Introduction to Atmospheric and Space Science
Prof. M. V. Sunil Krishna
Department of Physics
Indian Institute of Technology, Roorkee

Lecture – 51
Particle Motion in a Uniform Electric Field

Hello dear students. In today's lecture we will try to understand the basic description of plasma. By the word basic description what I really mean is, we have already seen in ionospheric discussions, we have seen that ionosphere is the entire layer of ionosphere acts as a conducting layer or acts as a mirror for the incoming radio wave propagation and depending on the suitability of refractive index of the medium in comparison to the electromagnetic wave refraction may happen. That is fine, but ionosphere as we have seen, is composed of electrons and ions.

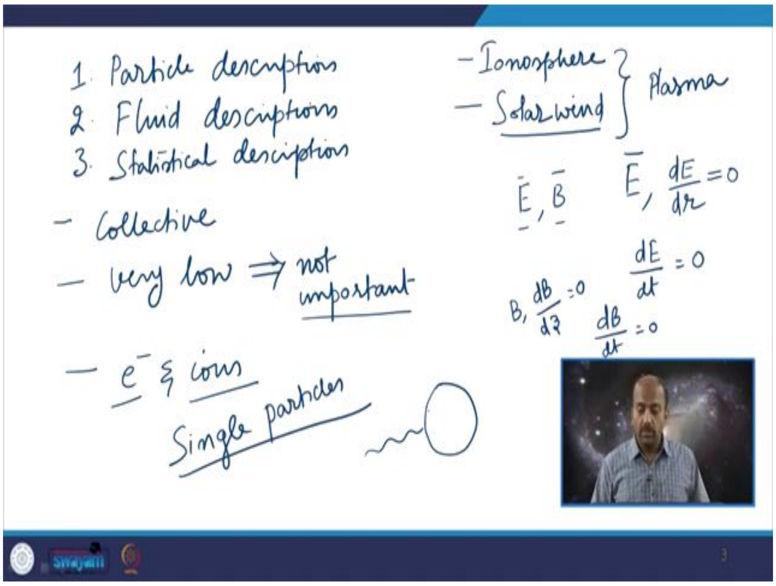
These electrons and ions are produced by these incoming solar radiation, the energic part of the solar radiation which ionizes, dissociates several atoms and molecules. It is generally expected that during the day time, the ionospheric densities are more because lot of production of electrons and ions is happening and during the night time, the electron density will be less. The typical electron density during the day time, let us say in the peak ionosphere will be of the order of 10 to the power of 12 per meter cube.

So, there are considerable amounts of electrons and ions present. Now, we can say whether depending on or applying a plasma criteria, we will be able to say whether this kind of plasma or this kind of charged densities of the order of 10 to the power of 12 per meter cube can be called as plasma or it should be called as a gas. That is a different story. But generally, it is well accepted that the entire ionosphere is plasma depending on the temperature or depending on the charge concentration.

Now, in order to understand various aspects of ionosphere, it is very important for us to treat ionosphere as plasma and use the laws of plasma physics for the ionosphere. So, in today's lecture, in continuation with our discussions on ionosphere we will try to understand how we can describe plasma or how what are the physical laws which the plasma will obey, what are the various principles the plasma will obey, how do you write equations of motion for the plasma, how do you write or how do you understand the governing equations for the motion of plasma from one point to another point, things like that.

So, this basically comes under the topic of a single particle or a particle description of plasma. So, what we will do is we will try to understand how we can describe plasma? So generally, plasma can be described in 3 different ways.

(Refer Slide Time: 03:10)



Plasma can be described in let us say for example: number 1 is a particle description of plasma where you treat the plasma as a particle. Second way is fluid description you treat the entire plasma as a fluid, then you apply the relevant equations or governing equations and the third way to describe plasma is statistical; that means, you treat it as a statistical entity, statistical description of plasma. So, these 3 are the three different ways in which you can describe plasma. Generally what happens is, when the particle densities are very very small, let us say we know very well that plasma has the ability to behave collectively.

So, plasma generally behaves as collective entity this is called as the collective behavior. So, plasma behaves collectively and when the densities are very low, very low plasma densities, we can say that the collective behavior is not important. We will say that the collective behavior is not important. So, what is the consequence of that? So in order to understand how to describe plasma in various ways, it is very important for us to understand how a single particle behaves.

That means, when the collective behavior is not important in a very low density plasma. So, we have seen that plasma is made up of electrons and ions. Insufficiently large numbers, that is what the basic description always says. So, electrons and ions should be present in large numbers to be able to give it the name plasma. When these electrons and ions are very low, then plasma does not behave collectively. I mean collective behavior would not be very important. Then you are going to treat the electron and ion as single particles.

Thus, so you are not going to treat it as a complete or single entity, you are going to treat this as single particles. That means, you have to understand how a single particle will be influenced when this plasma experiences different types of fields. So, for example, why do we talk about this? We talk about this because let us say, our examples have been systems of plasma for example, ionosphere. Ionosphere is a system of plasma which we are going to describe, which we are going to understand.

Then another system that we are particularly interested is let us say solar wind. Solar wind is also plasma. So, solar wind is extremely energetic plasma that is emitted by the sun and which travels towards the earth or at sometimes, enters the atmosphere of the earth,. So, these two are the 2 important plasma systems which are relevant for our course.

Now, you know what generally happens? Let us say, the plasma in the ionosphere or the plasma in the solar wind, experience electric field and magnetic field they experience these 2 different types of fields. Now, when the plasma is very rare so our idea will be to establish equations of motion when the plasma experiences electric fields and magnetic fields. Now, this electric field and magnetic field a combination of these 2 fields can be of different types.

Let us say, for example, now, when the solar wind is travelling towards the earth, let us say towards the earth; what kind of field do you think it will experience? Will it experience a electric field? Or will it experience a magnetic field? Or will it experience a combination these two fields? If it is the case then what is the nature of these 2 fields? Let us say. So, the electric field can be let us say, electric field can be homogeneous; that means, its not changing with respect to space or it can be static or it can be let us say it can be varying with respect to time.

That means electric field with respect to the space or time. let us say electric field is not changing with respect to time. That means, electric field is homogeneous, electric field can remain static let us say, dE by dt is 0. So, its not changing with respect to time, similarly you can have the magnetic field magnetic field is not changing with respect to space and magnetic field is not changing with respect to time. That means that you have if you consider ionosphere or if you consider plasma.

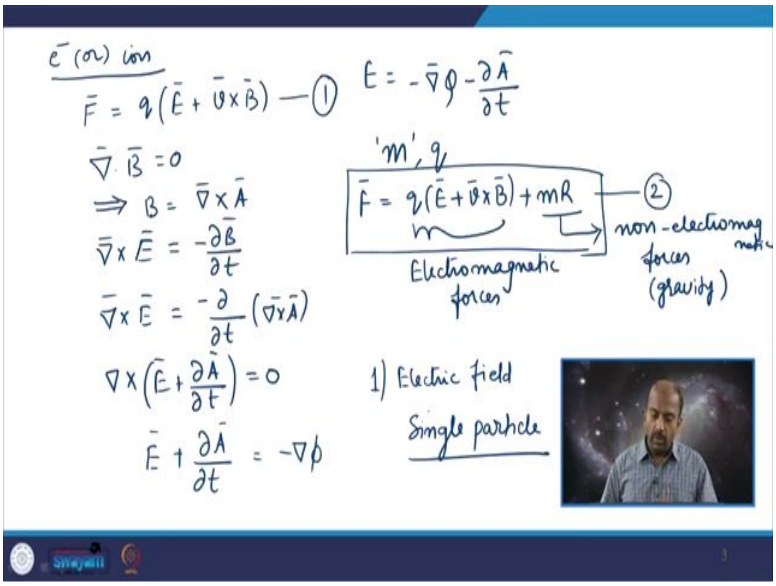
And if the plasma is very very low intensity, there is a possibility that this plasma is to be treated as a collection of single particles and if you want to understand how this collection of single particles behave, its very important for you to understand how a single particle, how one single particle will behave in the presence of a field, in the presence of an electric field or in the presence of a magnetic field. Then what you will do is, you will extrapolate this argument this description of a single particle all to the entire plasma itself and then you can say that the plasma may behave in this way,.

Now for us to go there we start from single particle description; that means, we will take one particle and we will say that this particle is going to experience these many different types of fields. So, in our discussions let us say from today, what we will do is we will try to put up different types of fields.

Let us say, time varying electric field, or time varying magnetic field, homogeneous electric field, inhomogeneous electric field and inhomogeneous magnetic field or a combination of any of these two these different types of fields and then we will try to establish how the plasmas behavior will change with respect to these fields. Let us say electron or ion.

Nowwe have to understand that we will derive the equations of motion and then we will try to describe how the particle will move, so that it satisfies these equations of motion that is the basic idea,. Now, let us start with a single particle a single electron or ion we will start with a single particle.

(Refer Slide Time: 09:44)



Electromagnetic force is given by

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{V}\times\mathbf{B})$$

Let us say electron or ion. Then we will say that when a charged particle experiences an electric and magnetic field, the force that it experiences is q times E plus v cross B. So, this is a very familiar expression that you must have been knowing. So what it says is that, if a charged particle experiences electric and magnetic field or if a particle is subjected to electric and magnetic fields, then it will experience this much amount of force.

Now here, what you see q is the charge, v is the velocity of the particle, B is the magnetic field strength and E is the electric field,. Now, let us say this is the basic starting point, now we will try to derive equation of motion based on this. So, in a very dense plasma if the plasma is dense, Coulomb forces generally couple the particle and the bulk motions become significant, that is what.

In a highly rarefied plasma, the particles do not interact well with each other and hence, each particle can be treated as independently; that means, when the plasma is very very rare, each the particular the particles interaction with the other particle does not become very significant. As a result, what you can say is that you treat each particle as a single entity and you apply equations of motion on this particle itself,. Now, this is the generalized expression.

So what you do is, I mean, we will also recollect some of other familiar relation from our electromagnetic theory lessons. So, what you say is that we will write, we know that very well that del dot B is 0 and that means, B can be written as del cross A and in terms of vector potential and del cross E will written as we can write it as minus dou B by dou t. So based on this, we can write that del cross E is minus dou by dou t of del cross A.

So, this is just a recollection of basic relations that we know already. So, now you can rewrite this expression as del cross E plus dou A by dou t as 0 or you can write E plus dou A by dou t as minus del phi. So, phi is the potential or ultimately you write E as minus del phi minus dou A by dou t. So, curl free electric field can always be expressed as a gradient of scalar potential. So, which you call as phi here. So, more general form you know. So, these are their set of relations that we will probably use in the discussions ahead.

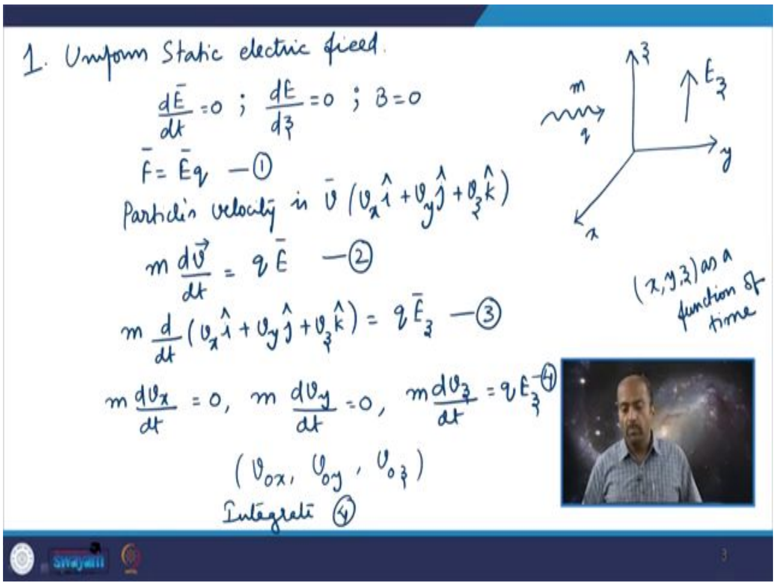
Now, for now let us slightly complicate this expression. let us call this as equation number 1 which is the Lorentz force expression where you have force given in terms of the charge of the particle the strengths of the electric field and magnetic field,. Now, let us consider a plasma particle with its mass m and charge q. What am I trying to do? I am trying to write let us say, the most general expression of the Lorentz force when the particles mass is also taken into account,.

Now, these are the electromagnetic forces and then you add another term. So, these are the electromagnetic forces and this term represents non- electromagnetic forces for example, such as forces such as gravity etcetera. So, when a particle experiences a combination of electric and magnetic field, and the particle also has some amount of mass then this is the total force that is going to be experienced by the particle.

Now, how do you justify this expression? The simple way is, let us say if the particle is entering the earth's atmosphere, it is going to experience electric field, its going to experience magnetic field, of course; and it is also going to experience the earths gravity. So, these 2 different types of forces are very much important. Now, the description of plasma is complete only if you take both the parts and combine them effectively,. Now, so this is the basic expression that we will be needing in all our discussions, let us say.

So, this is a very important expression. Now, so like I said, we will try to understand how the particles motion will be influenced by an electric field to begin with. So in today's lecture, we will start by saying , the plasma particle let us say a single particle, a single particle when experiences an electric field. How does the particles trajectory look like or how does the particle move?

(Refer Slide Time: 15:48)



Now, that means, let us consider the an electric field, let us consider the now, let us say this is y, this is x, this is z. Now, let us consider the electric field to be pointing in the z-axis. So, this is the direction of electric field. Now, let us consider a particle approaching towards this field, let us consider particle with charge q and mass m,. Now so, what is the nature of this electric field? this type of field is uniform static electric field,. So, what does it mean? It means that dE by dt is 0.

Because it is static in nature, then in addition, dE by dz is also 0; that means, along the z-axis, the strength of the field is constant as long as you go. So, the field is homogeneous with respect to the z-axis. I mean there is no field component along the y and x directions, t, we have taken the electric field only to be along z-axis. So, that is why you call it as Ez and this electric field is constant with respect to position z and time,.

So, electric field is only along z-axis. So, if there is no magnetic field, the magnetic field is 0. So, particle is experiencing only an electric field which is directed along z-axis. Now, what is the force? force that is going to be experienced by the particle is F is equals to E q that is it. Now, let us say the particle has a velocity, particles velocity is v which is v x i plus v y j plus v z k.

Now so, let us say this is equation number 1, we can write equation number 1 in differential form as m dv by dt is equals to q times E. Now, since the field is only along z-axis, we can write m d by dt of v x i cap plus v y j cap plus v z k cap is equals to q times E z. Now, let us say this equation number 2 is the differential form of equation number 1 and this is equation 3. Now, we can resolve this equation into 3 different component equations.

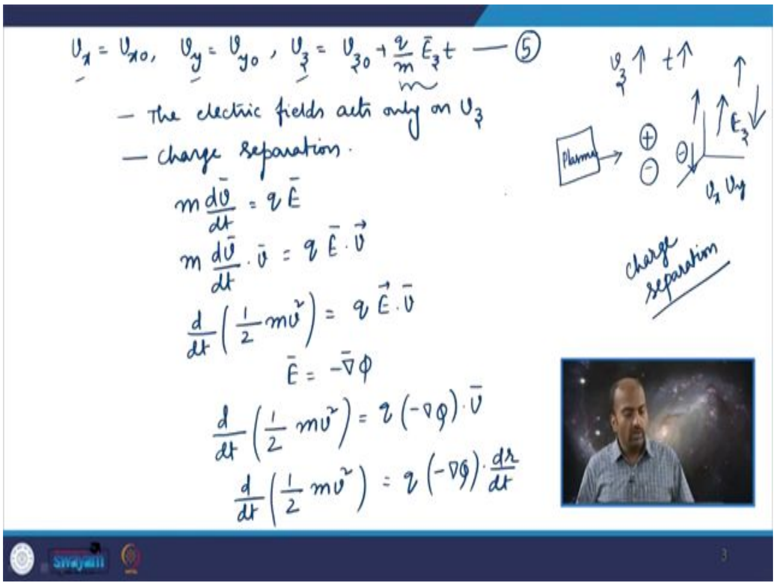
So, which is m dv x by dt is equal to 0, m dv y by dt is equal to 0 and m dv z by dt is equal to q E z. Let us call this set of equations as equation number 4. Now, let us say the initial coordinates of the particle now, what am I trying to achieve? So, I have started with the equation of motion, the force is equals to charge times electric field, the force is written as m dv by dt; now what I am trying to do is, I am trying to establish how velocity will change with respect to time.

That means, as the particle approaches this electric field or as the particle starts to experience this electric field, how will the particles velocity change with respect to time? So, ultimately I have a derivative with respect to time, if I integrate this equation, I will get velocity,. Now, velocity also has the information of position with respect to time,. So, then again if you find integrate, I will get the position.

So, what I am trying to do is, I am trying to establish relations for the particles position x, y, z as a function of time. So, I am trying to derive equation for the trajectory of the particle when it experiences this electric field,.

Now, we know very well how it is going to look like, let us see how,. So, let us say if I integrate this equation, I am going to get a constant. So, let us say the particles initial velocities are v 0 x, v 0 y and v 0 z. So, these are the velocities when the particle is not experiencing the electric field, when it experiences, they will change as a function v x, v y, v z,. Now, if it is the case, then let us integrate set of equations number 4,.

(Refer Slide Time: 21:31)



So, if I integrate this equation, we will get v x is equal to v x 0, v y is equal to v y 0, v z is equal to v z 0 plus q by m into E z times t. Let us call these equations as equation number 5. Now, here itself we have a very important outcome. The outcome is that velocity v x after the particle starts to experience the field, is same as the initial velocity. Similarly, along the v y it is same as the initial velocity v y 0, but along the z-axis v z is equal to v z 0, the initial velocity plus a term which is linear in nature. So, this is, so plus a term which is linearly increasing with respect to time.

So, I will say that v z is linearly increasing as the time goes, so this is constant. So, the motion of the particle along x and y directions will not be affected by the electric field. So, electric field is not able to influence these velocities v x and v y. But however, v z is only along the z component of velocity will increase linearly with respect to time,. Now what you can say is that. So, if the particle is entering into this field, its velocity increases linearly only due to E z. So, E z is not able to influence v x, v y.

Now, let us say is if the particle is now a positively charged particle or if the particle is now a negatively charged particle. So, positively charged particles will move along the electric field and negatively charged particles will move opposite to the electric field. So, it means is that if two types of particles enter into the electric field, all the positively charged particles will be going in this direction and all the negative charged particles will be going in this direction.

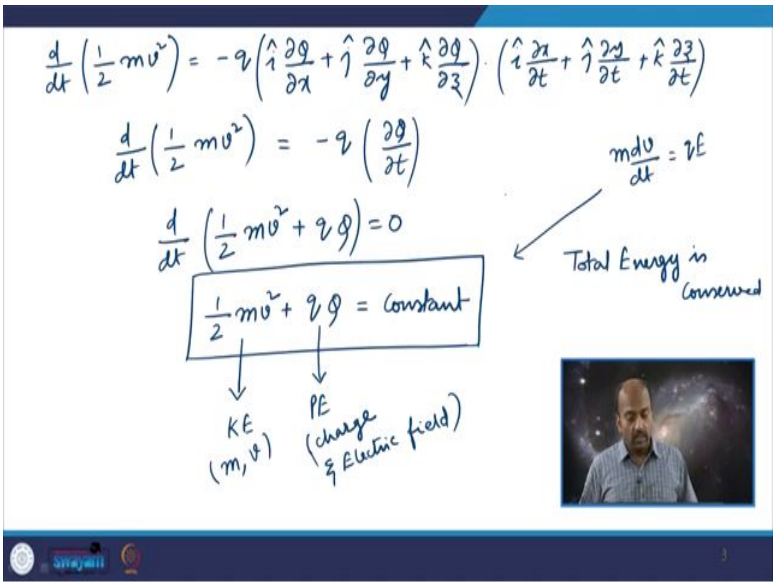
But how is the velocity of this particle is changing? It is linearly increasing with respect to time. That means, so before entering this electric field, let us say if there is a plasma which is coming.

So this is plasma which is moving in this direction. Before entering the field, the plasma particles are not separated. That means, after it enters, it starts to experience, the plasma particles are separated. That means, it is creating a charge separation. So, this electric field is able to create a charge separation. So, what is the consequence of this charge separation? Ultimately charge separation leads to the setup of a secondary electric field which will be opposite to the direction of the primary electric field and then we will see how it goes,.

So now, what do you see is that one important conclusion is that the electric field acts only on v z and it leads to charge separation very well,. Now; let us consider the original equation m dv by dt is q times E,. Let us take a dot product of this expression on the right-hand side with the velocity, or rewrite this expression as d by dt of half mv square is equals to qE dot v.

Now, we can bring in E as minus del phi. That means that d by d t of half m v square is equals to q times minus del phi dot v or d by dt of half mv square is q times minus del phi dot dr by dt.

(Refer Slide Time: 26:49)



In the uniform electric field,

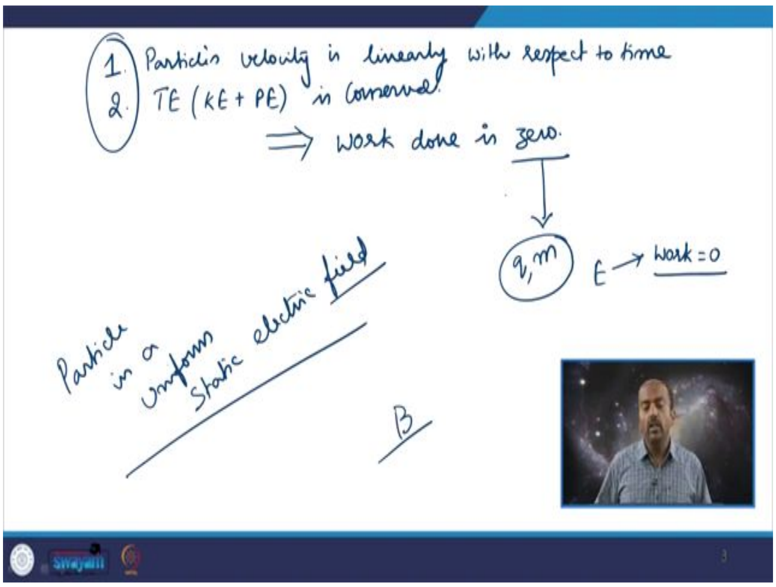
$$\frac{1}{2}mv^2 + q\Phi = constant$$

I am just rewriting it for the sake of convenience now I write as d by dt of half mv square as minus q times i dou phi by dou x plus j dou phi by dou y plus k dou phi by dou z. So, this is the del phi which I have written in the vectorial form dot I times dou x by dou t.

So, this is a dr by d t that I am writing j times dou y by dou t plus k times dou z by dou t. If I take the dot product, I can write d by dt of half mv square is equals to minus q times dou phi by dou t or d by dt of half m v square plus q phi is equals to 0 or half mv square plus q phi is constant. Now, what is half mv square? Half mv square is the kinetic energy which is by the virtue of the particles mass and velocity and q phi; phi is the potential,.

This is the q phi is also energy phi is the potential , potential is the potential energy per particle,. So, which is by the virtue of charge and electric field,. So, what have we got? Starting from the equation of motion m dv by dt is equals to q E starting from this equation of motion, what we have been able to achieve is a fact that if the particle is experiencing an electric field and if the particle has some mass and if the particle is having some charge, it will move in such a way that the total energy is constant; the total energy is conserved. That means that here we can say that the total energy is conserved. So, what is happening then? What is the role of electric field then?

(Refer Slide Time: 30:20)



So, we say that, so, one important conclusion is that: number 1 is particles velocity is linearly changing with respect to time, the second most important conclusion is that the total energy or let us say total energy; sum of kinetic energy plus potential energy is conserved,.

So, as a result so, when the total energy is conserved what we can say when the total energy is conserved the net this implies that the work done, what is the work done when the total energy is conserved? Work done is 0; that means, having a particle having a charge q and a mass m if it experiences an electric field, the role of this electric field is such that it keeps the network as 0. So, electric field does not do any work in changing the velocity of the particle or in any of the description.

So, these are the 2 important conclusions of single particle in a uniform static electric field. Now, so we will try to see how this description will change if you have a uniform magnetic field,. So, this is something about. So, let us just go back one-by-one, we will see where we have started. So, we wanted to understand the particle description of plasma because many times plasma is very rare and the collective behavior is not so important that we take it into account and when this particle is experiencing different types of fields, we will see.

So, this is where we are starting. So, this is a fundamental equation where in their electromagnetic effects and the magnet and the gravitational effects are to be taken into account. Then we consider simplistic type of field where the electric field is only pointing along the z-axis, in that kind of picture we allowed a particle to experience this field, we wrote the equations of motion we integrated to get the position velocity with respect to time.

We thought we were able to realize that this picture causes two things, I mean one is that velocity of the particle is uninfluenced along those directions in which electric field is not present and electric field will only be able to influence the z component by changing it linearly with respect to time,.

And the second consequence is that we realize that in this picture, the kinetic energy and the potential energy remains constant. So, phi is what quantifies the electric field strength and q is the charge, m is the mass. So, the mass is not to be neglected as such. So, this is the description of plasma or the description of a single particle when it experiences only electric field,. So, in the next class we will try to see how it changes when you have only magnetic field,.

Thank you.