Introduction to Atmospheric and Space Science
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Lecture – 53
Particle motion in a Uniform Magnetic Field and Guiding Center

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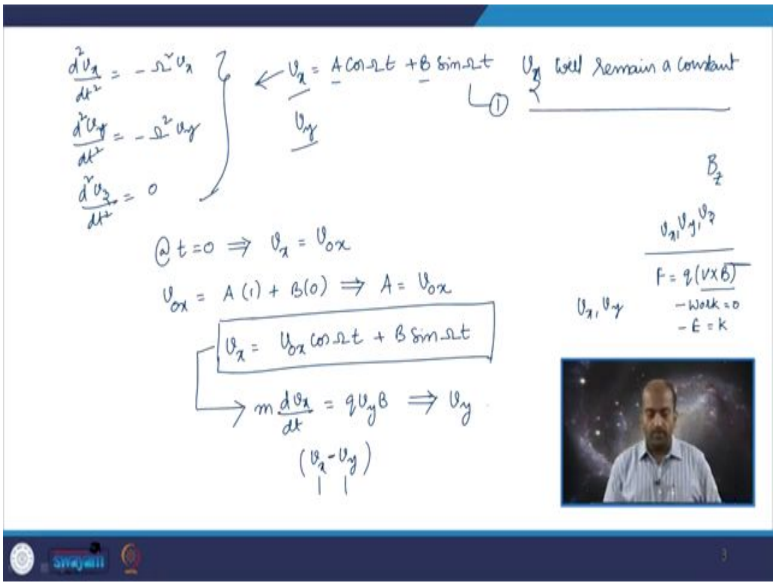

So, in continuation to the last class we have seen that if the particle is experiencing a magnetic field which is directed along z axis, the particles velocity is written as v x, v y, v z starting from the Lorentz force expression which is q times v cross B, we have realized that the work done is 0 and energy is constant.

So, when you break this cross product and, if you couple the equations in terms of v x and v y, we have realized three equations which are d square v x by dt square is equals to minus omega square v x d square v y by dt square as minus omega square v y and d square v z by d t square as 0.

We have also seen that these second order differential equations can take a simple harmonic solution such as, v x as A cos omega t and plus B sin omega t. We have realized that v x if you take a double derivative of v x and if you substitute this into these equations we will get the same, we will realize that this equation will satisfy the second order differential equation.

Now, we have to find out the values of these two constants A and B. And right now we do not want to assume, what will be the form of v y lets say instead of v x, we should also have the form v y. Now the objective is we want to see how will v x and v y look like, in the absence of these two constants, what can they describe? What can they describe about the particles trajectory, how do how will the particles velocity will look like?

Now, at this point itself we can say that from the second order differential equation v zwill remain a constant because the particle is traveling parallel to the magnetic field. So, it will not be influenced by the magnetic field. Now, if you want to calculate the values of these two constants, we need to use some conditions some let us say the boundary condition. So, at time t is equals to 0 we can say that velocity v x will be v 0 x.

If you use this condition in this expression, let us say number 1, we will say that v 0 x is equals to A into 1 plus B into 0; that means, the value of A is equals to v 0 x. So, now itself we can write v x is equals to v 0 x cos omega t plus B sin omega t. So, now we need to find out the value of the constant B.

Now, from the original expressions A B C, we can write that m d v x by d t is equals to q v y b. This expression means that the solution v x and v y are related. So, if you write a solution for v x you should very well be able to write v y if you know this expression. Now, what we can do is we can use this expression in combination with this expression to get the value of v y.

How do we do that?

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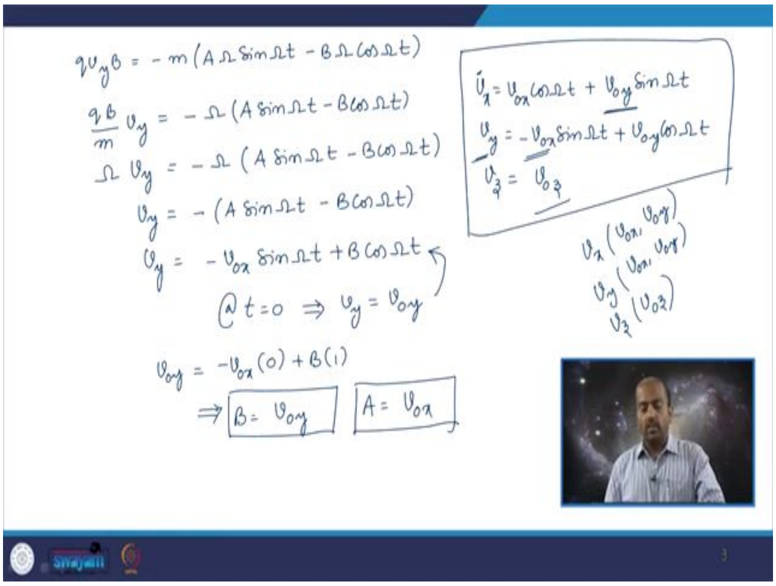

Now, we can differentiate v x, so we can write q v y B is equals to minus m times A omega sin omega t minus B omega cos omega t or we can write q B by m times v y. So, we have started from v x by the way is equal to minus omega A sin omega t minus B cos omega t. So, in that case we can write v y, q B by m is omega is equals to minus omega A sin omega t minus B cos omega t.

So, we write v y is equals to minus A sin omega t minus B cos omega t. So, having known the values of constants already we can write v y is equals to A is already v 0 x sin omega t plus B cos omega t. At this point we can write the condition as at t is equals to 0 v y should be v 0 y. Using this condition into this expression we can write v 0 y is equals to v 0 x into 0 plus B into 1, that implies B is equals to v 0 y.

So, we also got the value of the second constant. So the constant A is v 0 x and the constant B is v 0 y. Now the values of these two constants we can write v x is equals to v 0 x cos omega t plus v 0 y sin omega t and v y is equals to minus v 0 x sin omega t plus v 0 y cos omega t and v z is equals to v 0 z.

That means v z or the velocity component along the z direction or along the magnetic fields direction is uninfluenced. And second most important thing is the velocity along x direction has an initial component of velocity along the y direction and the velocity at any given time along the y direction has an initial component of velocity along the x direction. So, v x has v

0 x and v 0 y , v y at any given point of time has again v 0 x and v 0 y and velocity v z is just a constant along z axis.

Now, what we have to understand what are these equations telling us.

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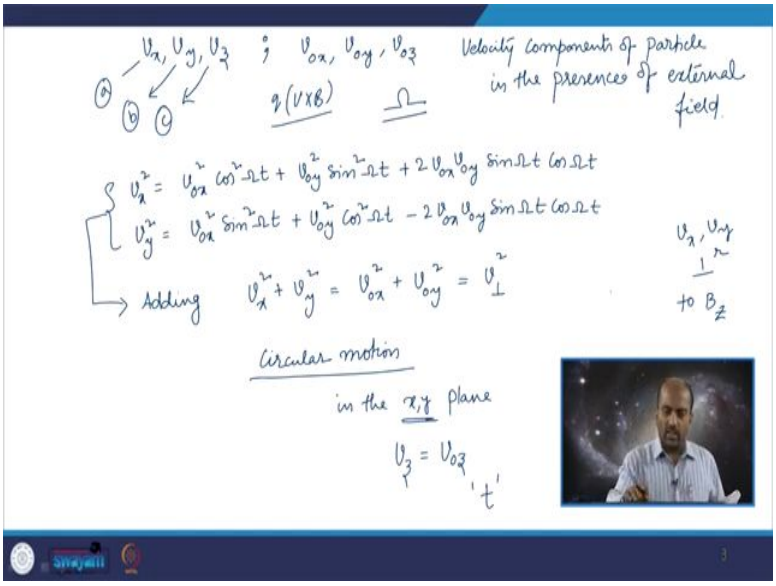

So, to understand let us say what do we have? We initially had v x ,v y and v z, then in combination with this we had the initial velocities as v 0 x, v 0 y and v 0 z,. Now what are these? These are the velocity components of particle in the presence of external field.

So you know these three components are out of this cross product, v cross B that is very important. How is the magnetic field coming into picture? It is coming into picture by the omega, which is q B by m. Now let us try to understand how this three velocity components can be combined for a physical interpretation of the result.

Let us square and add v x, v y and v z, v x square can be written as v 0 x square so this is cos square omega t plus v 0 y square sin square omega t plus 2 v 0 x v 0 y sin omega t cos omega t.

Similarly, v y square is written as v 0 x square sin square omega t plus v 0 y square cos square omega t minus 2 v 0 x v 0 y sin omega t cos omega t.

If you add these 2 let us call the expression for v x as equation A, expression for v y as equation B, expression for v z as equation C. If you square and add these expressions adding these two we will realize v x square plus v y square is equals to v 0 x square plus v 0 y square which we are going to call as v perpendicular square, because v x and v y are perpendicular to B z.

Now, from here what can you say about the trajectory of the velocity of the particle; it is circular, the particle is executing circular motion, because this expression looks like an equation of circle. So, v x and v y are combining to give circular motion along or in the x y plane, circular motion in the x y plane. What happened to the z axis? What is v z doing now, v z .

So, what is v z doing now? v z is a constant which has remained at the same value over the period of time. So, with time v z is a constant. Now what is v x and v y doing? v x and v y are executing circular motion in the x y plane and v z is a constant. Now let us consider how will the particle move. So, the particles trajectory can be more easily established if you find out the values of position with respect to time from starting from the velocities.

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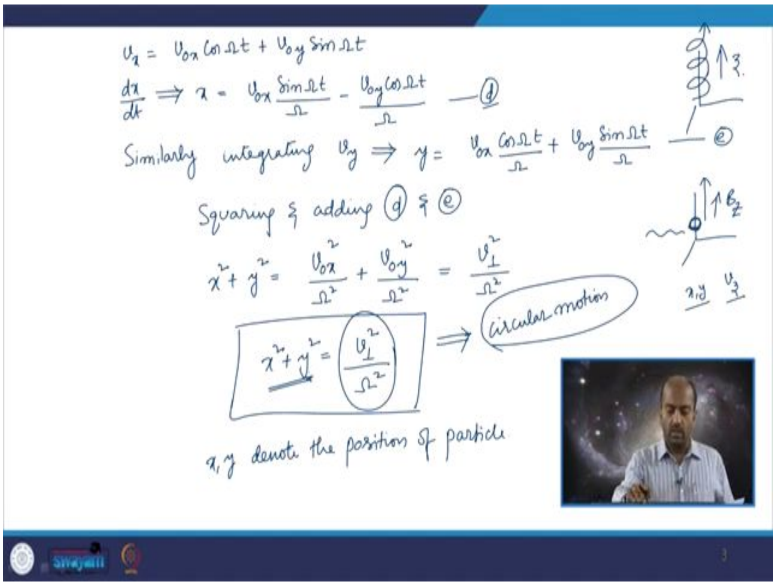

Let us say we write v x as v 0 x cos omega t plus v 0 y sin omega t. So, which is d x by d t so if you integrate this expression to get x is equals to v 0 x sin omega t divided by omega minus v 0 y cos omega t divided by omega, let us call this expression as d. Similarly, we can integrate similarly integrating v y we will get y is equals to v 0 x cos omega t by omega plus v 0 y sin omega t by omega.

Let us call this expression as equation number e. Now squaring and adding equation d and equation e, what we will get is x square plus y square is equals to v 0 x square by omega square plus v 0 y square by omega square which you are going to call as v perpendicular square divided by omega square.

Now, what do we have? We, we have x square plus y square as v perpendicular square divided by omega square. Now, what is x y? x y denote the position of particle, they are suggesting that the position of the particle is varying with respect to time as if it is executing a circular motion.

So, this equation represents a circular motion now. So, the particle has experienced the magnetic field, magnetic field was in this direction B z. Now the particle is experiencing circular motion like this,what is the circular is in the x y plane.

So, the velocity v z, what is velocity v z doing v z is constant; that means, the particle is having a non 0 component of velocity along this direction at the same time the x and y components of the velocity are executing circular motion. So, this gives you what we call as the helical motion of the particle. So, particle is executing circular motion and because of the non 0 component of the velocity along z direction particle is pushed in the positive z axis.

So, this kind of movement is generally called as the helical path or helical motion. Now we need to find if this is the circular motion I mean the on the right hand side this is of course, the radius of the circle and this is the plane in which the circling motion is executed. Now let us find the radius of the circle, of this circular motion.

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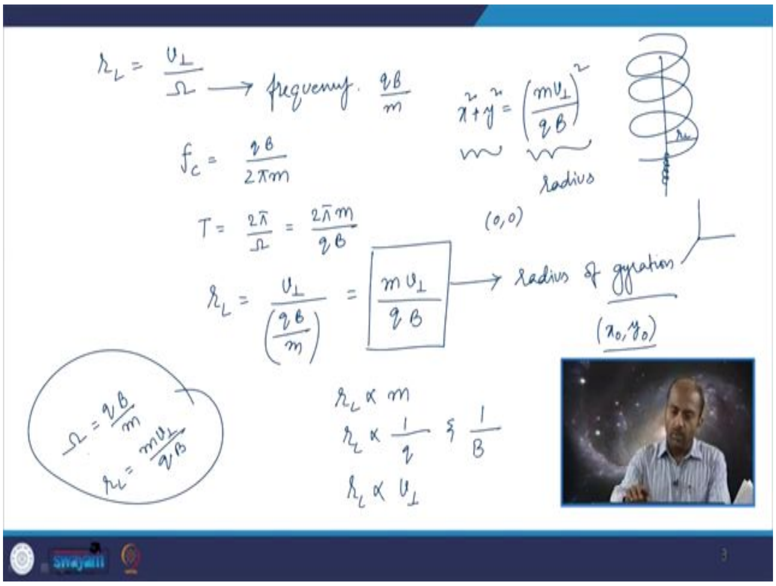

The radius of gyration (Larmor precession) is

$$r_L = \frac{mv_\perp}{qB}$$

Now, we can write the radius r L as v perpendicular by omega. So, v perpendicular is the velocity in the perpendicular direction omega is the frequency q B by m. What is omega? Omega is the frequency, angular frequency in terms of which is q B by m., if we recollect the relations. So, omega is the angular frequency which is written as q B by m so we can write the frequency as, q B by 2 pi m, and the time period of this execution is 2 pi by omega which is 2 pi m by q B.

Now the radius of this circular motion is written as m or, radius as v perpendicular by q B by m which is m v perpendicular divided by q B. So, this is the radius of circular motion, this is called as the radius of gyration, the particle is gyrating in this way and this is called as the radius, this is the radius of gyration.

Now, what is specific about this radius of gyration? Radius of gyration is proportional to the mass of the particle and is inversely proportional to the charge and to the amount of magnetic field that you apply, and radius of gyration is proportional to the velocity the perpendicular velocity of the particle; that means, more the mass let us say so; that means, that the ions will have larger gyration radiate, and the electrons will have a smaller radius of gyration.

So, electrons circle will look like this and ions radius of gyration will look like this. And now where are we? So, now we have so in terms of this. So, we can write x square plus y square is equals to m v perpendicular by q B square. So, this is the circular motion and this is the radius. Now from the circular motion is executed at the centre.

So, the particle is entering into the magnetic field at the origin. So, that is the idea otherwise the circular motion is executed around the centre which is x 0, y 0.

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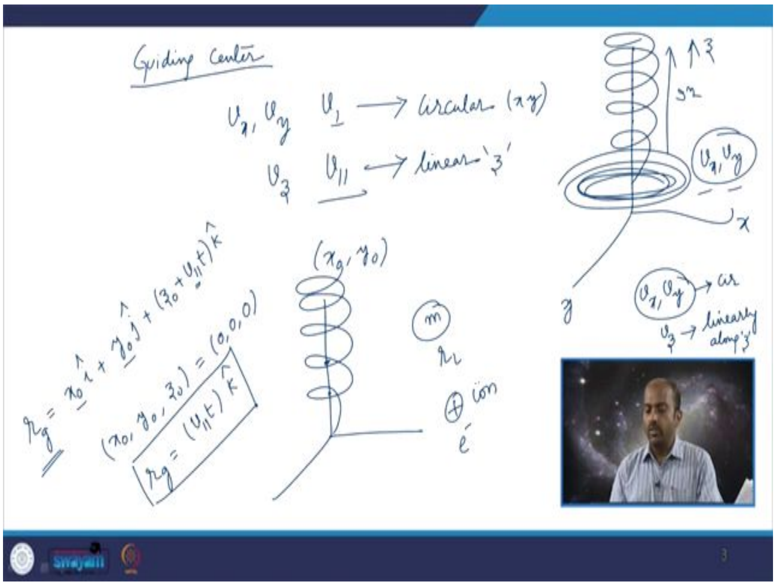

This point around which the circular motion is executed is generally called as the guiding center, guiding center is a imaginary point around which the circular motion is executed by the particle. So, the particle is executing a circular motion like this so the imaginary point around which circular motion is executed is called as the guiding center like this.

Now, if you observe carefully the x component of velocity and the y component of velocity combinedly are giving you the circular motion and the z component of velocity is pushing the particle linearly along the z direction. So, this movement that you see the particle to be going in this way is due to the z component of velocity, and this movement in this x y plane. So, this is the z direction. So, this movement in this x y plane this movement is due to v x and v y. So, this is the roll.

So, magnetic field is influencing only along v x and v y that is so that is the reason that we called v x and v y as v perpendicular components and v z as v parallel component. So, as a conclusion what you can say is that the perpendicular component will impart circular motion and the parallel component will impart linear motion along the z axis along the direction of magnetic field, circular motion in the x y plane.

Now, so what we can say is that. So, v x and v y are generally referred to as re perpendicular as they are perpendicular to the applied magnetic field, they are the components of velocity which are influenced by the magnetic field and they are executing the circular motion with a radius of r L which is called as the radius of gyration. The third component v z can be referred to as v parallel because it is parallel to the external field that is applied.

The guiding center around which the circular motion is executed is generally taken as x 0 y 0. The parallel component of the particle velocity allows it to move linearly with a velocity v 0 z a constant velocity along the z direction or the direction of the magnetic field.

So, at the same time the perpendicular component gets influenced and they move circularly around the guiding center. So, we can say that the particle moves in a helical path, so this movement that you see is a helical path in which the particle is progressing along the z direction by revolving around or revolving in the x y plane.

Now, we can write. So, what you see here is that what you see carefully, if you see carefully if you take the guiding center, if the particle is moving in this way the guiding center is moving in the. So, this is the guiding center around which the particle is moving this is the guiding center on which the particle is moving. We can write that the position of the guiding center let us say the position of the guiding center with respect to time because there is a linear velocity that is involved right. So, x 0 in the x direction y 0 in the z direction plus z 0 plus v parallel times t along the z direction.

Now you see x 0 , y 0 is the center of this radius So, they are constant and z 0 is the initial point at which the velocity, get starts to get influenced by the magnetic field. So, this is the parallel component of velocity which is parallel times t. So, with time this r g gives you the position of the guiding center with time. Now generally what if the particle enters the field at the origin we can say that x 0 ,y 0, z 0 are to be treated as 0 0 and 0. In that case the expression for r g simply becomes v parallel t along K cap.

So, hence we can say that the guiding center is moving along the z direction. Now the most importantly now comes the circling motion of the particle and the particles polarity. Now, here we have seen that the mass of the particle is coming into picture by to influence the r L. So, r L was defined as m v perpendicular by q B so omega is q B by m and r L is m v perpendicular by q B.

So, these two are very important relations, important in the sense that the mass of the particle is able to influence the frequency of this rotation and as well as the radius of this particular circular motion. So, here in addition to the mass where does the charge come into picture? Charge comes into picture in both the places more importantly charge. So, if you have a positive charge and if you have an electron or if you have an ion, the direction in which the circular motion is executed will be different.

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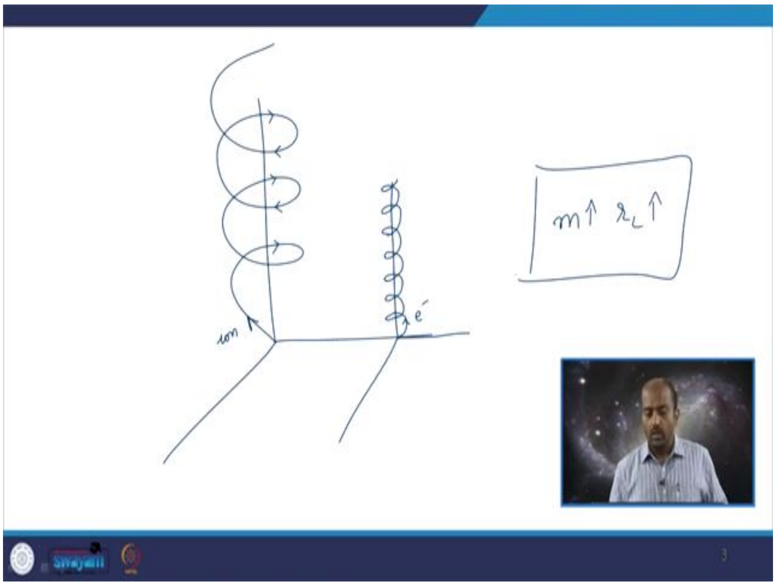

So, generally let us say what happens is, by applying the right hand rule you can realize for yourself that an ion will always drift or will always revolve to its left. So, this is an ion; that means, it will go like this. And an electron let us say if the origin is shifted here,. Now electron will revolve to its right. Now what I have done is I have kept the radius of gyration of electron small so that the effect of mass is reflected in the picture so this is an electron and this is an ion; that means, more the mass r L will be larger; so this is the idea.

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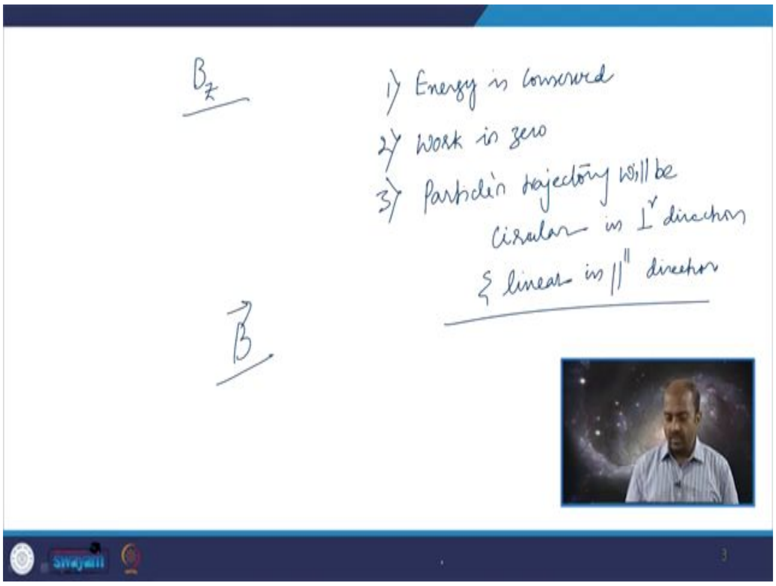

Now, what have we understood? We have understood that if the particle is experiencing a homogeneous static magnetic field then, number one; the energy is conserved, number two; no work is done, work is 0 ,no work is done by the field, number three; the particles trajectory will be circular in perpendicular direction and linear in parallel direction.

So, this is discussion about single particle motion in terms of, in the presence of a magnetic field ok.

Now, we will in the next lecture we will try to see how the particles motion will be in the presence of an electric and the magnetic field. Now we have first we have seen how will the particle move only in the presence of an electric field, then we have seen how will the particle move in the presence of only a magnetic field, now we will combine these two pictures and see how the particles motion will be influenced by both the electric and magnetic fields.

Thank you.