Introduction to Atmospheric and Space Sciences
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Lecture - 49
Debye's Shielding and Debye's Potential

Hello, students. So, today's class we will continue our discussion about the Debye's Shielding. So, we have constructed two thought experiments to understand the idea of shielding in plasma. So, these two experiments have made us realize that, plasma acts to prevent the external electric or magnetic fields; that means, the Debye's shielding is the fundamental let us say the length characteristic of the plasma.

Let us say if you keep a test charge inside the plasma; plasma will behave such that it will create a cloud of oppositely charged particles around this positive charge, such that only up to certain extent the effect of this positively charged particle the external particle can be felt and beyond which the particles or the charges will not be aware of the existence of the charge.

So, this is fundamental idea of plasmas collective behavior can be easily understood by introducing the Debye's shielding let us say. So, Debye's shielding.

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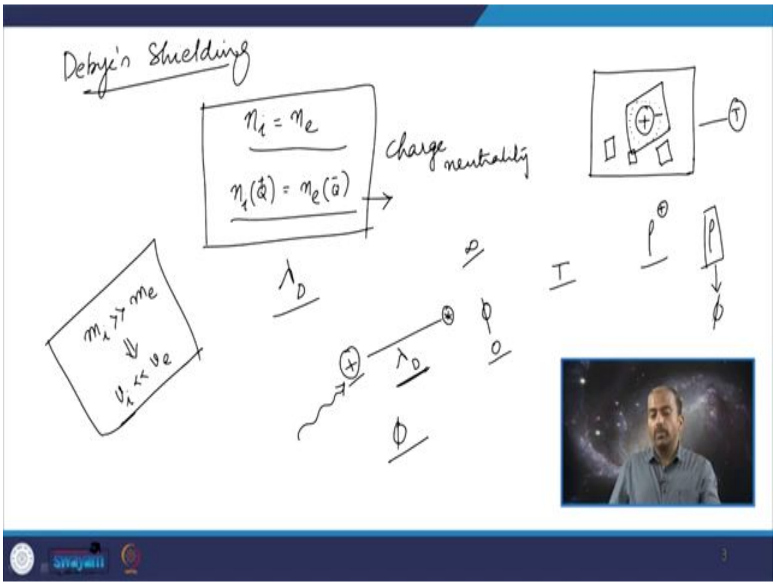

So, now what we will do is we will try to understand. So, in our experiment we have just taken plasma enclosure and we have kept a positive charge.

So, what happens is so there will be the accumulation of negatively charged particles around this positive charge and they will try to overcome or they will try to neutralize the positive charge. Now, in this picture the most important assumption that we have made is that plasma is neutral. So, plasma is overall the plasma is neutral; that means, the number of ions is equals to the number of electrons this may not guarantee neutrality because there is a possibility that the ions may be doubly ionized or triply ionized.

So, equating just the number of charged particles will not result into charge neutrality. So, charge neutrality or that means, the total effective charge let us say the total ions containing let us say, a positive charge and the total electrons containing a negative charge. So, this will give us charge neutrality.

So, the plasma is in a condition that the total number of particles of oppositely charged is equal and also the total magnitude of positive and negative charge is equal; that means, plasma overall is neutral. So, you may not realize charge neutrality in small pockets within the plasma. So, there is a definite possibility that within this pocket of plasma the there may be more number of positive charges or there more number of negative charges.

So, when there is more number of positive charges this will build up into charge density some let us say, charge density let us say charge density corresponding to positive charges, but if you consider the overall plasma as it is the total charge density is 0. So, total number of charges neutralize each other. So, there is no charge. So, as a result the plasma is electrically neutral entity.

But within the small pocket there is a definite possibility that there may be some charge density that exist within this plasma and this charge density may lead to many interesting things. But anyhow, so for this discussion we are trying to derive a mathematical relation for Debye's shielding. Debye's shielding is the length parameter is generally given as lambda D.

So, this should be basically in the in the dimensions of length. So, Debye's shielding fundamentally is the length dimension up to which let us say, if you keep a positive charge here. So, this Debye's length is the length dimension up to which the influence of this positive charge can be felt. So, if you keep anything here this is the point up to which this the influence of this positive charge can be felt this is kind of opposite.

Or this is kind of contradiction when you put this in comparison with the standard description that we study in electrodynamics where, wherever you have a positive charge this influence can be felt up to infinity. So, there is nothing which will hinder, but in plasma what will happen is this the length dimensional lambda D up to which the charges influence can be felt.

So, what do we want to understand from this. So, we have to derive an expression for lambda D let us say how does lambda D look like or. So, more precisely we want to understand how the potential of this positive charge will be inside this plasma. So, we want to understand what will be the nature of potential within this lambda D and what will be the nature of phi potential outside this lambda D.

So, what we have realized through our thought experiment is that potential will be felt I mean there will be an electric field due to this positive charge within lambda D of course, but beyond this lambda D the electric potential will be 0; that means, the total negative charges that surround is this positive charge will electrically neutralize this positive charge although they the particles may not come and stick to this charge, but they will behave so as to neutralize the effect of this positive charge.

Now, one more important thing that we have understood in the last discussion is that the mass of ion is much greater than the mass of electron. So, electron is a very small particle and as a result as a consequence what is expected is the velocity of the ion will be much smaller than the velocity of electron. So, this has to be combined with these facts. So, the basic idea is very simple the idea is you have a neutral plasma; that means, the total number of charges is equal.

So, there is no charge density inside the plasma. So, when there is no charge density, there is no resulting the electric field, but when you keep a positive charge inside due to the random thermal motions of the particles. So, this plasma is definitely at a particular temperature. So, there is characteristic value of temperature at which the electrons and ions are existing. So, they have some thermal velocity. So, they will move and because of the mass of ion being very high when compared to the mass of electron the particles velocity will be comparably very large for the electrons in comparison to the ions.

So, the point is there is a possibility now that in some space within this plasma many number of electrons are present. I mean, there is a large concentration of negative charges in a specific region. When there is a large concentration of negative charges in a specific region this leads definitely put to a charge density and this charge density may lead us to find out the electric potential.

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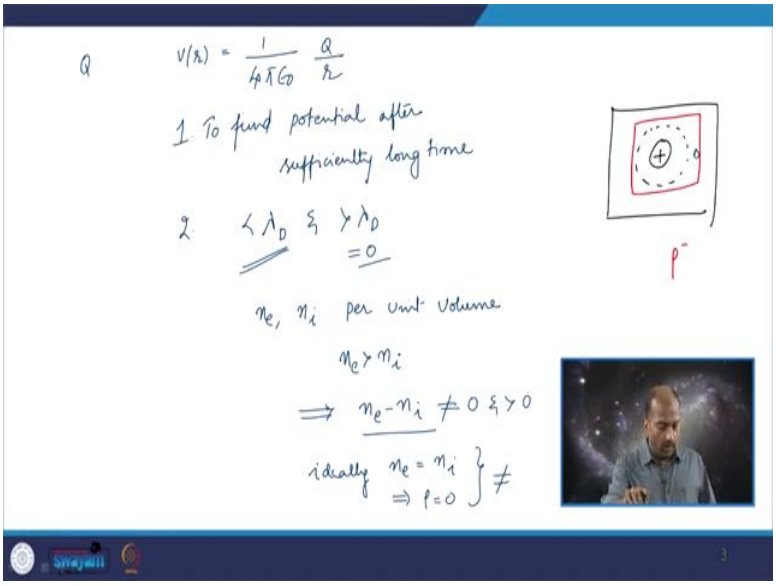

Now if so, immediately when you keep this when you take plasma at the immediate instance when you keep the positive charge the potential is 0 for a moment, but with a very long time if you give enough amount of time for the negative charges to accumulate or form a cloud around the positive charge.

Then within this small space what you will realize is that there is a nonzero charge density; which is basically negative in nature . Another most important idea is why do not these particles these electrons just come just stick to the positive charge and then just neutralize it. What is preventing them is that the electrons have a thermal velocity and because of the random thermal motions.

So, electrons will electron will not be able to stick to it rather electron will always try to stick to it and overshoot the positive charge and we will go and will again be bound by the Coulombic attraction force and will again comeback to the cloud. So, now, to begin with let us say if you have a positive test charge Q the potential to begin with will simply be potential in this small region will simply be v of r is equals to 1 by 2 pi epsilon naught Q by r.

So, what does this mean this is just a Coulombic potential. So, with a distance r the potential will look something like this; now our objective what is our objective. So, our objective is to find is to find potential after sufficiently long time. So, we want to find out potential after sufficiently long amount of time. Where do we want to find the potential, we want to find potential below lambda D and above lambda D both the conditions.

So, ideally our understanding says that there should be some potential some nonzero potential below lambda D and above lambda D the potential should be 0 because the charges outside the Debye's length or Debye's sphere will not be able to feel any existence of this charge; that means, that they would not be able to realize there is a charge here.

That means there is no potential due to this charge at this point outside now let us say. So, after having made these assumptions we will say that let us say in the vicinity of charge if we take n e as the number of electrons and n i as the number of ions, per unit volume. Let us say these are charge concentrations per unit let us say per unit volume.

So, in the vicinity of this charge after sufficient amount of time if you allow the cloud to form and after sufficient amount of time it is very easy to understand why n e will be very large when compared to n i this leads to effective charge. So, this will lead to effective charge as n e minus n i. So, ideally everywhere in plasma before you keep a test charge n e is equals to n i; that means, net charge density is 0.

But in this case its not. So, n e minus n i becomes 0 and greater than let us say greater than 0 becomes greater than 0. So, in that case, so, since there is a net charge available.

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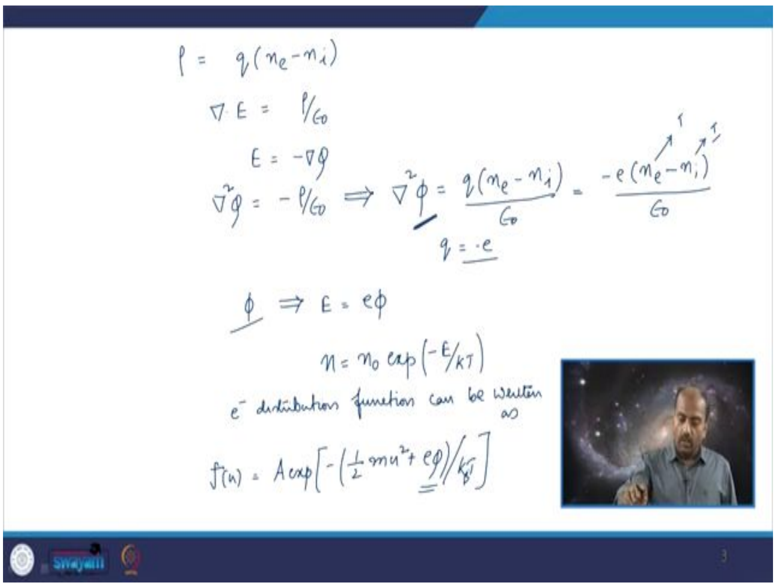

We can write the charge density rho as q times n e minus n i. We know from the electromagnetic theory the del dot e is equals to rho by epsilon naught.

Electron distribution function

$$f(u) = A \exp \left[\frac{-(\frac{1}{2}mu^2 + e\Phi)}{k_B T}\right]$$

So, which we know that e is equals to minus del phi. So, we can write del square phi as minus rho by epsilon naught; that means, del square phi is equals to q times n e minus n i divided by epsilon naught. Or in simple terms for the nature of this charge is going to be negative q is equals to minus e, we can write this as minus e times n e minus n i divided by epsilon naught.

Now, if you have a potential phi and if the particle experience this potential the energy will be e phi, e is the particles charge and phi is the potential. So, the where is the idea the idea is you have a nonzero charge density; this nonzero charge density will lead to the formation of a potential, and we are trying to find out the form of this potential as a function of distance.

So, now you do not see a distance here even let us say del square phi I mean del square if you open del square into r theta phi. If it is a spherically symmetric potential then you can probably get the dependence of r in phi in the analytical expression of phi you can probably get the dependence of r. So, now, if you want to do that you want to find out how n e is distributed with respect to temperature.

And n i is distributed with respect to temperature because n e is the number of electrons per unit volume and n i is the number of ions per unit volume. So, there must be some distribution which they will follow. So, that we can use it in our expression. Now let us say, assuming that the plasma is in thermo dynamical equilibrium and the charged particles are distributed according to Maxwell Boltzmann distribution.

So, let us say we say that n is equals to; according to the Boltzmann's law we write exponential minus E by K T; that means, at larger energies lesser number of particles are present under smaller energies more number of particles are present. So,the electron distribution function can be written as let us say where this is the distribution function.

So, this is the kinetic energy by the virtue of its velocity and e phi is the energy that is attributed to the electron by the virtue of this potential divided by K T or K B T. Now, since ions are far away from the electrons I mean near the vicinity of this positive test charge ions are very far and the electrons are very nearer to this test charge it is very reasonable situation.

Since the ions are very far from the electrons and they do not contribute to this charge within the space charge. So, the ions can be treated in the background.

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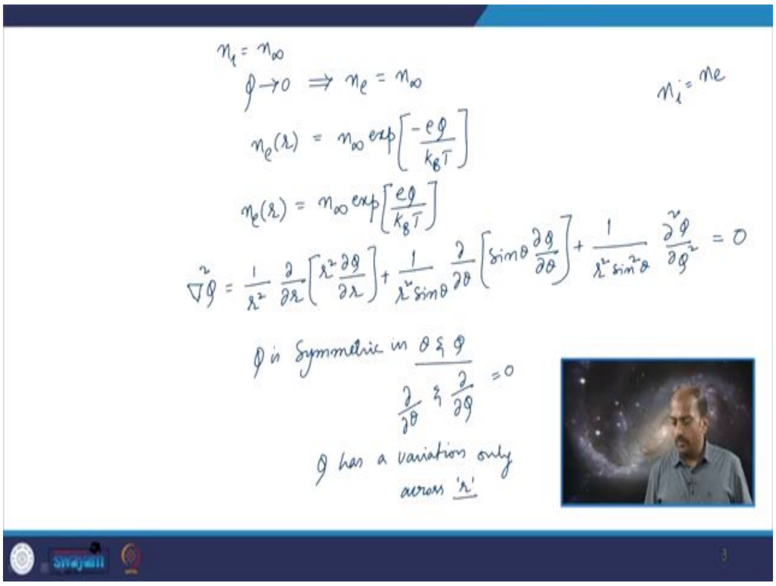

So, we write that n of i is equals to n infinity and when let us say when phi tends to 0 when there is no potential. Let us say for example, in that case n of number of electrons is simply equal to the number of ions.

So, in those cases n e of r can be written as n infinity times exponential minus e phi by K B T. Now since electron is neutrally charged particle we write n e of r is n infinity exponential e phi by K B T

So, del square phi can be written as the Laplacian can be written as 1 by r square dou by dou r of r square dou phi by dou r plus 1 by r square sin (theta) dou by dou theta of sin(theta) dou phi by dou theta plus 1 by r square sin square theta into dou square phi by dou phi square.

So, when n i is equals to when n e I can simply write del square phi is simply 0 in those cases. Let us say phi is symmetric in theta and phi, what does it mean it means that any derivative with respect to theta and any derivative with respect to phi can simply be made 0. So, phi has a variation let us say this again an assumption has a variation only across r.

So, in that case let us say. So, dependence only along the r.

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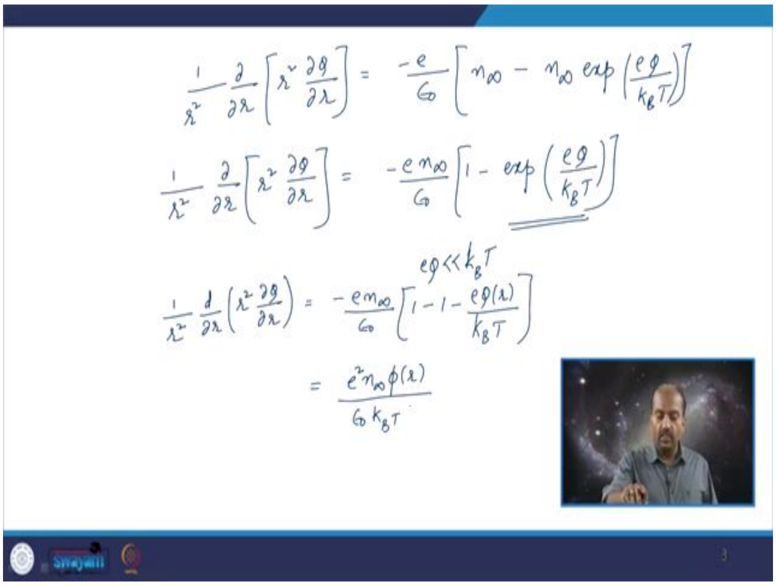

So, we will retain the r part of the derivative 1 by r square dou by dou r of r square dou phi by dou r is equals to minus e by epsilon naught times n infinity n i is n infinity minus n infinity exponential of e phi by K B T. Or 1 by r square dou by dou r of r square dou phi by dou r which is equals to minus e n infinity by epsilon naught into 1 minus exponential e phi by K B T.

So, we are trying to derive a second order differential equation for phi in terms of r. So, this is exponential x. So, which we can expand as per the Taylor series provided e phi is much less than K B T. So, in that case, I will simply write 1 by r square dou by dou r of r square dou phi by dou r is equals to minus e n infinity by epsilon naught times 1 minus 1 minus e phi of r now phi is just a function of r by K B T.

So, this can be written as e square n infinity phi of r divided by epsilon naught K B T. Now, let us just simply write the second order differential equation which looks like.

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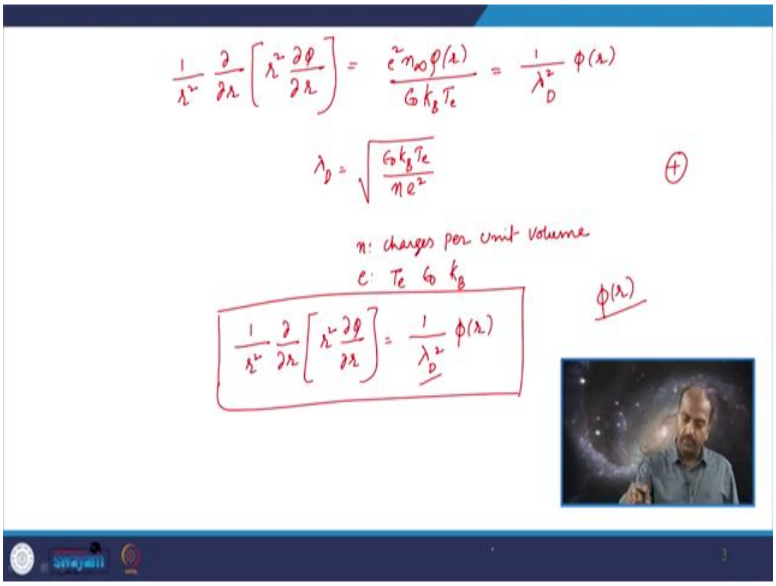

Debye's length

$$\lambda_D = \sqrt{\frac{\varepsilon_0\, k_B T_e}{ne^2}}$$

1 by r square dou by dou r of r square dou phi by dou r is equals to e square n infinity phi of r divided by epsilon naught K B T. Since, we are talking about electrons we can just say K B T e. Now this can be rewritten by identifying lambda D a constant times phi of r, where lambda D is simply epsilon naught K B T e divided by n e square. So, here n is charges per unit volume, e is the charge of electron, T e is the electron temperature, epsilon naught is the permittivity of free space, K B is the Boltzmann's constant. So, the differential equation is still intact I mean the differential equation is just now written in terms of a coefficient or in terms of a constant which you call as lambda D. Otherwise, we still have not solved the differential equation 1 by lambda D square times phi of r.

Now, if you solve this differential equation what will you get we will get phi of r. what is phi of r, phi of r is the potential that has been created by the positive test charge inside the plasma. So, will the phi of r depend on lambda D? Yes it is, it will depend it will certainly depend. So, now, this is a differential equation which you can solved by many different methods.

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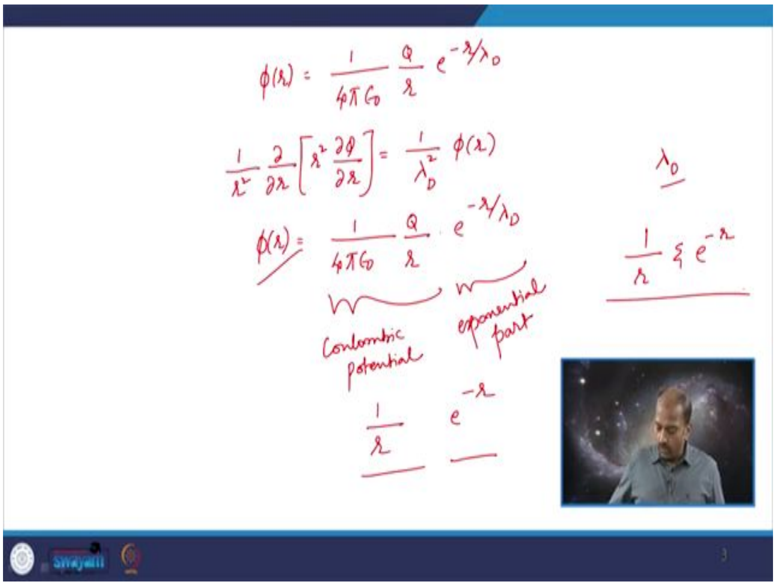

If you solve it let us say the differential equation solution will look something like this phi of r is equals to 1 by 4 pi epsilon naught q by r into e to the power of minus r by lambda D. So, the solution let us see you can substitute this into the differential equation which is 1 by r square dou by dou r of r square dou phi by dou r is equals to 1 by lambda D square phi of r.

Debye's potential

$$\Phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\ e^{-\frac{-r}{\lambda_D}}$$

Now, if you see. So, the potential is now is made up of two different parts 1 by 4 pi epsilon naught q by r into e to the power of minus r by lambda D. This is a coulombic potential created within by the test charge and this is an exponential part. Now, this varies this potential falls at 1 by r rate.

And this potential falls at exponential of minus r. So, you know which one is faster. Now, let us say so what is important about this potential. So, this is a very involved discussion how to identify the role of this potential within this plasma and where does the limiting condition of lambda D comes to satisfy that the plasmas ability to shield the electric field or shield the external charge.

We will discuss the detailed discussion about this limiting condition or the variation of the potential as a function of distance, Coulombic let us say as a function of distance as 1 by r and exponential minus r. So, this is where I will stop today. So, we will continue this discussion in the next class where we will try to understand the consequence of having the potential in a uniform like this.

Thank you.