Introduction to Atmospheric and Space Sciences
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Lecture – 50
Debye's potential – Continued

Hello, dear students. So, in today's class we will continue our discussion on Debye shielding. We will try to derive a mathematical expression for Debye's potential thereby, interpreting what is Debye's length.

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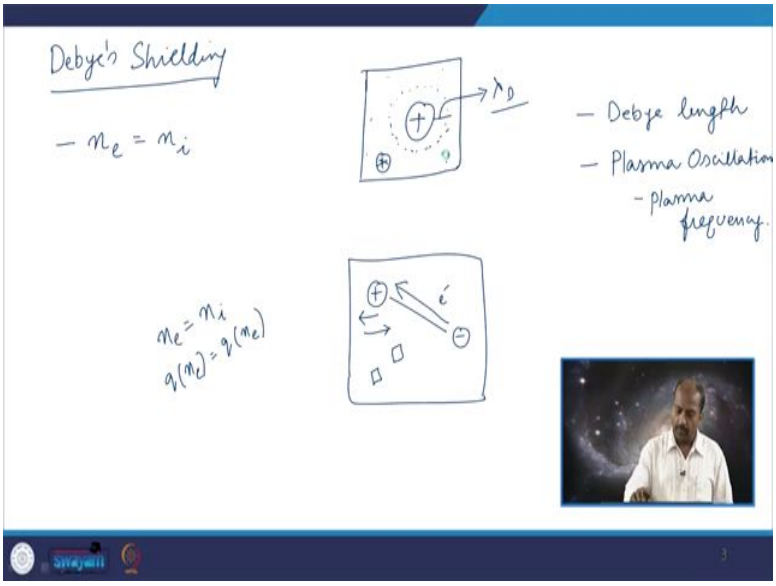

So, we will try to derive an expression for Debye length and then in characterizing plasma or in discussing the various parameters, a various properties of plasma, it is very important that we understand what is called as the plasma oscillations. So, we will in today's class we will try to understand what are plasma oscillations and we will derive an expression for plasma frequency.

So when we talking about the Debye's shielding, so we have seen that if we keep a positive test charge in a plasma. We have realized that this positive test charge will be surrounded by a cloud of electrons, not just one electron but a cloud of electrons such that the effect of this positive charge test charge will not be realized or will not be felt at any point beyond this cloud.

So, we defined this particular typical length scale as lambda D, which is also called as the Debye's length. So to define Debye's length, it is the length scale over which plasma tries to shield the effects of or the electromagnetic effects of a charge. So beyond this distance, the electromagnetic effects or the charges influence will not be felt.

So, basically when you keep it is charged in a plasma generally what happens? in the beginning it will be, it will start attracting the electrons. Now from here onwards what we can say is that, let us say the number of electrons and number of ions are equal. So, number of electrons is equal to number of ions macroscopically. Macroscopically, it is equal.

But when we were defining what is quasi-neutrality, we also made a mention saying that plasma allows the possibility of local or tiny deviations from the electrical neutrality only to the point that the electrostatic potential energy that arises due to the charge separation is roughly of the order of the kinetic energy of the electrons.

So, the basic idea was very simple. Before we get here its very important to understand the neutrality of the plasma as a whole. So if you have charge accumulation anywhere in the plasma, it will always be such that an electric field is generated by this charge accumulation or charge separation. And electric field will always try to compensate or let us say will always try to create charge movement, allow charge movement in the opposite direction of electrons.

Let us say electrons will move opposite to the direction of electric field and that movement of these electrons is such that it will try to nullify the existing electric field. But then we realize that this electrons in the in attempt to neutralize the positive ion, they will always overshoot the mean position and a restoring electric field is set up in the opposite direction, the role of this restoring electric field is to bring back the electron to the neutral position; in the process there will always be oscillations.

Now, when you look at the plasma, plasma is considered electrically neutral; that means, the total number of electrons is equal to the total number of ions. And, if you talk about the charge let us say total positive charge in terms of number of ions is equal to total number of negative charge in terms of electrons. But plasma being an electrical conductor is not just a gas with molecules.

So plasma should also allow the possibility of a Coulombic interactions between various constituents of the plasma. That means that plasma is neutral macroscopically, but microscopically in small regions of plasma, charge accumulation may always take place. Now,if you allow the possibility of a positive ion here, in the plasma what we can realize is that within sometime it will start attracting the negative ions and they will form a cloud around the positive ion when you keep a positive test charge.

Now what we are going to find out is let us say, we will be interested to find how this cloud is able to shield the electric field. So if you keep a positive test charge the electric field due to this positive test charge, by the means of Columbic interaction, should always be felt or can always be experienced at any point in the plasma.

Any point in the plasma should be able to experience this positive test charge. But what generally happens is, due to the concept of Debye's shielding, the electrons will surround the positive ion up to a particular distance and beyond this particular distance, electrons or any other charge will not be able to experience this positive test charge. So this is the means by which plasma shields out the external electric fields. so that means, there is only up to a limited spatial extent, the external fields influence can be felt in the plasma not the entire plasma itself.

Now the spatial extent to which the electric fields influence can be felt it can be called as the Debye's length. Now we are going to derive a relation for Debye's length and we are also going to see how the potential of a positive test charge will look like inside a plasma.

Then the question really comes I mean, what are the parameters over which this Debye's length will depend? Will it depend on the mass of electron? Will it depend on the number of charged particles per unit volume in the plasma? Will it depend on the temperature? Will it depend on any other property of the plasma? Let us say for example,.

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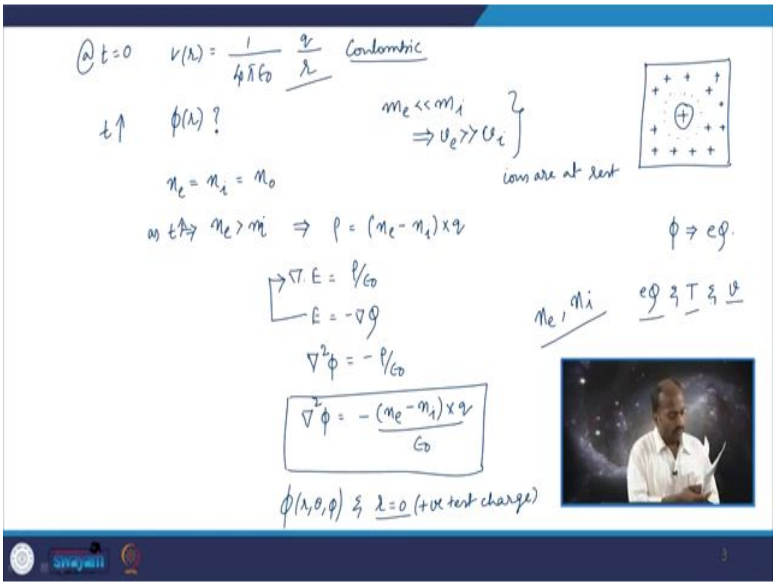

Now we will try to see all of that,. So for example, so what we can do is, let us say we can take plasma and we have a positive test charge in the plasma. Now so, we are saying the cloud of electrons will be surrounding the plasma. Now at let us say when at t is equal to 0, when the moment when you keep the positive test charge, I can write the potential due to these positive test charge at any distance as simply 1 by 4 pi epsilon naught q by r.

So what is the nature of this potential? This the nature of this potential is simply Coulombic. And what is the range of this Coulombic interaction? It is infinite. I mean, at any point in space should be able to feel the influence of this test charge. Now when you allow sufficient amount of time as t increases, away from 0, as t increases, we have to find the nature of this potential.

How will this potential appear? Now to begin with we are saying that this potential the potential due to this positive test charge cannot be felt after a particular distance, we are saying that. Now we have to see if we allow sufficient amount of time for this charge accumulation and the formation of cloud to occur, we have to find out the empirical form of this potential after sufficiently large amount of time.

So how will this potential change? Deviate away from the Coulombic potential is going to be the topic of our discussion,. Now let us say. So, to ease our calculation we say that the mass of electron is immensely small when compared to the mass of ion, which means that the velocity of electrons is very very large when compared to the velocity of ions,.

Why ere we making these assumption? We were making these assumption to say that the ions are not moving, ions are at rest. I mean so the ions are not moving. So they constitute a simple neutralizing background. At t is equal to 0, number of electrons in the vicinity of let us say the test charge is equal to the number of ions, just to begin with.

Let us call this concentration as n naught. As time progresses, as t increases, let us say t increases, what will happen is number of electrons in the vicinity of the positive test charge, will be slightly greater than number of ions. If it is the case then what we can do is so in within this sphere of influence of positive test charge, we can say that the total charge accumulated let us say is or the total charge density for example.

So the total charge density is number of electrons minus number of ions times the charge. So n is the number of particles per unit volume and q is the charge. So since the number of electrons is more in the vicinity of this test charge, the resulting charge density can be written as rho is equals to n e minus n i times q. Now we very well know that del dot e is rho by epsilon naught and e is equals to minus del phi.

So this charge accumulation if at all if there is a charge accumulation, there has to be a resultant potential due to that particular charge accumulation. And any particle should be able to experience this particular potential. And if there is a particle which is experiencing this potential, the potential energy if phi is the potential the potential energy that will come out of this configuration is just e phi. Where e is the charge of the particle and phi is the potential over which it is accelerated,.

Now substituting this into this equation what we can do is, we can simply write del square phi is minus rho by epsilon naught,. Which means that del square phi is equals to minus n e minus n i into q by epsilon naught. So now we know that within this small sphere of influence of the positive test charge there is a charge accumulation, the magnitude of this charge accumulation is known. The charge density there is available in this region is known, and the resulting potential is given the name of phi.

Now our objective is to solve this second order differential equation and see how the form of phi supports our argument saying that the potential will not stretch beyond Debye's length,. Now for simplicity we say that so phi is in the spherical polar coordinates r theta and phi,. And the positive test charge is situated at r is equals to 0. So this is the position of the positive test charge. Now, so in order to do this let us say.

So what we should know is we should know how n e and n i will look like in the presence of this potential e phi. And there is also a thermal energy which is attributed to the electrons by the virtue of temperature and in addition, we should also be knowing how the velocities of the electrons or ions are distributed,. So that means that n is the number of electrons and e is the number of electrons per unit volume. So how is this number of electrons per unit volume going to change as a function of r away from this positive test charge?

Because there is a potential, the electrons will experience a potential energy because there is a temperature the electrons thermal energy will be decided and these thermal energies are going to be distributed as per their velocities.

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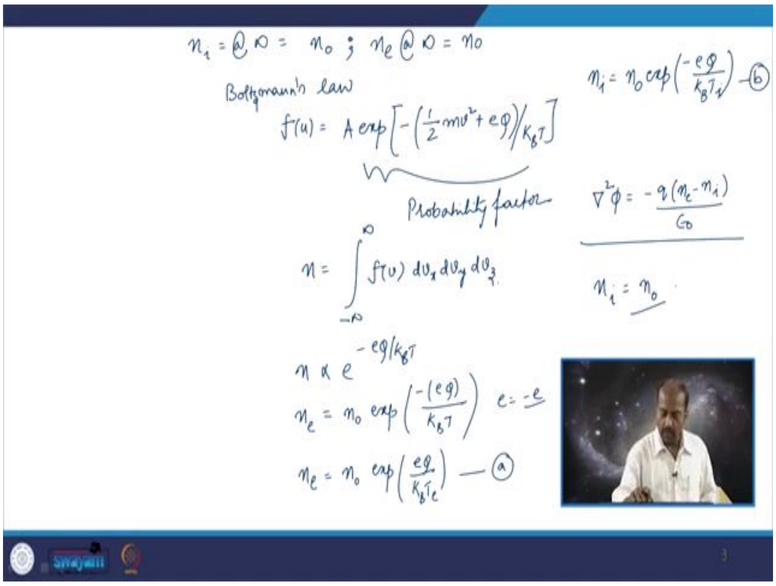

Now, let us say for example, so what we can simplify is that let us say so, at very far distance low n i at infinity. So that means away from the vicinity of the test charge is n naught and n e at infinity is equals to n naught. So, n naught in the vicinity of the test charge is natural to expect that these concentrations will be equal. Its only in the vicinity of this test charge things are not equal.

Now, let us say we say that. So they are following Boltzmann's law. So if the distribution is according to the Boltzmann's law, we can say that let us say, we should have a distribution function now. So their velocities are following this function A times exponential minus of half m v square m v half m v square is the kinetic energy of the electron, let us say of the particle.

Plus e phi is the potential energy which is by the virtue of the potential due to the positive test charge,So this becomes this entire factor becomes a probability factor,. Now, how do you get the number density from this probability factor? You just integrate this between minus infinity to infinity f v dv x dv y and dv z.

So, we will realize that n is proportional to e to the power of minus e phi by k B T. So we can write number of electrons is equals to n naught exponential minus of e phi by k B T which is equals to n e is number of electrons per unit volume is since e, the charge of electron is to be taken as minus e we will write that e phi by k B T.

So its more appropriate if you write k B T e,. So, similarly you can also write n i is equal to n naught exponential minus e phi by k B T i. Now we are trying to solve del square phi is equals to minus q times n e minus n i divided by epsilon we are trying to solve this,.

So, we can substitute let us say this as equation a and this as the equation b. We can also say that number of ions since they are just constituting a background, but not are distributed as per their velocities, let us say they are not moving anyway. So, the effects of potential being taken away we can say that the number of ions is simply n infinity or n naught we can also say that.

So, if you say that we will have to solve this second order differential equation to be able to get the functional form of the potential.

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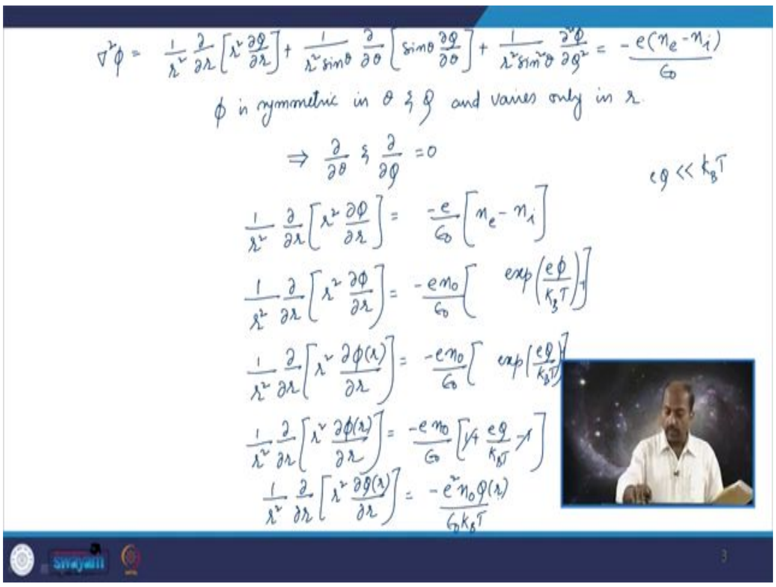

So, del square phi in the spherical polar coordinates is to be written as 1 by r square dou by dou r of r square dou phi by dou r plus 1 by r square sin theta dou by dou theta of sin theta dou phi by dou theta plus one by r square sin square theta dou square phi by dou phi square is equal to 0. Naught is equal to 0.

So this is equal to I mean minus e times n e minus n i divided by epsilon naught,. So let us say the potential phi is symmetric in theta and phi and varies only in r. That means, so derivatives with respect to theta and derivative with respect to phi are to be treated as 0. So what we will be left with is one by r square dou by dou r of r square dou phi by dou r is equal to minus e by epsilon naught times n e minus n i.

Which we can write as 1 by r square dou by dou r r square dou phi by dou r is equals to minus e n naught divided by epsilon naught into 1 minus exponential e phi by k B T or more. We can also write so, since now phi is just a function of r, so we will write 1 by r square dou by dou r of r square dou phi of r divided by dou r is equals to n e minus n i.

So, its going to be this minus 1. So this is going to be minus 1. So now, if you substitute 1 by r square. So if you say that e phi is much less than k B T, see if the potential energy is much less than the thermal energy, we can write r square dou phi of r divided by dou r is equals to minus e n naught by epsilon naught into 1 plus e phi by k B T minus 1.

So what we have is 1 by r square dou by dou r, r squar dou phi of r by dou by dou r is equals to minus e square n naught phi by epsilon naught k B T. So here when phi is a function of r,.

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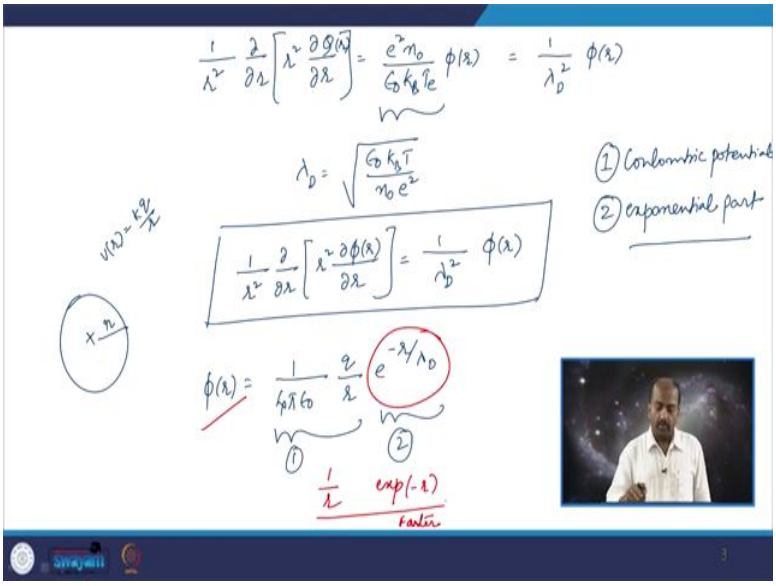

Now, if you substitute the electrons charge being negative we can write this equation as one by r square dou by dou r of r square dou phi by dou r is equals to e square n naught by epsilon naught k B T e into phi of r. So we can write this as a constant as 1 by lambda D square into phi of r. So how did I define lambda D?

Lambda D is epsilon naught k B T divided by n naught e square. So I have this second order differential equation. What is this equation? This is 1 by r square dou by dou r of r square dou phi by dou r is simply 1 by lambda D square phi of r.

Now the solution of this differential equation is going to be the functional form of the potential. So the solution looks something like this. Phi of r is simply 1 by 4 phi epsilon naught q by r into e to the power of minus r by lambda D. So this has 2 parts actually. This has let us say we call Part 1 and Part 2. So Part 1 is Coulombic potential, you see it clearly just 1 by 4 phi epsilon naught q by r.

The part 2 is an exponential part. Now how do we understand the effects? I mean what we have been able to do is we have been able to get the functional form of the potential that exists within the, if you have a positive test charge and if this line indicates the presence of the electron cloud, we have been able to establish how does the potential vary with respect to r within this sphere influence inside the plasma.

So, what it means is that if you have plasma so how does r vary? I mean if you do not have plasma, simply Coulombic in nature which is just k q by r. But if you have plasma, you have the additional appearance of this particular exponential part. So this is very important. I mean this will decide how the potential within the Debye's sphere will vary with respect to space.

So ultimately what we have derived is phi as a function of r of course and so, this is Part 1 and Part 2. So, I mean you see that Part 1 is 1 by r dependent and Part 2 is exponential minus r. So, both of them are decays 1 by r decay and exponential minus r decay. So, 1 by r is kind of slower in comparison to exponential minus r. So this decays much faster. So exponential decays much faster than the 1 by r decay,. Now let us say let us try to understand the more important features of this particular potential.

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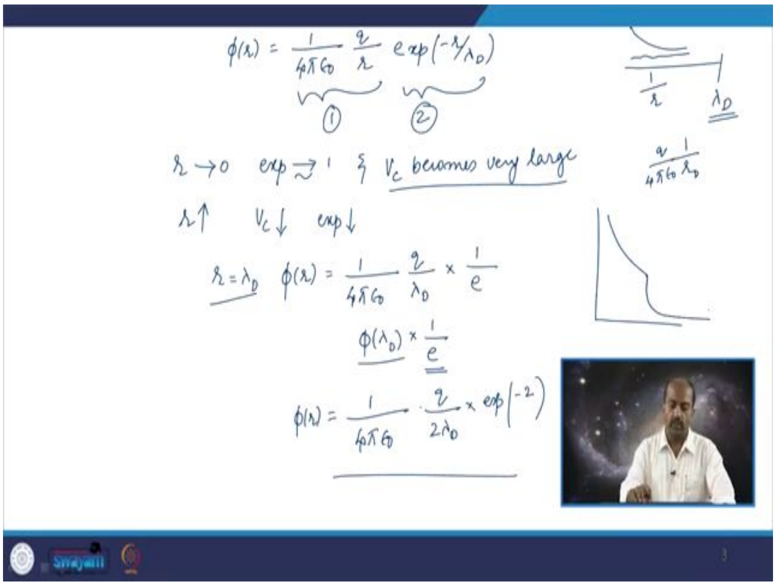

Let us say. So we have defined phi we have been able to derive is 1 by 4 pi epsilon naught into q by r exponential minus r by lambda D. Let us say, let us observe this. So this is Part 1 and this is Part 2. So when r tends to 0. That means, in the closest proximity of the test charge when r tends to 0 which is the closest proximity of the test charge, what will happen? So at the exponential term, simply becomes nearly equal to 1.

But at the closest proximity of the test charge, the exponential term becomes 1 and the Coulombic term becomes very large. So, let us say V c the Coulomb potential becomes very large. So, what do you infer? So within the closest proximity of the test charge the potential is clearly Coulombic in nature. So r tends to be very small. So, potential tends to be very large.

So, the potential is just very large and the nature of potential is just Coulombic in nature. When r increases, what happens? When r increases, this particular part the second part will start becoming smaller. So V c will decrease and exponential will also decrease when r increases. So how does it decrease? Let us say for example, when r is taken as lambda D. When r is equal to lambda D. So the potential is phi of r.

So if it is just Coulombic potential, we can simply say that phi of r is 1 by 4 pi epsilon naught q by lambda D. I mean lambda D is a limit , is just a distance this is lambda D. So here everywhere here the potential is varying as 1 by r. So the potential is decaying as 1 by r here. When lambda D becomes I mean, this is the potential should just be 1 by 1 by q by 4 pi epsilon naught, 1 to 1 by r d the potential should be just that.

But due to the additional second term, so this second term was just 1 s when it was very close to the positive test charge. So here, when r becomes equal to lambda D it simply becomes 1 by e. So, the potential is just potential at lambda D into 1 by e. So, what you can say is that so when at the Debye's length, the potential suddenly drops to 1 by e th of the potential which should be present at lambda D. So, what happens even after that?

So even after that. So, the decay will be faster. So if you cross the limit of lambda D, the potential will become even more . Let us say so the potential is 2 lambda D let us say phi of r then now becomes let us say 1 by 4 pi epsilon naught into q by 2 lambda D into exponential minus 2. So, this is even a smaller potential than this. So what you observe here is that Debye's potential or. So, Debye's potential is just I mean the potential inside this Debye's sphere is not varying as 1 by r, but is varying 1 by r up to a certain limit and beyond this certain limit it is falling even more faster.

So, the potential should ideally be let us say should be like this and then now the potential is dropping suddenly like this. Now, let us see if we draw it more carefully.

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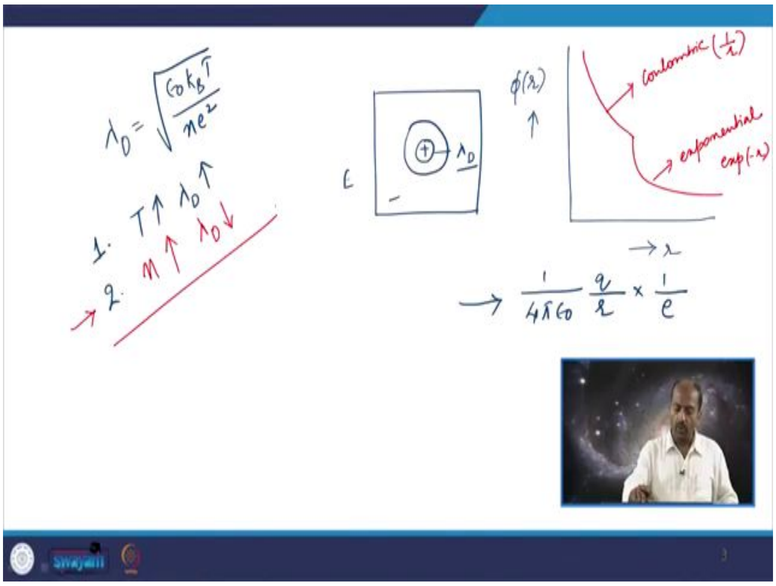

So now the potential is if you take r on the x-axis and phi of r on the y-axis, it will look something like this. So this is the 1 by r potential and suddenly it becomes exponential and decays even more faster. So, this decay is Coulombic and this decay is exponential. So this is 1 by r decay and this is exponential minus r decay. So you can see that the exponential decay is very fast in comparison to the Coulombic decay.

So what does it say? I mean it says that if you have plasma I mean this kind of supports our understanding of Debye’s shielding anyway. So how does it support? It supports in the sense that if you have plasma, then the influence of this positive test charge propagates by the means of creating a potential. Generally, in electromagnetics we have learned that the range of the Coulombic potential is infinite, it extends. I mean there is no stopping.

But if you have plasma, the potential will not stretch beyond a certain limit and this certain limit is called as the Debye’s length. So in terms of this potential, we have been able to realize that one up to this Debye’s limit, the potential is simply Coulombic. So, there is the natural order of things. So but when you cross this, when you when you are just approaching the Debye’s length itself, the potential suddenly drops to one by e th of the original value. So, this is the beauty of Debye’s shielding.

So, what we can conclude is that. So Debye’s shielding is the mechanism by which plasma shields out the electromagnetic effects of electric fields on the plasma. So, if you keep a test charge inside, the little field that is created by the positive test charge is shielded out for the rest of the plasma. And if you keep an electric field outside, it will do the same thing again. The electric the plasma will create a will create a mechanism by which the effects of this electric field are shielded out for the rest of the plasma. The depth up to which the electric field will penetrate into the plasma is called as the Debye's length.

Now, more importantly if you look at the expression of Debye's length, there are few very important conclusions that are to be drawn. So, if you look at the expression lambda D is the Debye's length. It is simply epsilon naught k B T by n e square. So, what it says is that , what we can realize from this is that so as the temperature increases, the Debye's length will increase.

So, if you have a plasma which is very hot in nature, it will have a larger Debye's length. That that is a very very important conclusion. We will try to understand how does it, I mean why does it vary like that. If you have more so, the second thing is if you have particles per unit volume as the number of particles per unit volume increase, the Debye's length will kind of decrease.

So, these two are the main otherwise, the rest of the things are just constants. So, Debye's length will not depend on the mass, Debye's length will just depend on the temperature and will depend on the number of particles charged particles per unit volume. So let us look at this, what does it say? I mean it says that as there are more number of particles in a given plasma, the Debye's length will be smaller. I mean, its kind of contradictory to our idea that when the particles are more, the Debye's length should be larger.

But, what it means is that as the number of particles are more, it takes I mean the available particles to neutralize a positive test charge are more in a given volume. So within a small volume itself, there are more number of electrons present. So it would not take much of the space for the shielding to happen. So in a denser plasma, the positive test charge or the electric field is shielded in a very minimal extent of space this is the basic understanding of Debye's length,.

So now, we will we have 1 more topic for discussion, which is called as the plasma frequency. So we have seen how the plasma tries to restore or maintain electrical neutrality. So, plasma oscillations are the means by which plasma maintains electrical neutrality.

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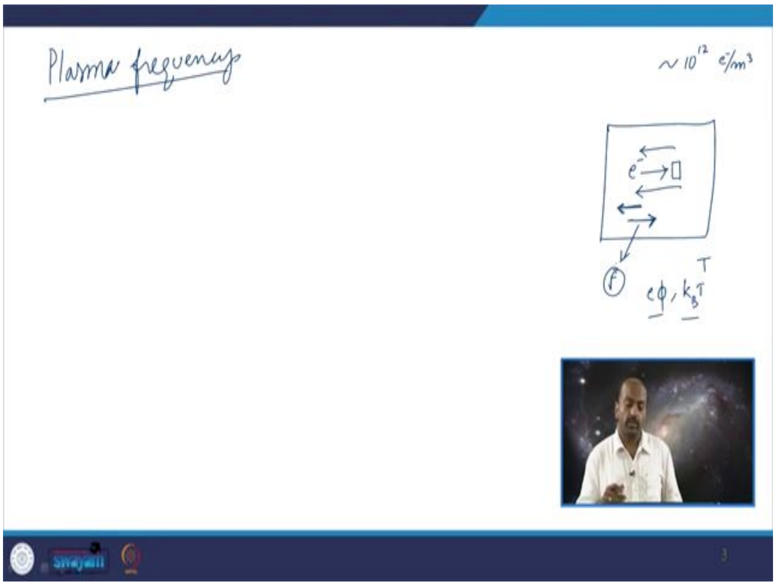

So let us see how it does it, plasma frequency. So plasma frequency is a very fundamentally very important parameter. So plasma frequency is let us say for example, when you look at the ionosphere let us say ionosphere has nearly 10 to the power of 12 electrons per unit volume.

So what it means is that. So due to the number of charged particles present, there is a frequency associated with the plasma. So the what is the idea of frequency is that let us say if you have plasma and if you displace an electron away from its position and create a charge density somewhere, this charge density will try to create an electric field and such that the displaced electron has to be restored to its original position.

So this is the idea of maintaining equilibrium. So at any given temperature now, very important thing is, at any given temperature there is always a an amount of thermal energy that is associated with the particle. So particle inherently has some energy. So because of which it is moving around. So now, if the electric field is making the particle to go in a direction. In addition to the electric field, the potential energy created by the electric field, the particle also has some energy by the virtue of its temperature. So, these 2 things are there with the electron.

Now what happens is, the electron of course, will try to go back to its original position, but in the process it will overshoot the position because of the inertia the particles the particle has some mass, so it has some inertia. So it will always overshoot the position because of its velocity,. When its overshoots again, there is a restoring electric field that is set up and this restoring field will just try to bring the electron back to its position.

So, this to and fro movement will create an oscillation. We will be interested to find out what is the frequency of these oscillations and this frequency of the oscillation is a characteristic of the plasma itself. So this frequency will be dependent on the characteristic on the number of particles that are present in the plasma itself. So every plasma is characterized by its frequency. So in this example what we will try to derive the plasma frequency,.

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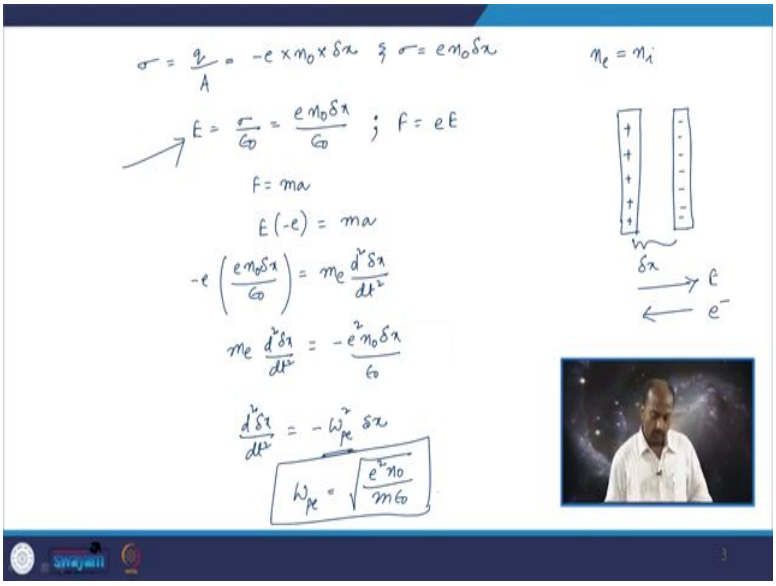

Plasma frequency

$$\omega_{pe} = \sqrt{\frac{e^2 n_0}{m\varepsilon_0}}$$

So let us consider an example let us say that there are equal number of electrons and ions in the plasma. Let us say by some mechanism, we have been able to displace all the electrons away by a small displacement of delta x. So I have kept I have been able to by some mechanism I have been able to displace all the electrons onto a slab by a distance separation delta x. So naturally, all the positive charges are separated from the negative charge , negative charge accumulation by this small distance delta x.

Now this arrangement does look like a simple parallel plate capacitor,. So what we have to do is, now we have to find out what will be the frequency associated with the restoring forces in this particular picture. Now, let us say if there is a slab on which all the electrons are accumulated we can write the charge density of this slab as q by a which is minus e times the number of particles per unit volume times delta x.

So this is the number of particles per unit volume. So you are getting sigma is the surface charge density on this plate. So q by a q is the charge, a is area. So if this is the surface charge density, the resulting arrangement leading to the formation of an electric field is simply sigma by epsilon naught which is equals to e n naught delta x divided by epsilon naught. So, this is for the electrons and for the ions it is e n naught delta x.

So thedirection of electric field is in this direction and the movement of electrons these e and the movement of electrons is in this direction,. Now we can use the Newton's second law of motion let us say. So what is it? f is equals to m a. If there is an electric field, the amount of force that is experienced by the electron is f is equals to e q.

So now that means, that e times minus e is equals to m a. Minus e times e n naught delta x by epsilon naught is equals to m e d square delta x by d t square which means m e. So d square delta x by d t square. So this is m a. This is d square delta x by d t square is the acceleration times mass is equals the force which is electric field times the charge. We have already got the electric field here. So is equals to minus e square n naught delta x divided by epsilon naught.

So this looks like a simple harmonic motion, we can write d square delta x by d t square is equals to minus omega p e square into delta x. So this is angular frequency. So that means, that omega p e is equals to e square n naught by m epsilon naught. So if the particles are displaced and if the particles are executing something called as a simple harmonic motion, the angular frequency of these motions should be of this order.

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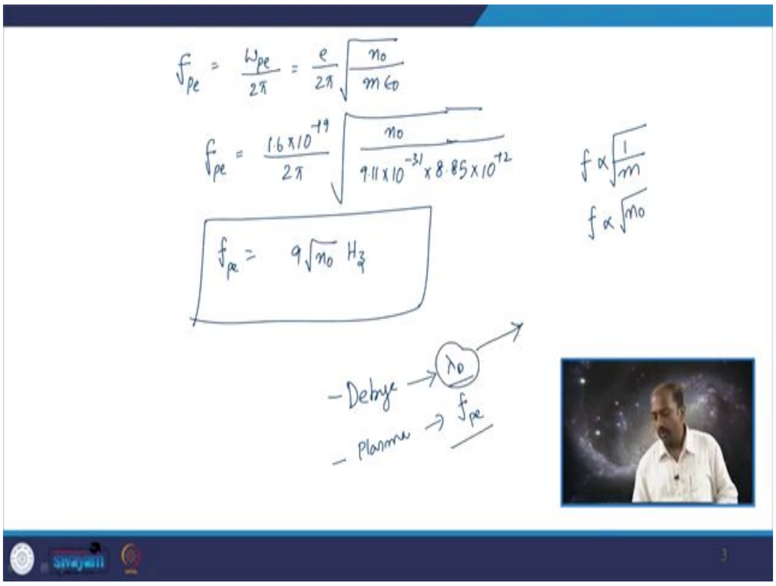

Plasma frequency for electrons

$$f_{pe} = 9\sqrt{n_0} \qquad \text{Hz}$$

So let us say; so this now if you want to calculate the frequency. So the frequency of electron plasma, plasma frequency is simply this which is e by 2 phi into square root of n naught by m epsilon naught. So, f pe is 1.6 10 to the power of minus 19 divided by 2 pi times, n naught can be anything. Times 9.11 10 to the power of minus 31 times 8.85 10 to the power of minus 12 which will reduce to 9 times square root of n naught Hertz. So this is the electron plasma this frequency is called as the electron plasma frequency. So, what are the features of this plasma frequency?

Plasma frequency depends on the mass and plasma frequency depends on the charge per unit volume. So plasma frequency I mean if the particles are more denser the plasma frequency is going to be smaller. Now again here, we always say the electron plasma frequency, but the ions are masses they cannot move. So its the same thing that we say again. So this is something about the Debye's shielding and plasma frequency.

So now, Debye's shielding has given us a length dimension lambda D and the plasma frequency has given a frequency term which we call as f pe.

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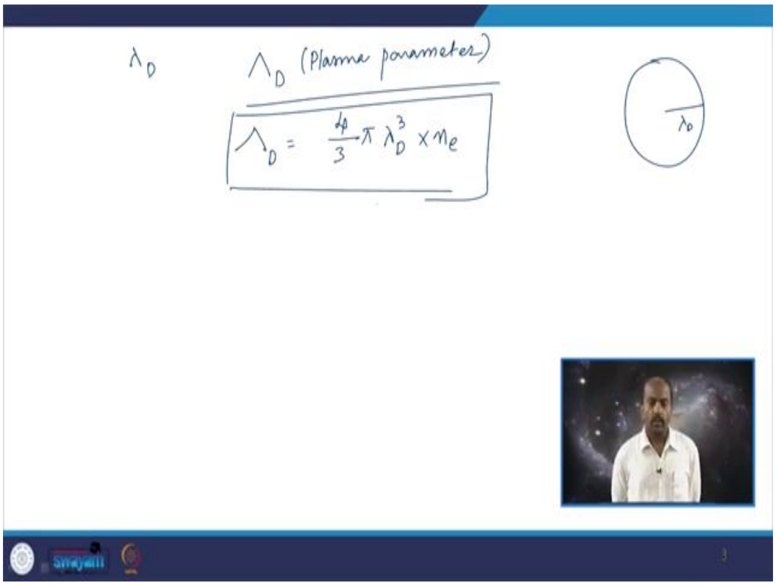

Plasma parameter

$$\Lambda_D = \frac{4}{3}\pi\lambda_D^3 n_e$$

So in addition to this, so we can also define using lambda D as the length dimension, we can also define a parameter called as capital lambda D which is called as the plasma parameter. What is the importance of this plasma parameter? So, if you have a sphere of radius lambda D the volume of that particular sphere is going to be 4 by 3 pi lambda D cube. So this is the volume.

So the number of particles in this spherical volume of radius equal to Debye's length is going to be the plasma parameter. So n is the number of particles per unit volume you are multiplying with the volume itself. So which will give you the number of particles which are present in a sphere which has the radius equal to Debye's length. So, plasma parameter is a very important characteristic of the plasma.

So depending on the plasma parameter, you can say or quantify what type of plasma it is? What could be the effects of this plasma? And so on,. So now like I said before, so every plasma differs greatly in comparison to any other gas. So plasma is of course, a gas but with charged particles. So, the charged particles are enough in number so that we can take into account the electromagnetic effects,. So this is going to be the conclusion about our discussions on the introductory plasma physics.

So, these topics will be very much useful for you to appreciate the various aspects of ionosphere, how the ionosphere responds to various types of solar energy input and how the ionosphere is coupled because ionosphere is charged entity, how the ionosphere is coupled with the magnetosphere of the earth and the magnetic field of d that is created by the sun.