Introduction to Atmospheric and Space Sciences

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Lecture – 46
Chapman's Alpha Layer

Hello dear students. So, today we will continue our discussions on Chapman's theory of layer production. We have mathematically derived the production rate as a function of altitude and as a function of solar zenith angle. So, the idea was to mathematically derive an expression which will give the shape of what is called as a Chapman production layer which will vary with respect to solar zenith angle and various other parameters.

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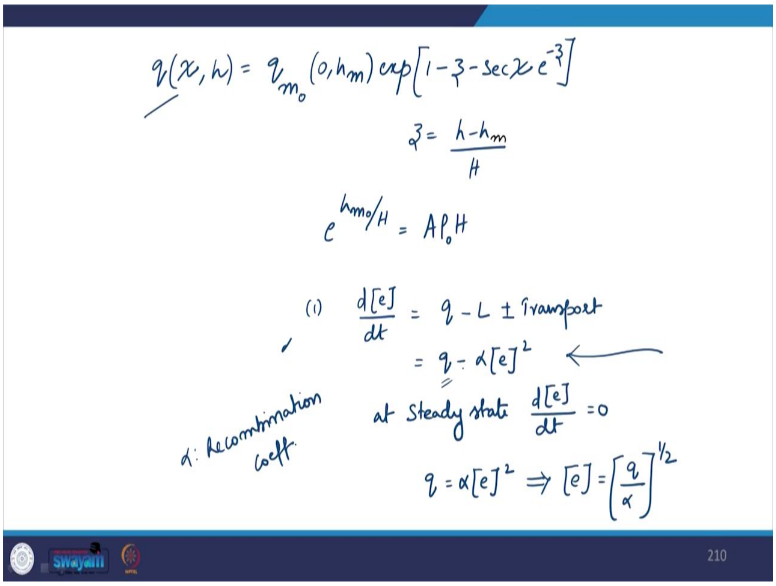

So, we have stopped deriving q which is the production rate as a function of the solar zenith angle and the height as q m let us say 0 of 0 h m into exponential 1 minus z minus secant chi e to the power of minus z. Where the height is now given as z, where z is the height above the peak production. So, the height is normalized with respect to the scale height which is above the peak production. Let us say h naught or for more generalized way we can say h m; h m is the peak production height, h m divided by H, capital H is the scale height.

So, we can simply say that e to the power of h m 0 by capital H is A naught H; where h m is the height of peak production. So, from using the simple form of continuity equation let us say we write the rate of change of electron density with respect to time. We know that it will be production minus loss plus minus any transport. This is the general form of continuity equation.

If we implement the charge neutrality you can write the production minus alpha times e square, so where q is the rate of production that means, the rate at which electrons are produced at any given point. The rate at which the electrons density will change with respect to time let us say in a cubical volume will be the rate of production of electrons in that cubical volume, the rate of product where the rate of loss of electrons from that volume and any amount of electrons that come in or go out due to the transport.

So, let us say at steady state when things do not change with respect to time. At steady state, we write the rate of change of electron density with respect to time as 0; that means, the same things are not changing with respect to time. So, we can write that q is equals to alpha times e square. It is a familiar form of continuity equation that we have used or we will be using in our discussions of ionosphere.

So, t the electron density we can simply be written as q by alpha raise to the power of half. So, what is alpha anyway? alpha is the recombination coefficient. Now, alpha can be different for different types of recombination electron may combine with a molecular ion, leading into dissociation where electron is eventually lost. That means, electron has been removed from the ionosphere. Electron can combine with them with an atomic ion leading to the formation of a neutral species where again electron is lost.

So, depending on the species which contributes for the recombination; the rate at which this recombination may proceed or the coefficient the corresponding coefficient will be different.

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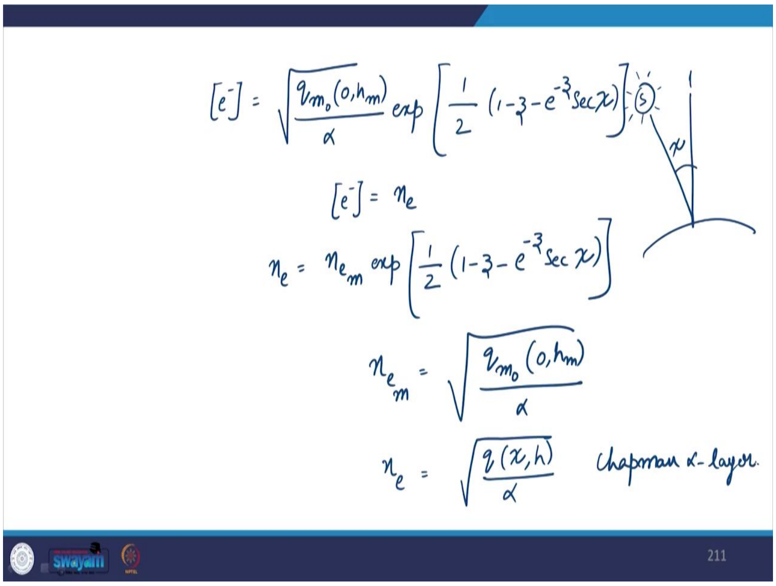

Electron density in Chapman's alpha layer

$$n_e = n_{e_m} \exp\left[\frac{1}{2}(1 - z - \sec\chi\, e^{-z})\right]$$

Where $n_{e_m} = \sqrt{\frac{q_{m_0}(0,h_m)}{\alpha}}$ and α = recombination rate.

So, if you use this equation into the production equation, we will be able to write the electron density now we have q by alpha as q m 0 at normal incidence at the peak production height divided by alpha times exponential half 1 minus z minus e to the power of minus z. I have just substituted the formula that we have derived for the production rate.

So, let us say the electron density is now denoted with let us say n e. So, we can write n e as n e m; the peak, the maximum production the maximum electron density times exponential half times 1 minus z minus e to the power of minus z into secant chi. Where the n e m is simply q m 0, h m divided by alpha. q m 0 is the peak production at normal incidence. So, we have gotten rid of chi, chi is 0 this term. So, chi is the angle at which the sun is at a given time.

So, this is the 0 position and this is the sun let say. So, the angle at which you see the sun away from the afternoon is the solar zenith angle. We have already discussed this. So, n e m, n e is simply q the rate of production at any angle or at any solar zenith angle and at any given height divided by alpha. So, this is called as Chapman alpha layer.

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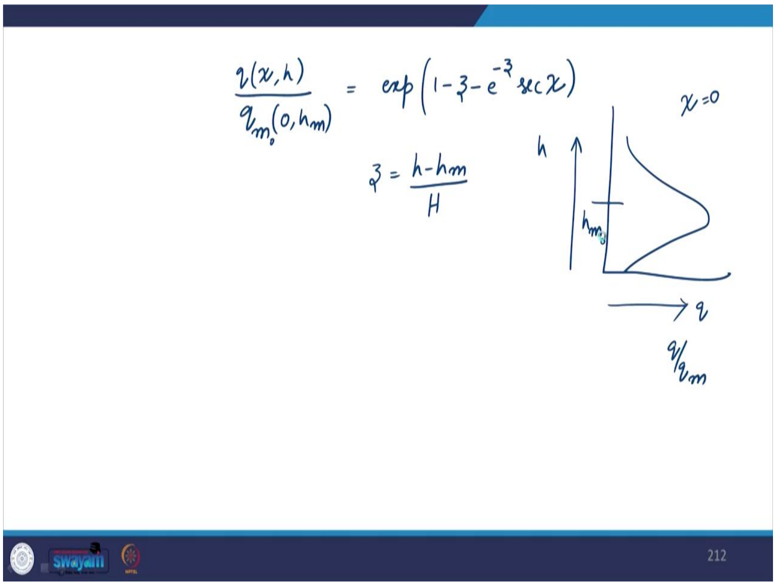

Or we can write the normalized production, the production at any given angle and at any given height with respect to production at 0 angle and the peak production height. As exponential 1 minus z minus e to the power of minus z secant chi.

Where you should always remember z is a normalized height with respect to the peak production height. So, the idea is you are trying you have now derived a relation for this, this layer, this is the form of the layer. So, we have already discussed the various characteristics pertaining to this let us say q or you can also say q by q m and this is h. Now the height at which this peak is appearing is the h m.

Now, if you fix the angle chi is equals to 0 then you will have to call this h m as h m 0 that is it. However, what is the basic idea? I mean let us say before we lose track of what is what is going on? What we have done so far. Let us give you a quick recap of the various things that we have derived.

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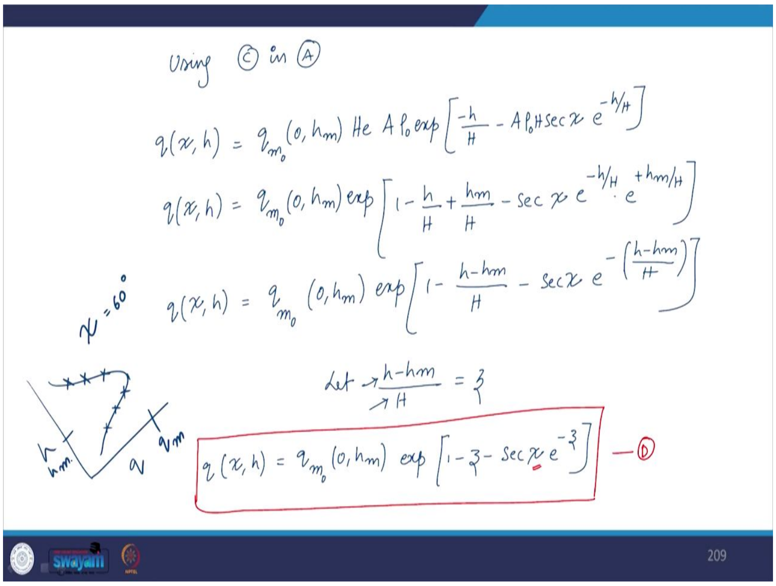

So, this is what we have derived. We have derived the rate of production at any angle chi and at any height h. That means, so, what we have our intention was to be able to if this is q on your x axis and if this is h on your y axis. Our idea was to be able to derive a mathematical relation to calculate the value of q at any given height and for any given angle.

Now, the most important thing is the curve that I have drawn is for a given value of , let us say chi is 60 degrees; that means, the shape of this curve will remain the same for different values of chi, but let us say the scale over which this curve will span or the peak production the height can be different. Now here the peak production is this one q m, the maximum production is happening here. And where is this maximum production happening? It is happening at a height h m.

Now we have derived a relation for this shape of the production layer. So, this is called as a production layer. Why is it in this particular shape? The answer is simply because, the intensity of the radiation that comes from the top decays exponentially as it travels towards the surface and at the same time the number of available species to initiate ionization also decays exponentially from the bottom. So, these two exponential rates balanced each other or become equal to each other at a particular point. Where, maximum amount of energy is utilized by ionizing the maximum possible species that are existing at that level.

So, you call this height as the peak production height. And the production that happens at this height is called as the peak production. Now we have derived a relation which will accommodate not just the peak production rather any given height and any given production ,so we have done that. Now we have simply modified this expression, we have the generalized expression in that.

So, we have used the continuity equation in this famous form where our idea was to replace q with this Chapman production layer we done that and we have derived a simple rearranged relation to get the number density of electrons at any given time as a function of the height and as a function of the angle.

So, there are some important consequences which come out of this mathematical treatment. Which tell you so many important points about how ionosphere behaves or how the ionospheric electron density is dependent on certain variables during certain times and how it is not dependent on certain other variables. So, let us look at these consequences one by one it is a very important in the sense that the mathematical treatment is important, but the consequences are even more important.

So, the consequence number 1 is you always define so far. The treatment has been such that all the species whether atomic or molecular have been taken to be of the same cross section. So, the absorption cross section, the cross section over which a photon is absorbed or photon is kind of scattered is assumed to be the same for all the species which is ideally not true. So, different gases have different cross sections and they also decide what kind of rate the reaction given reaction should happen.

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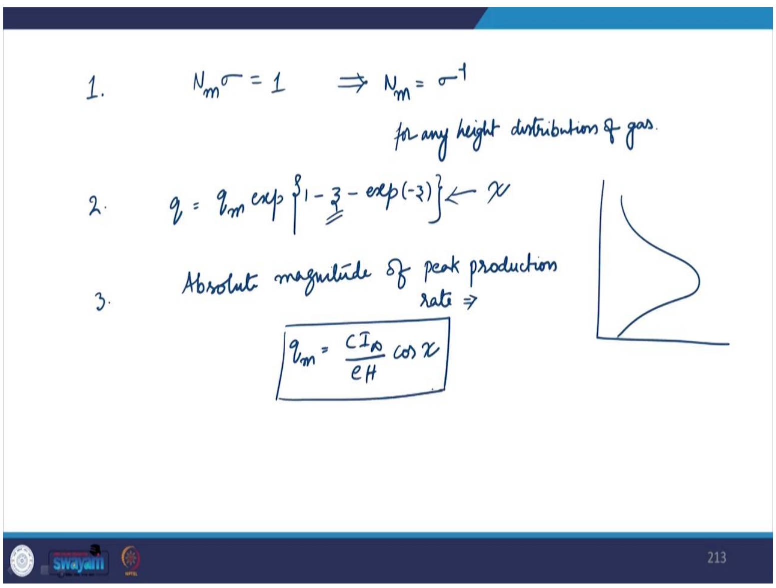

Absolute magnitude of peak production

$$q_m = \frac{CI_\infty}{H\,e} cos\chi$$

So, the consequence number 1; you should always remember that the peak production is always observed at a height where this product is equals to 1. So, peak production happens when the number of atoms or let us say number of atoms of molecules in a given cross sectional area becomes equal to the total cross section of the all contributing atoms and molecules. So, this is the basic idea.

Now the peak of production is at a height where the number of molecules in a unit column drawn from that height towards the sun is equals to sigma inverse. So, that means; which implies that N m is equal to sigma inverse. So, I have by mistake written 0 instead of one. So, it should be 1 becomes equal to 1. So, this statement is true, whatever may be the height distribution of gas? This is true for any height distribution of gas.

Consequence number 2; the rate of production q is written as q m in simple terms the same exponential 1 minus z minus exponential minus z where q m is the is the production peak production rate at a height and height is measured above the high above the peak production normalized with respect to the scale height.

So, the production layers appropriate to any set of parameters can be presented by one curve if heights are measured from the peak production height in the units of scale height and production rates are measured as a fraction of the total value at the particular peak. So, how will they look like? Now, what I am saying is the shape of the curve should remain the same for any kind of let say for any kind of angle or the shape the shape depends only on the value of z and that is indirectly on the scale height and is independent of the angle.

So, the basic shape is not dependent on the angle it is dependent. So, you see you do not see a chi dependence here right. So that means, that the shape of the production is dependent on the height, but nothing else and that to height normalized with respect to the scale height. So, let us say the absolute magnitude of the peak production per unit volume. So, what is it? The absolute magnitude of peak production rate is equal to q m is C I infinity divided by e capital H cos chi.

Now the absolute magnitude of peak production is dependent on the angle, but not the shape of the total production layer itself.

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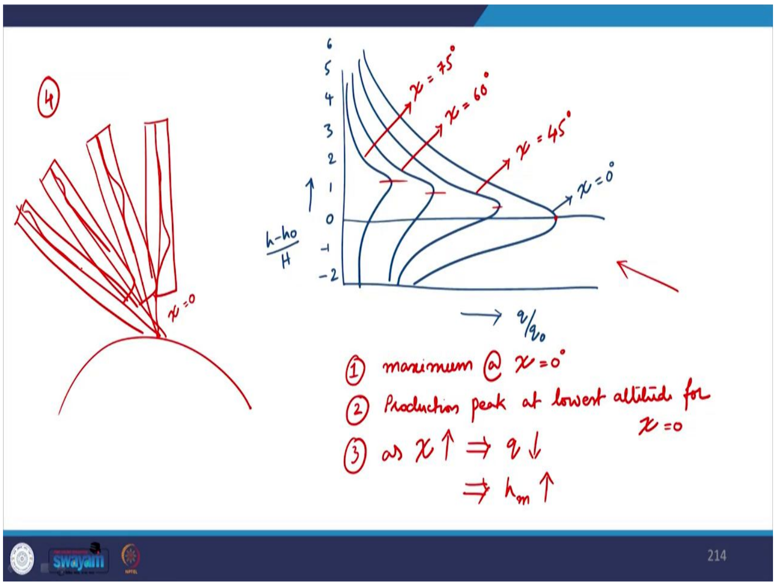

Now, if you draw a curve, let say how does it look like let us say we call this as four. So, take the q by q naught, what is q? q is the production at any given height. q naught is the production when there is a normal incidence. And you take on the y axis you take h minus h 0 by capital H.

So, you identify 0 here.So, this is the Chapman layer, now this is for an angle chi is equals to 0 degree and this is for an angle chi is equal to, let us say 45 degree, this is for chi is equals to 60 degree and this is for chi is equals to 75 degree. What do you see? This is the most important result of this Chapman's production layer, what do you see? You see, number 1; again within number 4 of the consequences we have few important observations that we need to make.

So, let us say 1, production is maximum at chi is equals to 0, chi is equals to 0 is the normal incidence. 2; production peak at lowest altitude for chi is equals to 0, what does it mean? The production peak, this is the production peak. Where is it lying? It is lying in the lowest altitudes if you see on the y axis you have the altitude. What does it mean?

So that means that; at the normal instance not only peak production is achieved, but at the same time the radiation travels to the deep or travels farther into the atmosphere towards the surface. As the angle is increased so, as chi increases as the angle at which is incident increases it goes up. What happens? one consequence is q will decrease. The second consequence is that the height of peak production let us say not h 0 rather goes up.

This is the most important consequence you see this. So, this height is going up, you see this trend which is going up. So, what does it mean? it means that as now you imagine the surface of the earth like this you draw a line which is an angle chi is equals to 0.

So, for any angle you see that the radiation is traveling more distance when it has to reach the surface; that means, the radiation is encountering more volume of the atmosphere before it gets completely attenuated. That means; so, if this is the column of atmosphere that this particular ray of light is encountering, because this is less area this is less distance that it has to travel it will travel farther with little attenuation and it will redeposit most of it is energy at a particular point.

But when the radiation the same radiation is travelling in a very oblique incidence so, it has to travel more atmospheric column. So, the radiation will keep getting attenuated and the peak production will keep appearing much higher in the altitude. So, this is the basic reason why the production happens at a much higher altitude when the angle of incidence or the solar zenith angle is greater than 0 degrees. So, this is one important conclusion from the Chapman's production layer. So, this is basically has to do with the amount of atmospheric column this ray of light encounters when it makes a oblique incidence.

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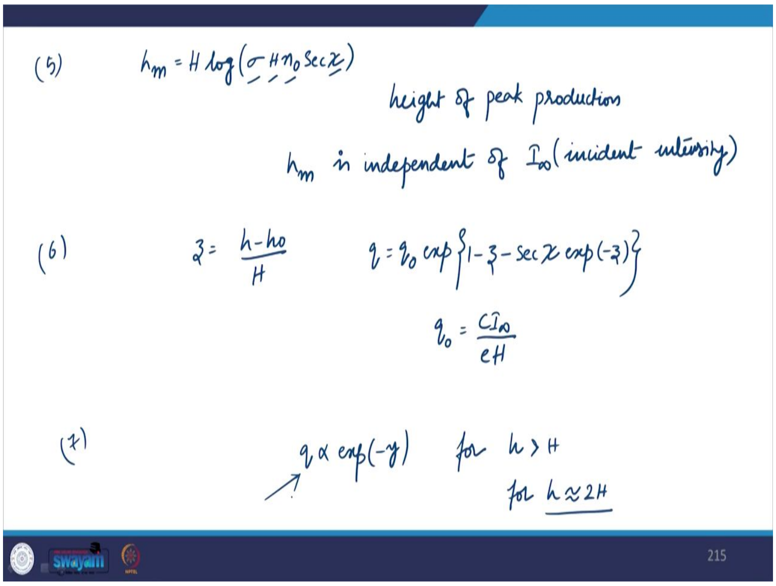

Now, conclusion number 5, let us say is the height of peak production h m is given by h log. So, we have to get h outside that equation n 0 secant chi. What is this? This is the height of peak production. So, the height of peak production depends on the cross section the scale height which is again scale height you can take individual scale height for different gases. The concentration n naught and the angle chi and is independent of the incident intensity.

So, this is the major conclusion the height of peak production is independent of I infinity or incident intensity. Next, let us say if the heights are measured like this z is equals to h minus h 0 by capital H. You can simply write q as q 0 times exponential 1 minus z minus secant chi times exponential minus z, where q naught is equals to C I infinity by e capital H.

So,this is the peak of magnitude of q for chi is equals to 0 q naught is the peak value for chi is equals to 0. So, for different-different angles this will be a deviation from this magnitude. So, we can write you know another important conclusion is we can write that q is proportional to exponential minus y for heights for h which is more than the scale heights or for heights which are of the order of two scale heights. We can simply write the rate of production is proportional to exponential minus y.

So, this is some discussion about the Chapman's theory of layer production. So, if we just recall where we started this discussion, the discussion was that ionosphere consists of four different layers D, E and F layer is again F1 and F2. So, it is a single layer in a crude sense that ionospheric density, electron density starts appearing somewhere it around 40 or 50 kilometers and it goes up till 350 or 400 kilometers.

So, throughout this region you see that the electron density peaks at the particular points. we call these inflection points or the region that is separated between two points as a particular layer of ionosphere and we have named them as D, E and F layer. So, we wanted to understand why this kind of layered phenomena appears in the ionosphere, why cannot it just be treated as a single continuum of electrons and ions from bottommost height to the uppermost height.

So, we realized that due to the opposite exponential decay of the intensity and the number density of the species we see something called as a Chapman layer. So, we derived the mathematical formulation for this Chapman layer. So, we did all this mathematical treatment by substituting and we also made some assumptions and ultimately we realized that the shape of a production layer. It should be of this kind and if this is the shape of the production layer.

We wanted to see what and all parameters does this shape depend on and what are the major important conclusions this layer the shape will tell us.

So, this was something about the Chapman's theory of layer production which is a very important topic for understanding ionosphere. So, we will stop here, we will have a discussion on something called as the F layer, how differently layers are formed whatever, what are the basic characteristics of each layer, what kind of chemistry goes into each layer, what kind of continuity equation you will be able to write in each layer.