of magnetic field is in the positive z direction.

So, what is being drawn here is as you see the magnetic field strength is increasing in this particular direction. So, what you can say? So, the gradient of the magnetic field is pointed in the y direction. So, what is the gradient? Gradient is the maximum rate of increase or change.

So, generally it points towards the increasing direction. So, the magnetic field gradient is in this direction del B.

So, this is the gradient of these so, if you want to write magnetic fields coordinates, you can write magnetic field is 0 along x direction, there is no magnetic field, along y direction there is no magnetic field, but along only along z direction the magnetic field is existing. So, it is B z whose strength is changing across the y direction. We have taken simplest types of fields, let us say for example, homogeneous static fields, but in nature we do not generally come across homogeneous fields.

So, let us say for example, if you consider a simple bar magnet, the magnetic field due to this bar magnet will change at each and every point. So, we cannot expect a homogeneous magnetic field to be present around this bar magnet. So, it may I mean it may be possible for you to create homogeneous or uniform magnetic field within some space, but generally the nature of magnetic field is such that it is not homogeneous or it changes always with respect to the space, because somewhere the magnetic field has to be stronger and somewhere the magnetic field has to be weaker. So, it is always like that.

So, now, we are trying to implement a natural or feasible type of magnetic field which is inhomogeneous in nature. So, now, the inhomogeneity of course, is let us say if this is the scale of inhomogeneity is this let us say capital L.

So, what it means is that? So, as long as you are within this distance capital L the magnetic field is homogeneous. But if youcross the length scale capital L, the magnetic field changes. So, what we have seen in the case of a homogeneous magnetic field, what we have seen is that the particle will experience a force which is basically F is equals to m d v by d t is written as q times V cross B.

Now, with this type of magnetic field if you resolve this cross product what you can see is that F x will be equals to q times v y B z, F y is equals to minus q times v x B z and F z will be 0. So, this has to be in combination with our understanding that v has a component of velocity v x, v y and v z. The v x and v y components can be called as the perpendicular components and v z component is the parallel component.

Now, since v z is parallel to the magnetic field itself. So, the velocity across the z direction will not be altered by the magnetic field. So, there will not be any acceleration that you can see in the direction parallel to the magnetic field so; that means, the magnetic field will not do any work l in the parallel direction. So, what I am trying to say is this is the velocity of the particle to begin with if this particle makes an entry into the magnetic field, it will experience a force like this.

Now, what we did was; these are 2 differential equations which are kind of coupled. So, we solved these differential equations and we assumed solutions for the velocity. So, this is m dv x by dt; it is F is m dv x by dt. So, then we obtained a solution which was implicated that the particle is executing circular motion in the magnetic field. Now if you can use this approximation or you can use this solution as it is for this picture, how do you use it? I mean; what you can say is that you have defined the particle to be executing a simple harmonic motion or circular motion with a radius of gyration which is mv perpendicular by qB.

So, if you make plasma to enter into this magnetic field. What you can realize is that the particles at this lower end at the bottom end will have the magnetic field strength beam being, lower here will have a radius of gyration which is larger and as the particle moves or if the particle is here the magnetic field strength is larger here.

So, the radius of gyration will become smaller; that means, just it is kind of consistent with our idea of the drift of the guiding centers which is like we have seen that when the particle is experiencing an E cross B field, we have seen that as the particle travels or tries to gyrate across then due to the effect of the electric field on. And in combination with the initial velocity of the particle the radius of gyration in the first half cycle will not be same as the radius of gyration in the second half cycle.

So, because of the change in the radius of gyration, we say that the guiding centers are drifting in a particular direction with a particular velocity and this guiding center of drift velocity is irrespective of the charge and the mass of the particle.

So, similarly here also what you are able to see now itself is that the radius of gyration as the particle moves from here to here changes by some magnitude; that means, the particles radius of gyration starts becoming smaller and smaller. So, what does it mean? Due to this radius of gyration changing, we can experience some drift velocity in this direction drift of the guiding center, let us say we call this as v del B.

Now, why is this drift velocity coming into the picture itself has it been a uniform magnetic field the radius of gyration would have been same all the while. So,there is no idea of the drift velocity. But, since there is a drift velocity that is developing due to the radius of gyration being larger at a point and smaller at a point and this radius of gyration changing is mainly due to the fact that the magnetic field is having a gradient. So, we call this drift velocity as the gradient drift velocity.

So, the particle experiences in this situation the particle experiences what is called as the gradient drift. Now so, let us try to derive an expression for the gradient drift velocity,. Now what you have to do is we have to find out we already know, if we have a generalized force F then the drift velocity associated with this is we have derived this in our earlier class as v E is equals to 1 by q F cross B by B square.

Now, what is the role of this F? This generalized force is able to gives a particle with a charge and a mass m some initial velocity. So, this is a generalized expression, generally the relation that we have derived is v E is equals to E cross B by B square. So, this is the relation that we have derived and then we made a small modification in this expression and then we said that the generalized expression. So, you can substitute the force to be an electric force f is equals to E q you can as well substitute F is equals to mg.

Or you can as well substitute F is equals to mv square by r anything. But the point is when the particle is experiencing a generalized force in combination with the magnetic field the drift velocity will be given by this. So, what is the simplest example? The simplest example is the earth itself. Let us say earth has a magnetic field due to the revolving solid molten core of iron that is at the center of the earth due to which a magnetic field is present along with the earth.

So, the magnetic field of the earth is something like this. So, the magnetic field lines let us say magnetic field lines diverge away from the geographic South Pole and converge at the geographic North Pole. So, this is the geographic north and this is the geographic south and the geographic north is actually coinciding with the magnetic south and the geographic south is coinciding with the magnetic north.

So, let us say it is N m and S m. Now what you see here is that as let us see if you put a particle at rest, if you are able to do it. If you put a particle at rest the particle is able to see the magnetic field, but the particle does not have its initial velocity, but the particle certainly has some mass. so, it will experience a force mg towards the earth and this force gives it an initial velocity, then the magnetic field will take over and then as the particle travels closer and closer to the earth the magnetic field strength becomes strong.

So, there is a magnetic field gradient which is going towards the earth and the particle is experiencing some force due to the gravity as well . So, this is a realistic example where the plasma or the solar wind that is coming towards the earth will experience some kind of drift due to the gradient in the magnetic field. This is one example.

So, now what we have to do is. So, if the particle is experiencing a magnetic field gradient we have to calculate what will be the average force that will be experienced by the particle as its radius of gyration decreases along the gradient. So, if you know the force; let us say if you calculate the force, you just have to substitute that force into this generalized drift velocity expression; then you will be able to get the, original expression for the drift velocity so, for us to do that we have to make some approximations.

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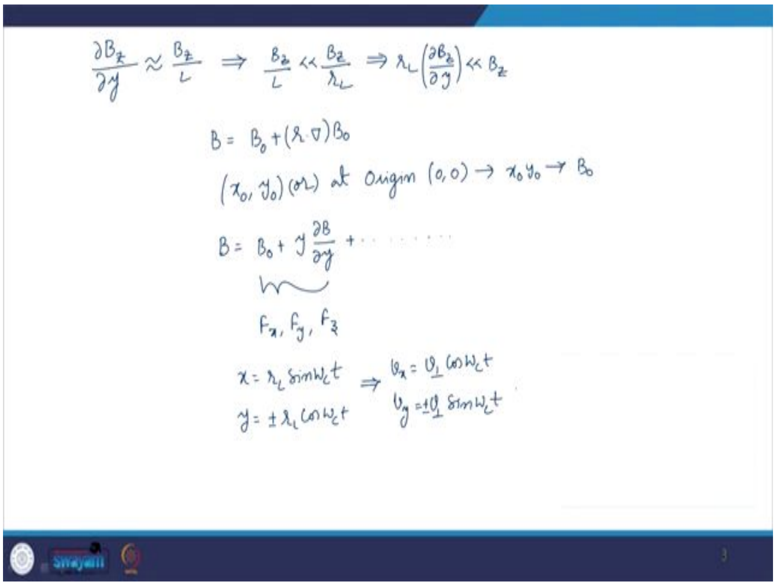

So, since the gradient is present only in the y direction, the magnetic field is along the z direction. So, this dou B z and since the gradient is present only in the y direction. The magnetic fields derivative with respect to other space coordinates does not mean anything. So, this can be approximated to let us say B z over L. Since L is the space dimension within which the magnetic field is actually not changing.

So, this can be equated saying that B z by L must be much less than B z by r L which implies r L times dou B z by dou y should be much much less than B z itself. Now what we do is; within the limit of this scale distance over which field is homogeneous we make a Taylor expansion of the magnetic field. So, how do we do it? B 0 is the magnetic field strength at the origin or let us say at the guiding centers coordinate x 0, y 0 or at origin.

So, where the particle is initially present at the origin, the path the position is x 0, y 0 and if the magnetic field strength is B 0. Now if the magnetic field strength at the guiding center is B 0, the magnetic field strength at a distance r can be approximated by Taylor series expansion which is is B 0 plus r dot del times B naught.

Now, what are we trying to do? We are trying to see B 0 is the magnetic field strength at the origin or at the center of the radius of gyration or at the center of orbit of gyration right. So that means, so, we have to find out what will be the magnetic field now the magnetic field is not uniform.

So, the magnetic field has to be calculated at any distance r away from this center. Do you know the magnetic field strength at the center? Yes, we know the magnetic field at the center to be equal to B naught. Now if you know the B naught then at any distance r knowing how much B changes over distance r we can find out what is the magnetic field at any point. So, this point is at r. So, again so, this is a very familiar form of the Taylor series so, what we can do is, we can expand it saying that B 0 plus y times dou B by dou y plus the higher order terms.

So, mathematically we do not deal with the higher order terms we say that the higher order terms are too small to be considered in calculations so, we ignore them. So, we will take only the first two terms in our calculation. So, what are we trying to do is; we are again trying to find out or calculate the expressions for F x, Fy and F z.

Now just to recollect so, we have in the case of a uniform static magnetic field, we have seen that if the particle experiences a magnetic field which is uniform in space ,we have seen that x is equals to r L sin omega c t and y is equals to plus minus r L cos omega c t which gives you kind of v x is equals to v perpendicular cos omega ct and v y is equals to plus minus v perpendicular sin omega ct. Now this plus minus indicate the direction in which the particle will gyrate, some to the left and some to the right.

So, this is we have we know this already. So, if you want to substitute this into the expressions for F x and F y.

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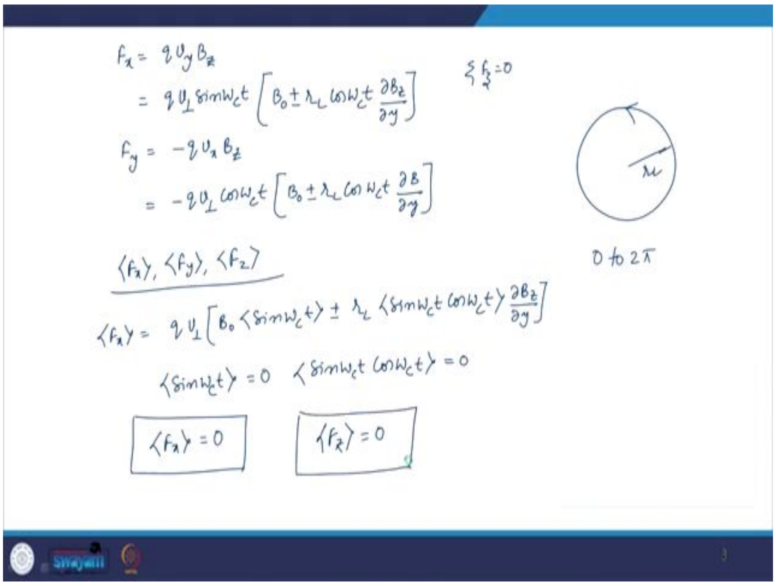

We know that F x is q v y B z. Now if you substitute q v perpendicular sin omega c t times B z at B z at a distance r is Taylor expansion of B z plus minus r L cos omega ct into dou B z by dou y. Similarly, we can write F y as minus qv x B z which is equals to minus q v perpendicular cos omega ct times B naught plus minus r L cos omega c t dou B by dou y. And what is F z? F z is 0 and F z is anyway 0.

Now, what is different here in comparison to what we have done earlier. The difference is that the particle is experiencing this force earlier the particle was experiencing this force just because of the magnetic field B z, there was no substitution for B z. Now we are substituting something for bz; that means, what is the force that is experienced by the particle as by the virtue of the magnetic field.

Now, since the magnetic field is not constant, but it is changing with respect to space this variation, with respect to space so, the magnetic field is constant at the guiding center you know the magnetic field at the guiding center you know, you want to find out what will be the magnetic field at a distance r naught at a distance of r L, but at a distance r. So, then if this magnetic field is changing so, you have this original term this original term is q v perpendicular B naught sin omega ct is still there. In addition to that you also have a second term which has come into picture only because of the gradient in the magnetic field.

Now because of this so, F x, F y and F z are the magnitudes of forces which are experienced by the particle. Now what you want to do is at this point? We want to find out what is the average force the particle experiences over 1 gyro orbit; that means, 0 to 2 pi.

So, between 0 to 2 pi in within 1 gyro orbit, let us say within 1 gyro orbit, what is the average force? What is the average value of F x? What is the average value of F y? And what is the average value of F z? So, the point is you want to find out what are the average forces the particle will experience within 1 gyro orbit, then we can probably use it to calculate our drift velocity. So, the average of F z is anyway 0, average of F x, if you want to calculate will be q v perpendicular times B naught average of sin omega ct plus minus r L sin omega ct cos omega ct into dou B z by dou y.

We know that average of sin omega ct in a full cycle is 0 and average of sin omega ct times cos omega ct which is half sin 2 omega ct is again 0 so; that means, that the average force in the x direction is 0. So, the particle is not experiencing any force due to the change or due to the gradient in the magnetic field in the x direction which is natural, which is kind of expected already. And we also know that F z is also 0 the average F z is also 0.

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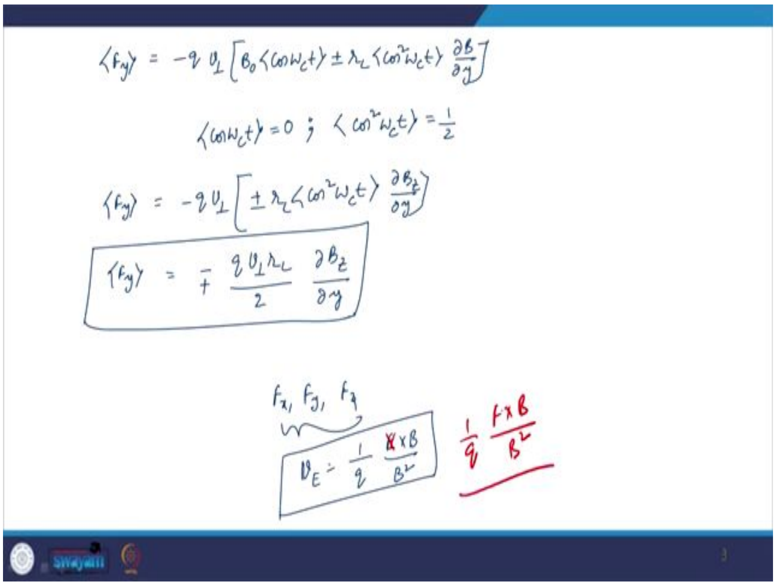

Now, let us find out if there will be an average force across the y direction, across the y direction, it will be minus q v perpendicular B naught cos omega ct plus minus r L cos square omega ct dou B dou y. Now the average of cos omega c t is 0 ofcourse, over the full circle and the average of cos square omega c t is half.

So, the average force F y will now become minus q v perpendicular plus minus r L cos square omega c t dou B z by dou y which is equals to minus plus q v perpendicular r L divided by 2 dou Bz by dou y. So, the average force along x direction is calculated to be 0, the average force along the y direction is calculated is to be this and the average force along the z direction is also calculated to be 0.

Now so, we have calculated F x, F y and F z. So, we can use these 3 in the expression for the generalized drift velocity as 1 by q F cross B by B square. So, we will stop here. Then in the next class, we will try to see how we can substitute this into the drift velocity formula and how we can get the gradient drift velocity and what are the consequences of such a drift velocity.

So, from here what we can see is that if the particle is experiencing a magnetic field which is changing with respect to space, we have been able to calculate 3 different components of the forces, the force the mainly the origin is again the Lorentzian force. We have been able to calculate F x, F y and F z some of them are 0 already, but so, now what we are trying to achieve is that we know the force. Once we know the force we have a generalized expression for the drift velocity in terms of the force and the magnetic field. We can use this expression to calculate the generalized drift velocity in the presence of a.

So, this should be 1 by q F cross B by B square where F is the generalized force. Now let us see how we can utilize these components, the average components of the force that we have derived and then substitute into this equation and let us see how it will give us the drift velocity.