**Introduction to Atmospheric and Space Science**
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**Lecture – 52**
**Particle Motion in a Uniform Megnetic Field**

Hello, dear students so, in continuation with our earlier discussions of single particle description of plasma. We already seen that if the plasma is very weak or if the number of particles per unit volume in the plasma is very small then we have understood that the collective behavior of plasma becomes unimportant.

And then we can treat the plasma as a collection of single particles each of them behave on its own; that means, there are no interactions or the interaction between the particles between the individual particles is not very important. So, then what we do is if this plasma is experiencing an electric or magnetic field or a combination of these we can describe the plasma by saying or by analyzing the behavior of a single particle in the plasma.

And then we can extrapolate this behavior or these equations of motion for the entire plasma. So, this approach was called as single particle description of plasma where we treat plasma to be a single collection of single particles.

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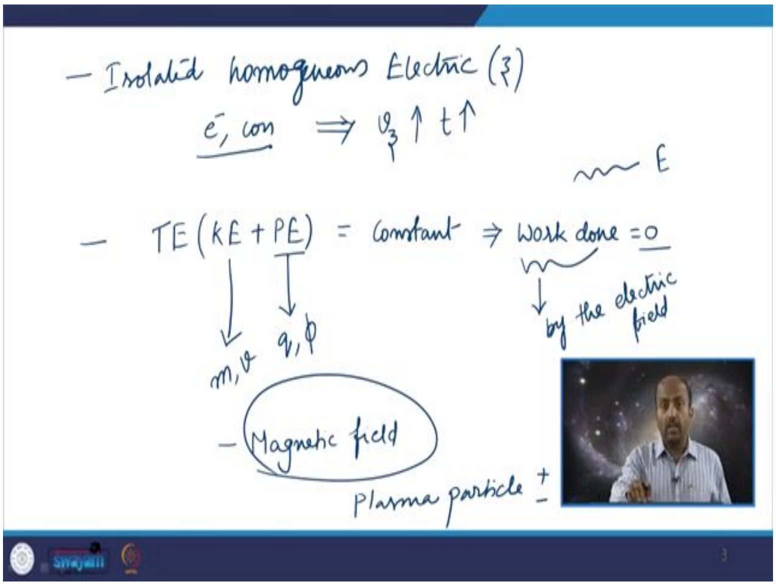

So, in our last discussion we have seen how the plasma or how single particle will behave in a isolated; that means, there is no magnetic field isolated homogeneous electric field. So, we have considered an electric field along z axis and we have consider the movement of let us say electron and ion in this fields. So, what we have realized is that we have realized that the velocity of the particle along the direction of electric field will vary linearly with respect to time this is what we realized.

And the velocity of the particle in other directions remains uninfluenced ,in addition to this we have also realized the sum of the total energy that is the sum of kinetic energy plus potential energy. The kinetic energy was a function of the mass of the particle and velocity of the particle the potential energy is basically a function of charge of the particle and the electric field strength or the potential that is offered by the electric field will remain a constant.

So, this is what we have learned I mean if a single particle is subjected to an electric field which is homogeneous in nature; that means, it is not changing with respect to space and static; that means, it is not changing with respect to time. Then in that case the total energy will remain a constant. a consequence of this is that the net work done will remain is 0. So, if there is no change in the energy the work done is 0; that means, what is doing this work? The work is done by the field by the electric field.

So, there is no amount of work that is done by the electric field. So, this is an important conclusion that we have derived in the last class. So, in continuation to this what we will do today is that we will try to understand how a magnetic field will influence a plasma particle. So, this plasma particles polarity could be positive as well as negative; that means, we can take an electron to be the particle which is experiencing a magnetic field we can as well take an ion to be experiencing the magnetic field.

Let us see how this mathematical approach will look like.

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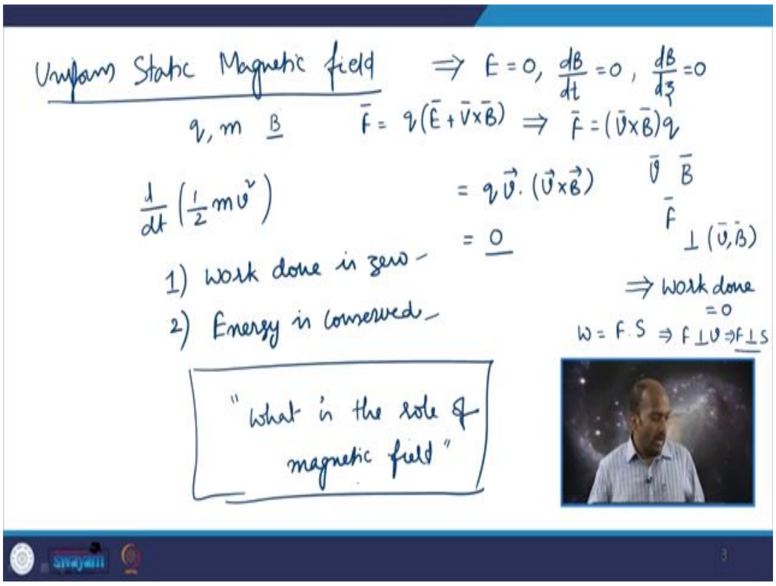


Now, let us consider this case two which is uniform static magnetic field. So, this is the case in a continuation so; that means that uniform static and isolated magnetic field. So, how do you write it? So, in this description the electric field is taken to be 0 the magnetic field is assumed to be constant with respect to time. So, there are no fluctuations.

So, it is not a time varying field it is a static field. So, it remains constant with respect to time it also remains constant with respect to position let us say; that means, it is homogeneous in space as well as in time. Now, let us consider a particle of charge q and mass m to be experiencing this particular field B. Now, from the generalized expression the force that is experienced is the Lorentz force is written as q times E plus V cross B. V is the velocity vector, E is the electric field vector and B is the magnetic field vector. So, this expression reduces to F is equals to v cross B times q; that means, the force will be in a direction.

So, if you consider v to be in one direction velocity to be in one direction and the magnetic field to be in one direction the force will be in a direction which is perpendicular to both velocity as well as the magnetic field to both this. Now, what is the consequence of this? The consequence of this is the work done what should be the work done in this picture work done is 0 why?

So, work done is. So, how do you define work done? work done as is the force times displacement no velocity; that means, if the force is perpendicular to V it is perpendicular to the displacement as well. Since, this force is perpendicular to the displacement the work done will be 0; that means, the effect of the magnetic field on the particle is such that it is having some effect I mean the magnetic field does influence the particle the particle does experience some amount of force, but the net work done in this picture by the field is 0, whatever the force that is imparted by the magnetic field is not doing any work as such.

Now, how can we put it let us say we take again we take the rate of change of kinetic energy of the particle mv square which is equals to qv dot v cross B. So, from the vector identities we can say that this is equals to 0. So, here what we can say is that number 1 work done is 0. So, we are saying this in the beginning itself in the electric field case we said this at the end of the discussion. So, work done is 0 and the second thing is energy is conserved.

So, energy is conserved in this picture and the work done the total amount of work done is 0. Now, what the scope of this discussion is to understand. So, if there is a magnetic field and if the magnetic field is not doing any work and it is not changing any energy of the particle then what is the role of magnetic field? So, this discussion will be an effort to understand what is the role of magnetic field how does it influence the particle.

So, this is the idea. we have to understand by the means of simple mathematics we will try to establish what is the role of magnetic field what does the magnetic field do in this kind of a picture.

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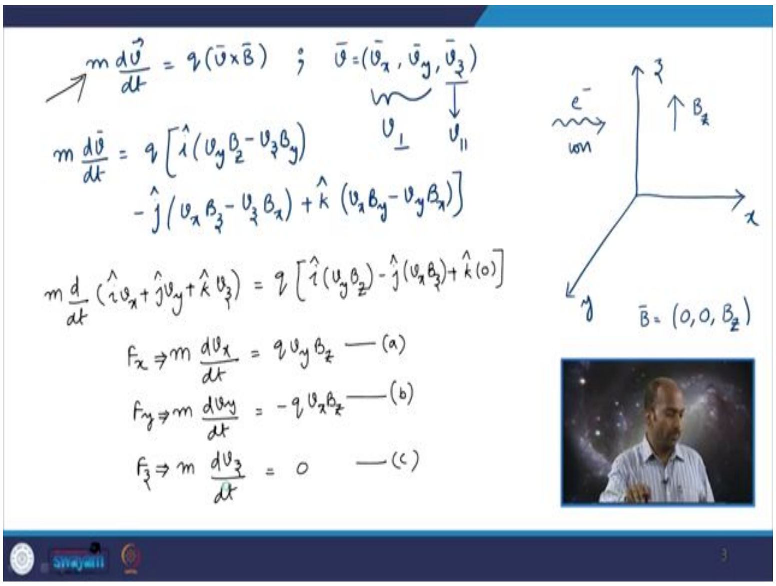

Now, for this example we will consider a magnetic field in this particular geometry. So, what we will do is. So, we will consider  this is the coordinate system that we take let us say x, y and z.

So, the magnetic field is in this direction. So, this magnetic field is B z. So, the magnetic field is written as 0, 0, 1 or more easily we can simply write that you can simply write the magnetic field as 0, 0, B z. So, magnetic field has only one component. Now, we have to understand if a particle comes under the influence of this magnetic field, it can be an electron or it can be an ion whatever it is what will be the role of this magnetic field to change this particles velocity or to change this particles trajectory.

So, that at the end of this description we have the energy conserved and the net amount of work done as 0. So, this is the geometry of magnetic field that we take. So, in this example we can simply say that we can start from the general force expression. So, the force is m dv by dt is equals to q times v cross B. Now, the velocity of the particle. So, this is the expression for the force velocity of the particle is in three components v x, v y, v z.

Now, the velocity of the particle v z is going to be called as v parallel the reason is v z is in the direction parallel to the magnetic field v x and v y are going to be referred as v perpendicular because they are perpendicular to the magnetic field. Now, what we can do is we can resolve this. So, we have the magnetic field given as this and we have the velocity given as this we can resolve this curl or cross product as I can write m dv by dt is equals to q times i into v y B z minus v z B y in the next line minus. j cap the unit vectors v x B z minus v z B x plus k cap v x B y minus v y B x this is the full expansion of the cross product. Now, we substitute the actual field that we have got then  what we will get m d by dt of i v x plus j v y plus k v z is written as q times i v y B z minus j v x B z plus k cap into 0. So, this is a three component equation. So, we can write an equation for x component, y component and z component of velocity from this single equation.

Which will look like m dv x by dt is equals to q v y B let us call this as equation a. Then m dv y by dt as minus q v x B let us call this as equation b. And m dv z by dt is equals to 0 let us call this as equation c. So, what we have done? What we have done is we have simply written this equation the Lorentz force equation into component form. So, we have now we get force along the x axis force along the y axis and force along. So, this is this can be force along the x axis, this is the force along y axis and this is the force along z axis.

So, before we go ahead we can now itself we can say that now, B is simply B z anyway B is B z. So, force along the x axis is coupled with the force with the velocity along the y axis similarly, force along the y axis is coupled with the velocity along the x axis. And force along the z axis is 0 because particles velocity is parallel to the magnetic field because of the curl. So, it is parallel.

So, the curl will make it as 0. So, what do we infer? We infer that we can break the Lorentz force expression into these three component equations and these three components component equations indicate that the force along the z axis is 0.

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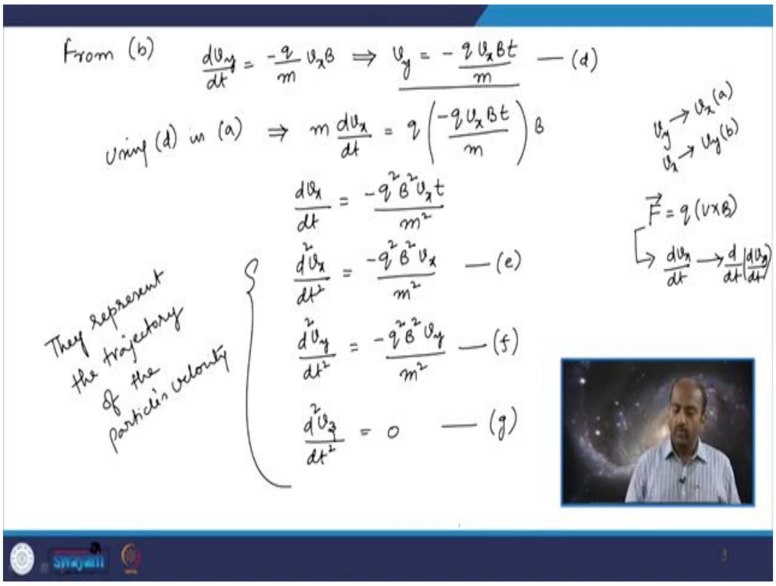


Now, so, let us say  from equation b we can write that dv y v y by dt is minus q by m v x b which implies v y is equals to minus q v x b times t divided by m.

In the uniform magnetic field, particle's motion is represented by the following equations –

$$\frac{d^2v_x}{dt^2} = \frac{-q^2B^2v_x}{m^2} = -\Omega^2 v_x$$

$$\frac{d^2v_y}{dt^2} = \frac{-q^2B^2v_y}{m^2} = -\Omega^2 v_y$$

$$\frac{d^2v_z}{dt^2} = 0$$

having solutions

$$v_x = v_{0x}cos\Omega t + v_{0y}sin\Omega t$$

$$v_y = -v_{0x}sin\Omega t + v_{0y}cos\Omega t$$

$$v_x = v_{0z}$$

Where, $\Omega = \frac{qB}{m}$, is angular frequency of rotation of the particle.

Now, let us substitute this result the v y result into v x which is given using let us call this as equation d using equation d in equation a what we can write is m dv x by dt is equals to q times minus q v x B times t by m into B. Which is equals to dv x by dt is equals to minus q square B square v x t by m square. Now, if you differentiate this we will get d square v x by dt square is equals to minus q square B square v x by m square.

So, now what we have done is we have coupled we have v y which we substituted into the expression for v x. Similarly, we can substitute v x into the expression for v y which is equation number b and this was equation number a. Then we will get another second order differential equation which is d square v y by dt square is equals to minus q square B square v y by m square. Let us call this equation as e and this equation as f.

Now, we already had one expression for dv z and we can write d square differentiating it again with respect to time d square v z by dt square as 0 let us call this expression as g. Now, so, by starting from the expression for the force q times V cross B and from starting from there getting the value of dv x by dt and differentiating this expression again we have got three second order differential equations what do they represent? They represent the trajectory of the particles velocity they simply represent the trajectory of particles velocity how does it look like. Now, so, simple I mean we can say that this is a second order differential equation and any harmonic solution for v x or v y will satisfy these equations.

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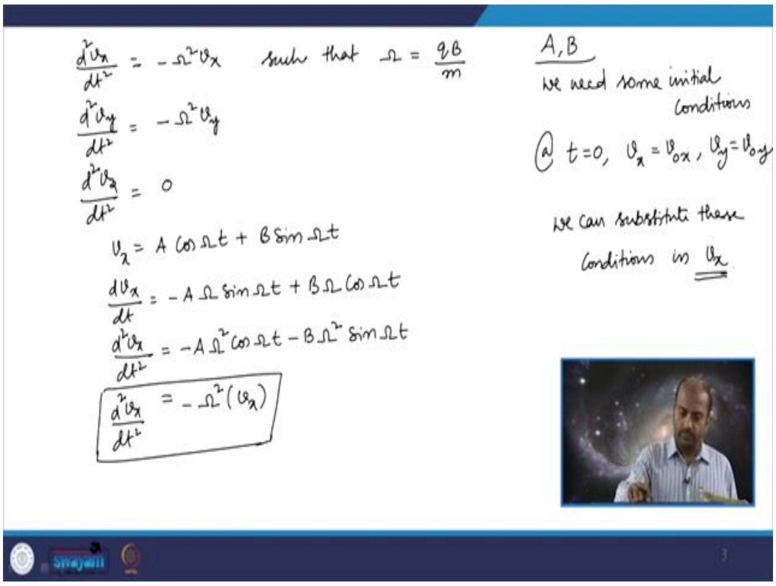


d square v x by dt square as minus omega square v x such that omega is written as q B by m. Similarly, d square v y by dt square as minus omega square v y and d square v z by dt square as 0.

So, what is the change? The change is that we have calling this this q square B square by m as omega square. So, in this expression. So, we have this expression in terms of d square v x by dt square is equals to omega square v x. Now, these equations look like they can take a simple harmonic solution which we will write as v x as now we will write v x as A cos omega t plus B sin omega t. Now, let us say if this is given as this we can write dv x by dt as minus A omega sin omega t plus B omega cos omega t.

And d square v x by dt square is minus A omega square cos omega t minus B omega square sin omega t which is simply d square v x by dt square as minus omega square v x; that means, the choice of solutions we have taken holds for the given differential equation. Now, if it is the case now, there is a second order differential equation and we have two constants the what are the constants? The constants are A and B we need to find out the values of these two constants before we go ahead.

Now, what are the values of these two constants, what can be the values of these two constants? So, in order to find the values of these two constants we need some conditions we need some initial conditions. So, what are the conditions let us say at t is equals to 0 the velocity v x is taken as v 0x and the velocity v y is taken as v 0y. Now, what we can do to find out the values of these constants? We can substitute these conditions in v x ok. Let us see.

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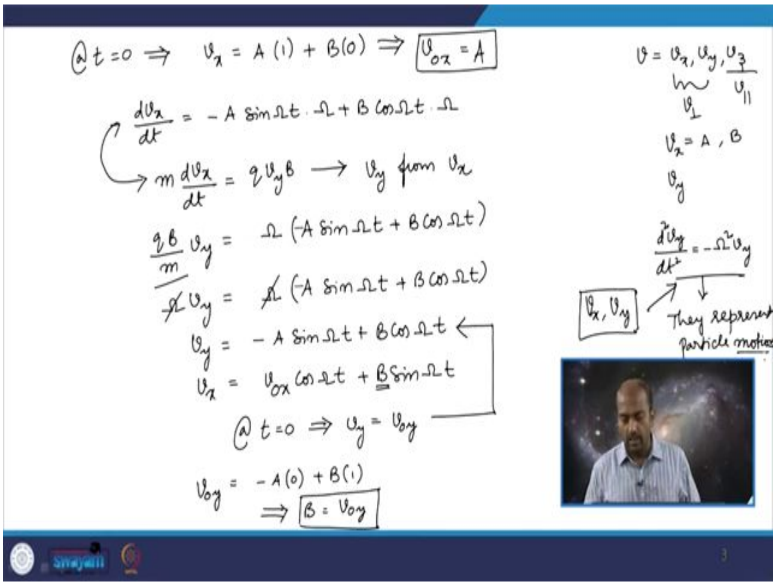


So, let us say at t is equals to 0 in the beginning v x is equals to A into 1 plus B into 0 that implies at t is equals to 0 v x is v 0x is equals to B. So, this is the value of B. So, by applying an initial condition we got the value of v 0x as because or B is equals to v 0x. Now, we already have dv x by dt which is written as minus A sin omega t times omega plus B cos omega t times omega.

And in addition to this we have one equation which is m dv x by dt is equals to q times v y B this is from the original equation that we have this equation was the product of cross product, resolving the cross product.  So, we have taken the v to be v x, v y and v z. Now, we said v z is not going to be influenced by the magnetic field and v z was called as v parallel v x and v y were called as v perpendicular.

Now, we have taken a solution for v x in terms of constants let say A and B, we should also have a solution for v y which will also satisfy the second order differential equation which is d square v y by dt square which is in terms of minus omega square v y in addition to this now we get a relation which is in between v x and v y. So, now, it gives us a possibility wherein we can get the value of v y or get the functional dependence of v y from v x itself this is the same without involving two more constants.

So; that means, that if you use this if you combine these two equations what you can write is q B by m v y q B by m v y because dv x by dt is qv v y b. So, now, we have one m which comes here is omega times minus A sin omega t plus B cos omega t. Now, we know that q B by m is omega. So, omega v y is omega minus A sin omega t plus B cos omega t we cancel omega and you can write v y as minus A sin omega t plus B cos omega t.

Now, with respect to v x, v x is v 0x cos omega t plus B sin omega t. There is a small mistake it is that now if it is a case. Now, we need to find the value of B the value of A was suitably substituted as v 0 x from applying the initial condition, now we need to find out the value of B. Now, similarly let us say at the beginning at t is equals to 0 v y would have been v 0y.

Now, using this into this equation we can write that v 0 y is equals to minus A times 0 plus B times 1 which implies B is equals to v 0 y. Now, we have both A and B. So, A is the initial velocity along the x axis and B is the initial velocity along the y axis for this kind of a solution. Now, we will simply have to substitute the values of A and B for getting the full functional form of v x and v y such that they satisfy this type of second order differential equation.

Now, what is particular about this type of equation? They represent particle motion. So, we will continue this discussion in the next lecture how we can interpret this two v x and v y and how they can tell us how the particle will move with respect to the with respect to time in the presence of an external magnetic field.

Thank you.