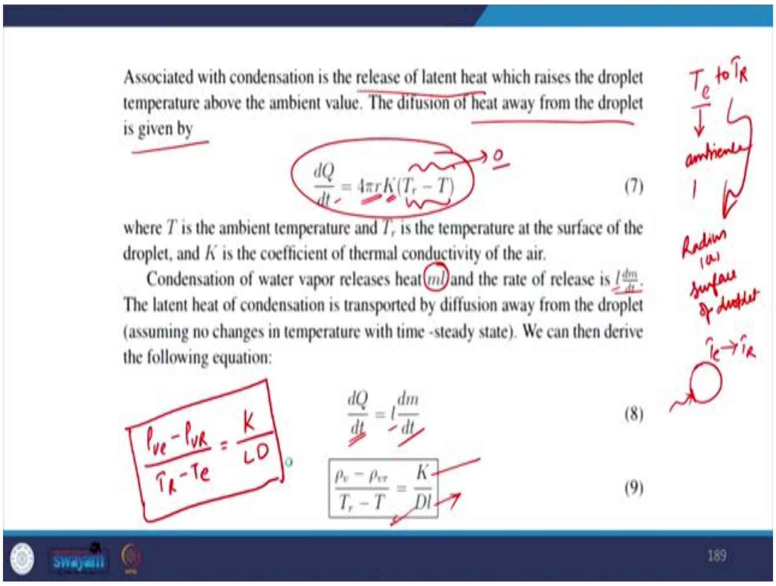
Introduction to Atmospheric and Space Sciences
Prof. M. V. Sunil Krishna
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 40
Radial Growth of Droplets by Diffusion-Continued

Hello, dear students. So, we will continue our discussion to derive the Radial Growth of a Droplet, subject to the conditions that are existing in the ambience. So, we will take the diffusion to be the main process by which the droplet is growing in size right. So, we have stopped at this point, where we have derived an expression for the density change with respect to the density of the ambience at saturation in terms of temperature changes latent heat and the specific gas constant.

So, if you say that the diffusion is the main process, let us say associated with the condensation is the relation with the release of latent heat which rises the droplets temperature above the ambient value.

(Refer Slide Time: 01:22)



So, in the integration while we were identifying the limits of integration, we have seen that the integration is to be carried out from a temperature T e to temperature T R, where T e is the temperature of ambience and T R is the temperature at the radius or surface of the droplet.

So, why is the temperature different between these two places?, what is the physical process that is happening? In order to understand this. When the condensation happens release of latent heat is of course, is there when the condensation happens latent heat is released and this heat is attributed to the droplet which rises the temperature from let us say T e to T R above the ambient value.

So, the diffusion of the heat away from the droplet so, once there is a temperature gradient, once you say that the temperature of the droplet or the surface of the droplet is larger there is; obviously, a reason for this temperature to go away or for the energy to conduct. So, how does the energy the gradient in the temperature compensate itself by conduction, I mean the heat is given out.

So, the diffusion of heat away from this droplet is given by this. So, this is a simple conduction equation, where the rate of change of temperature of course, the rate at which heat is given out with respect to time is of course, something to do with the radius or the size of the droplet, the coefficient of thermal conductivity and the difference of the temperature. So, T R is the temperature of the or the surface of the droplet capital T is the temperature of the ambient.

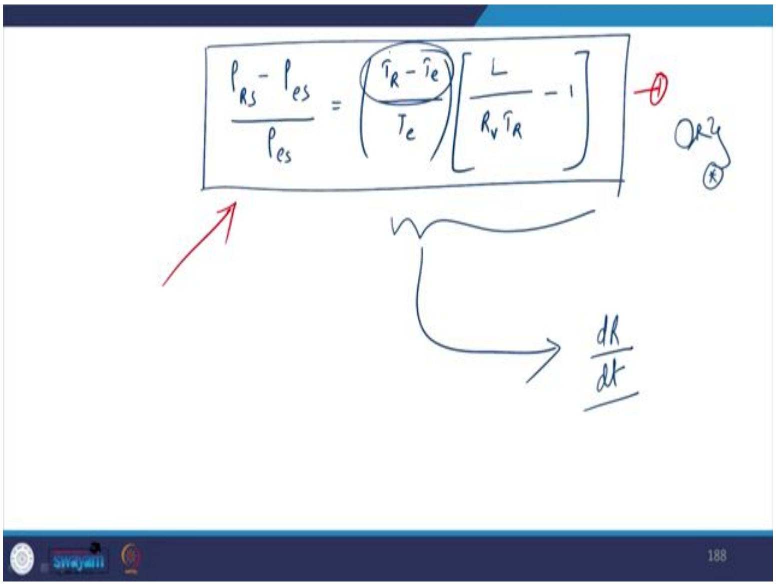
So, if this difference is 0 then there is no heat conducted of course, absolutely, but since there is a difference, why does the difference come in first place? Because it has to accommodate the latent heat that is released, when the condensation happens so, this is a simple conduction equation, where the T is the ambient temperature T R is the temperature at the surface of the droplet capital K is the coefficient of thermal conductivity.

So, the condensation of water vapor it gives out a heat ml , ml is the amount of heat and the rate of release is l dm by dt. The rate at which this is released l is; obviously, not a function of time. So, we keep it out of the differential. So we say that l dm by dt is the rate of change of release of the heat.

So, the latent heat of condensation is transported by diffusion away from the droplet. Assuming that the no changes in temperature with steady state, we can simply derive these two equations. So, this is of course, the rate of heat that is being conducted or being released from the droplet is l dm by dt

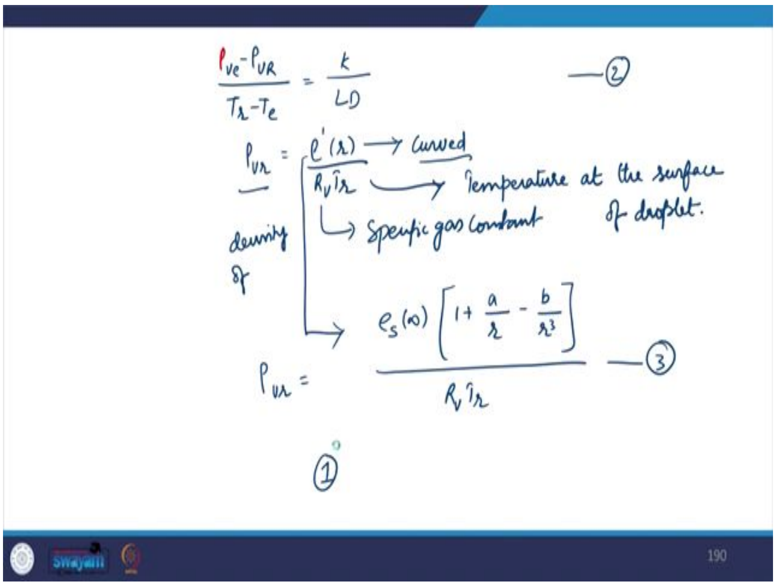
So, if you substitute this we can simply say that the density of the vapor minus the density of vapor at the radius by T R minus T e is the coefficient of thermal conductivity this is the coefficient of diffusion and this is the latent heat. So, in our terminology, we can write this as rho V e minus rho v R divided by T R minus T e is k by L D.

(Refer Slide Time: 04:49)



So, this is one equation, we are going to supplement with our earlier equation. So,let say this is equation number one from here. So, we call this as equation two. So, how many equations do we have now?

(Refer Slide Time: 05:02)

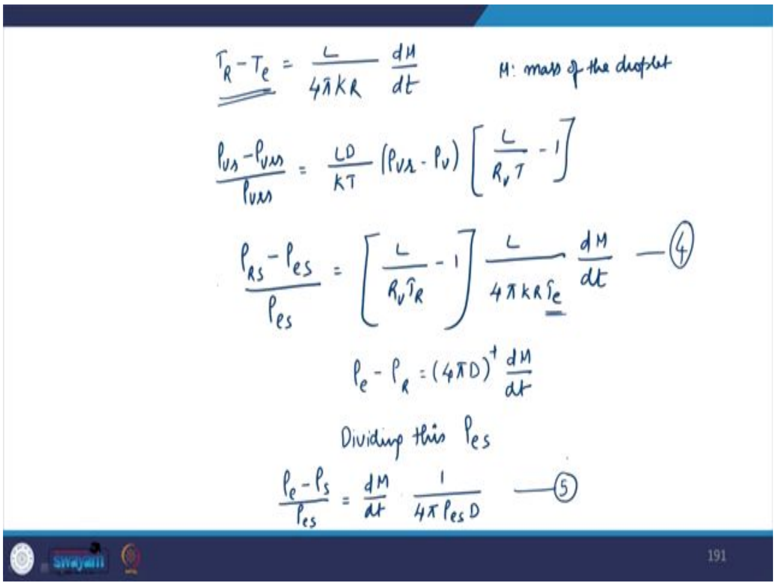


So, we have, for example, rho v minus rho v R divided by T R minus T e is equals to k by LD. Again simply the ideal gas equation rho v R is e prime r. Now, we are talking about a droplet right by R V T r what is this? The density this is very important this of course, this is just ideal gas equation this is the density of vapor at the surface of the droplet. This is the curved at the curvature term involved here, this is the specific gas constant and T of course, is the temperature at the surface of droplet.

Now, so, here now we can involve. So, if this is with respect to the curvature, we have to use the (Refer Time: 06:26) relation or the curvature and the solute effect. So, we can write this expression as e s of infinite to 1 plus a by r minus b by r cube divided by r v T r right. So, let us say this is equation 2 now and this is equation by the way this is rho vr this is equation 3.

So, we have equation number 1 which is this equation 1, we have 2, we have 3 right. Now, from these 3 equations we will do some rearrangement of these 3 equation and we will try to see how we can derive an expression.

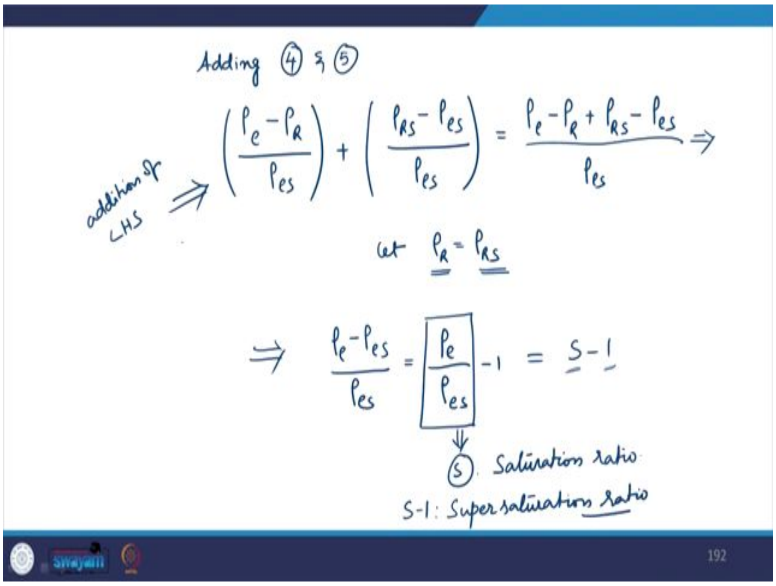
(Refer Slide Time: 07:28)



So, we can say that T R minus T e is L by 4 pi k R into dm by dt. where M is the mass of the droplet. So, we can now combine these 3 equations as rho v s minus rho v rs divided by rho vr s is equals to LD by KT to rho vr minus rho v times L by Rvt minus 1 ‘ or rho rs minus rho e s divided by rho e s is equals to L by R vTr minus 1 times L by 4 pi KR Te dM by dt. So, this is T R minus Te that, we have used in this expression. So, T e is still remains the same. So, T e is still here.

So, let us say this, we will now try to simplify this equation. Now, we know that rho e minus rho r is 4 pi D inverse into dm by dt dividing this expression with rho es, we will get rho e minus rho s divided by rho e s is equals to dm over dt into 1 by 4 pi rho e s capital D. Let us say we call this equation as, in this order, what we call this equation as 4 and we call this equation as 5 ok. So, this is 4 and this is 5.

(Refer Slide Time: 10:18)

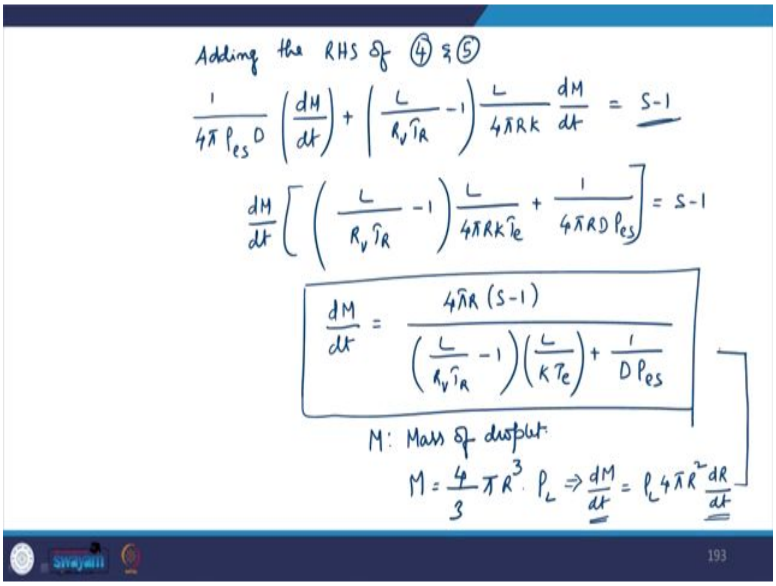


Now, let us add equation number 4 and equation number 5. We will get on the left hand side rho e minus rho r divided by rho es plus rho rs minus rho e s divided by rho es which is equivalent to rho e minus rho R plus rho Rs minus rho e s divided by rho es. So, let us say rho R is equals to rho Rs; that means, the density at the surface is equal to the density at saturation.

So, in this case this expression reduces to simply rho e minus rho es divided by rho e s that is also equal to rho e divided by rho es minus 1. So, the vapor density of the ambience with respect to the density of saturation can be defined to be equal to S minus 1. So,this term is equal to S. What is S? S is the saturation ratio and S the maximum value that the saturation ratio is expected to possess is ideally equal to 1. So, the quantity S minus 1 is of course, indicates the super saturation. So, you have already. So, S is the saturation ratio which is 1 ratio if it is the ratio its 1 or if it is saturation its 100 percent.

Now, this term itself can be maximum 100 and you take a 100 out of it I mean 1 out of it. Then you will realize, what is this super saturation ratio. So, now, this was the addition of LHS alone.

(Refer Slide Time: 12:50)



Now, let us adding the RHS of equation number 4 and equation number 5. We will realize 1 by 4 pi rho es capital D times dm over dt plus L by Rv TR minus 1 times L by 4 pi R k dm by dt. Now it isequal to S minus 1, we have already done this.

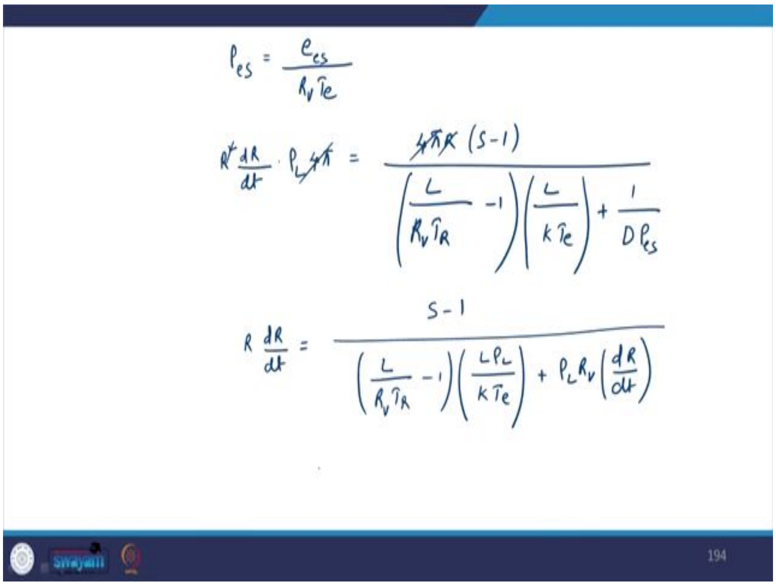
So, take d m over dt outside then, we will realize L by R v T r minus 1 to L by 4 pi R K Te plus 1 by 4 pi RD rho e s S minus 1 or dm by dt is equals to four pi R into S minus 1 divided by L by Rv T R minus 1 into L by K T e plus 1 by D rho es.

So, what is M? M is the mass of droplet. So, M can be written as 4 by 3 pi R cube times the density of the liquid, mass is density times volume. So, this is the volume of the droplet, mass of the droplet and the density which is of course, the density of the liquid.

So, if you write similarly dM over dt ,the rate of change of mass with respect to temperature. So, what should it depend on is the density of course, which remains a constant and of course, 4 pi R square into d R over dt. So, rate of change of mass will only depend on the rate of change of the radius, the remaining are constants.

So, we will have to substitute this, I mean the point of making that taking this derivative is to be able to substitute this into this.

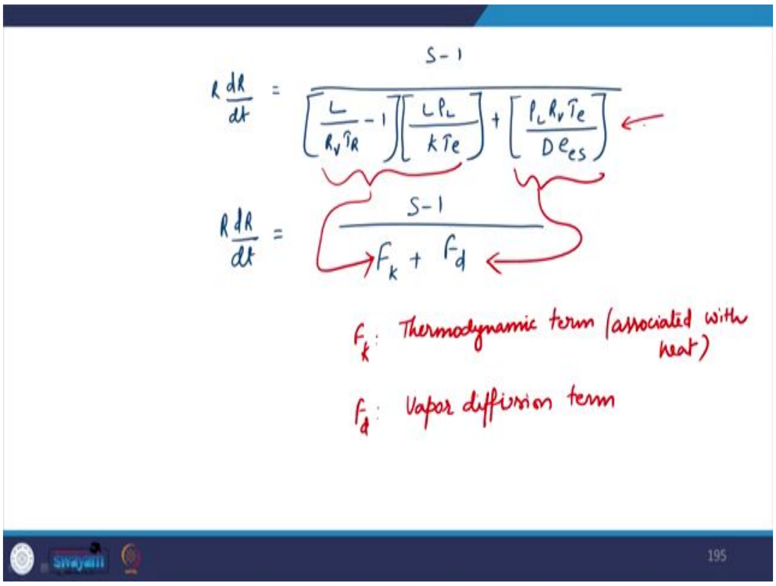
(Refer Slide Time: 16:18)



So, let us say we know that from the ideal gas equation rho es e es divided by Rv T e what is rho es ? rho es is the density of water vapor at saturation, is equal to the vapor pressure at saturation divided by Rvt just the ideal gas law .

So, let us say, we use this density change R, where is it R square dR over dt into rho L times 4 pi yeah 4 pi is equals to 4 pi R times the super saturation ratio divided by L by R v T R minus 1 into L by k Te plus 1 by D rho es. If you cancel this simply be able to write R dR by dt is S minus 1 to L by R v T R minus 1 to L rho L divided by k T e plus rho L Rv dR over d t.

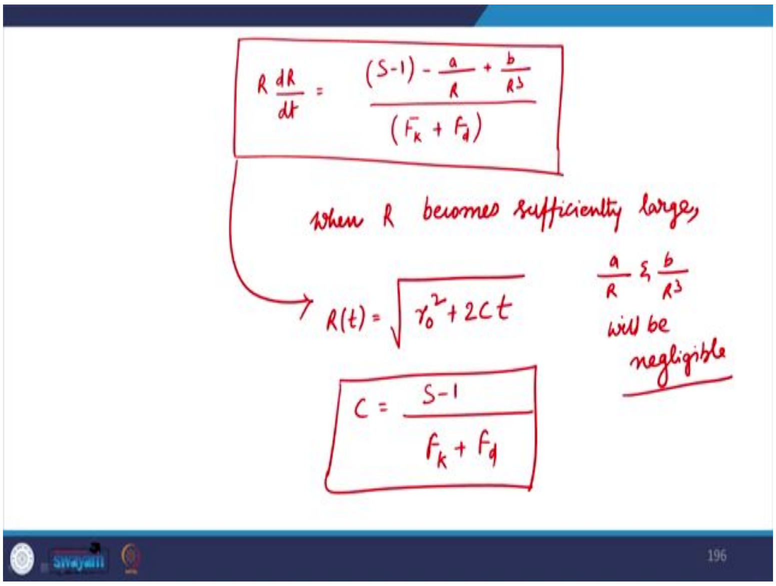
(Refer Slide Time: 18:16)



Or we can write it as R dR by dt is equals to S minus 1 divided by L by R v TR minus 1 L rho L divided by k T e this is just simple algebra, I am just trying to rearrange the terms for the sake of convenience right. So, it should not be any worry if you do not understand you just have to look at the previous slides get all the equations together and rearrange them or substitute according with the method that we have followed you will be able to get it.

So, this we are going to call it as. So, first in simple terms, we are going to call it as d R d R by dt is S minus 1 divided by F K plus F d. So, what is this F k?. So, this is just a notation. So, this entire term is F k and this term is F t . So, what is Fk, why do why are we calling it as F k? So, F k is the thermodynamic term associated with heat and F d is the vapor diffusion term. So, this term is associated with the diffusion and this term is associated with the thermal term .

(Refer Slide Time: 20:16)



Now, at this point again, we can introduce the solute and the curvature effects as R d R by dt is equals to S minus 1 minus a by R plus b by R cube divided by F k plus F d. So, this is the generalized expression for the growth of a droplet how does it change with respect to time.

So, we have all the constants all the variables that will tell you what is the concentration of liquid, what is the concentration of ambience, what is the density because of the ambient vapor, what is the gas constant, what is the temperature everything every parameter is given here which will characterize or which will tell you the characteristics of the vapor and the droplet.

Now, given this so, we can find how the radius of a droplet changes over time that is the basic idea . Now, let us say if this is the case then when the radius R becomes sufficiently large. Then, we can simply say that a by R and b by R cube will be negligible. So, in that case we can reduce this expression as R of t , I mean we are going to integrate this and R a function of t will simply be r naught square the initial radius plus 2 C t where the constant C is S minus 1 divided by F k plus F t.

So, this is the expression which tells you how the radius of the droplet changes with respect to time. So, what we have seen is just to recap, if you see the mathematical treatment. So, far for us to understand or appreciate the expression. what we have done is we have simply taken the Clausius Clapeyron equation with the vapor and the liquid variables .

Then we have rearranged it in terms of density getting rid of the alpha . Where we saw that first time how the vapor pressure at saturation changes with respect to temperature, we will depend on the density at saturation and the temperature and the latent heat. Then, we wrote the vapor pressure at saturation in terms of the ideal gas law and we took it derivative and we equated the first two terms and then we got this equation .

After thus we take an integration of this resulting equation from the ambient temperature to the temperature of the droplet. then we got this expression then, we made a simple approximation when z is very very small, very small values of z log z 1 plus z is equals to eventually equal to z naught not 1 plus or not something it is just.

So, we applied this approximation on this parameter this ratio, then we derived how the density changes between the saturation between the radius between the surface and the ambience in comparison to the ambience . So, then we got this expression right, then we introduced the variables of thermal conductivity and the coefficient of diffusion saying that the diffusion is happening at the droplet surface and this diffusion the rate at which the diffusion is happening the rate at which the molecules are getting attached to the droplets must release some latent heat.

And, this heat must be must build up a gradient by allowing a heat conduction to happen from the surface of the droplet to the ambience. So, we took all these parameters and we later introduced the solute and the curvature effects into this. And then; we thought the mass has the droplet to be M and we try to find out the rate at which the mass of this droplet will change with respect to time. So, that is where the time dependence actually came into the picture .

So, we took the mass to be equal into density times volume, we took a derivative of the mass and that is where actually the time dependence has come into the picture otherwise that there is no time dependence so far in the picture . So, eventually we algebraically simplified all this things and then, we realized the rate at which the radius of the droplet changes is equal to simply these two terms S minus 1 is the super saturation ratio divided by F k plus F d, where F k is the thermo dynamical term F d is the diffusion term .

So, this concludes the discussion about the diffusional growth of a cloud droplet how it changes how it its radius changes with respect to time . So, we will stop here. I suppose you can understand this .