Introduction to Atmospheric and Space Sciences
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Lecture - 33
Atmospheric Stability Conditions

Hello dear students. So, in today's class we will continue our discussion on Atmospheric Stability. So, the last time we have seen that we have taken an air parcel under the assumptions that, the air parcel we will also have the same pressure as the environment at whatever height that it is present, right. So, today we will continue the same discussion. So, it is going to be atmospheric stability.

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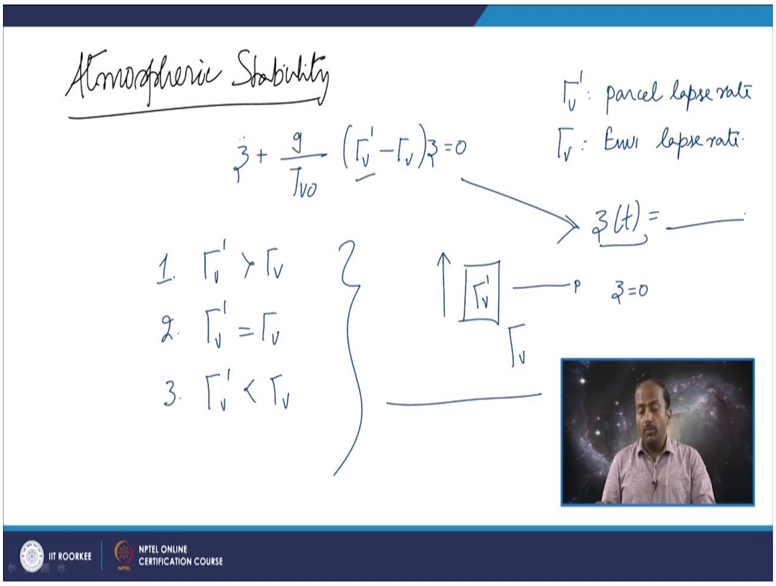

So, we are going to see the mathematical treatment of the various stability conditions; how they will represent the motion of an air parcel in an atmosphere with varying lapse rates between the environment and the parcel, how the atmosphere will allow the parcel to sink or how the atmosphere will allow the parcel to rise further in the atmosphere, right. So, basically the second order differential equation that we have derived in the last class goes something like this.

So, this is T v 0 gamma v prime minus gamma v times z is equal to 0. So, if you remember. So, z is the vertical coordinate, T v 0 is a virtual temperature at the surface or at the zero level; gamma v prime is the rate at which the virtual temperature varies with respect to height inside the parcel and gamma v is the rate at which the temperature varies outside the parcel or environment. So, we say that, gamma v prime is the parcel lapse rate and gamma v as the environment lapse rate.

So, if you see last time we have seen that, this will the various different types of solutions that this equation can predict, will decide the nature of motion that the air parcel will undergo, right. So, if it is the case, let us see case by case what different types of motions that we can see for the air parcel, right. So, there are three possibilities here. So, here gamma v prime becoming greater than gamma v; the second possibility is gamma v prime is equals to gamma v; the third possibility is gamma v prime less than gamma v.

Physically speaking what do these three conditions mean? These conditions mean that, if you have an air parcel let us say in with respect to the surface; the air parcels temperature is varying at a rate gamma v prime with respect to the height, and the environment is varying at a rate gamma v. So, you may now ask, why do you think the parcel lapse rate is different from the environment?

It is due to the fact that, the parcel is an adiabatic entity; that means, it is not allowing energy or mass transfer between the environment or through the walls of this air parcel, it is a small control volume which is not fixed in space, but it is allowed to move, right. So, if you expect this air parcel to be existing at the same pressure as the surroundings; the air parcel existing at the same pressure.

Let us say if you create a mechanism by which you slightly perturb this air parcel. So, air parcel is steady at rest let us say at z is equals to 0 for instance take the reference to be Z is equals to 0; at this point the air parcel is not moving anywhere, it is not going up, it is not going down. So, if you now displace the air parcel with respect to height slightly in vertically upward direction; then the aspect of stability is that, we have to see under what conditions the displaced air parcel will try to return back to z is equals to 0 or it will try to move away from z is equals to 0.

Naturally if you see that the air parcel is returning back to its original position or z is equals to 0, you would simply say that the atmosphere is stable. So, atmosphere is not allowing vertical movement. So, it is also the atmospheric stability also is generally is known as the vertical stability, right. So, basically if the parcel is raised in height; so naturally it is going to a lower temperature region, where the pressure is lesser. So, if the pressure is less, the air parcel should also be at the same pressure. So, it should also be at the lower pressure.

So, for the pressure to become less, the air parcel has to expand; and if this expansion is at the expense of internal energy of the air parcel, then naturally the temperature of the air parcel will decrease, right. So, if the air parcels temperature is decreased so; that means, it is moving away from the equilibrium position, right. So, this is the basic idea of stability.

Now, let us see how do we arrive at these solutions; I mean basically the solution of this equation is z as a function of time is the solution of this equation is this equation. Now, so if we can write the solution for z of t by any means; we will under various situations described in 1 2 3, z of t will take different forms and these different forms will explain different kind of scenarios, right.

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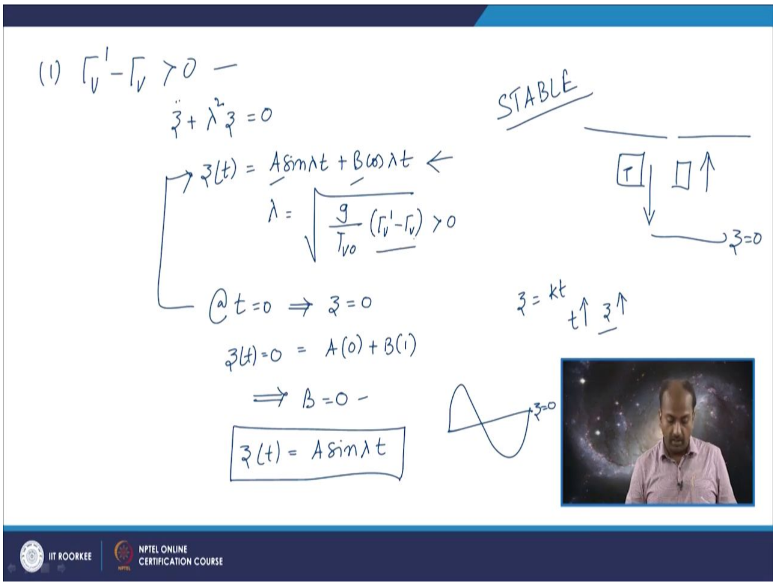

So, let us see, let us take up the first case in which gamma v prime minus gamma v is greater than 0. So, before I do the simple mathematics, let us try to understand what does it mean. So that means that, the lapse rate, the temperature inside the air parcel is decreasing at a faster rate than the environment.

So, what does it mean? At any given height if the air parcel is has risen to a point, the environment temperature is decreasing slowly; but the parcels lapse rate is increasing faster. That means, so the parcel is more likely to experience lower temperatures at a given height at the same pressure. So, any air parcel which is at a lower temperature is bound to travel towards the earth; any air parcel which is at a higher temperature is bound to move away.

Now, you have kept your reference at z is equal to 0. So, if you push an air parcel from z is equals to 0; if it is temperature decreases faster in comparison to the environment, so environment is warmer in comparison to the parcel. So, parcel which is cooler is naturally has a tendency to sink, right. So, this is the idea of stability. So, the second order differential equation can simply be written as z double prime plus lambda square z is equals to 0.

A simple solution for this equation can be assumed to be z of t is A sin lambda t plus B cos lambda t right; such that lambda is equals to square root of g by T v 0 into gamma v prime minus gamma v, which is now greater than 0 according to this, right. Now, if this is the solution, so what kind of solution is this? I mean this solution can be explained, when you resolve the constants A and B.

Now, let us say at t is equals to 0 that means, in the beginning at t is equals to 0, let us say there's the position z is assumed to be 0. If you just substitute this into this equation, you will realize z of t is equals to 0 when t is equals to 0. So, A into 0 plus B into 1, because you have substituted t is equals to 0 in the solution, right.

So, if it is the case, then you will realize that B is 0; now A is not 0, but B is 0. So, if you put this into this equation. So, we will write z of t is A sin lambda t, right. So, z of t, the position of an air parcel with respect to time appears to be a sinusoidal function in time. So, now, you can find out the frequency of this oscillation.

So, what does it mean? So, the position is not linearly changing with respect to time. So, if you write let us say z is equals to some constant k times t; what does it mean? As t increases, you will simply realize z also increases. So, what does z indicates? z indicates the position of the air parcel at a given time. So, as you go on increasing along the time, as you travel along the time; the position is never coming back to the same value, it is always increasing with respect to time, right.

But here you do not see like that, I mean whatever the extreme values of the time that you put; it will always be an oscillation about the mean position. So, the parcel may appear to be deviating or may appear to be traveling away from z is equals to 0. So, this is your z is equals to 0.

So, if you have an environment in which the temperature of the environment is decreasing at a slower rate in comparison to the temperature inside the air parcel; then no matter whatever the displacement that you create. It will always be such that it will appear the parcel is moving away from the equilibrium position, but in time it will come back to it is mean position. So, what kind of situation is this; I mean do you think it is an unstable situation or a stable situation. So, it I think it is very clear that, the atmosphere is not allowing the parcel to move away from a position, it will it is making the air parcel to come back to its original position.

So, this kind of situation is called as stable. So, now, we say in simple words, you say that the atmosphere is stable; atmosphere is stable for vertical movement of the air parcel, right. So, the atmosphere is not allowing the air parcel to move.

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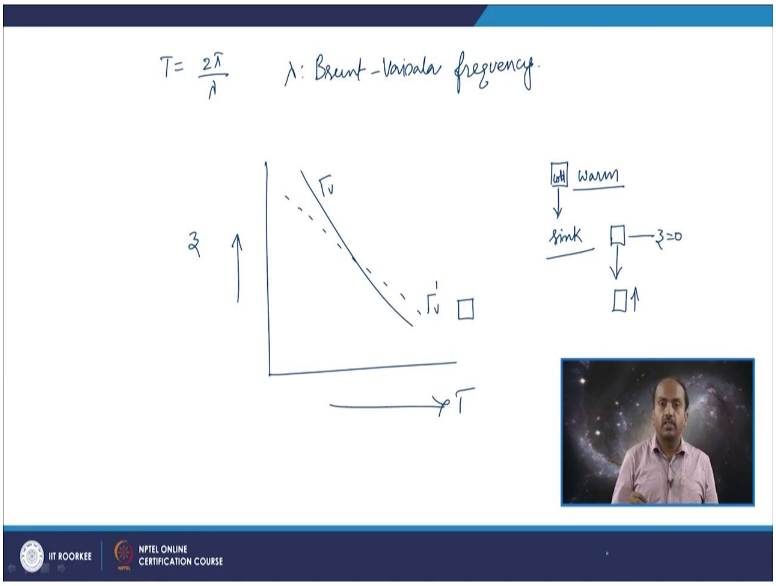

So, we can calculate the time period of these oscillations. So, simply T is equals to 2 pi by lambda, where lambda, is a very important parameter, lambda is called as the Brunt Vaisala frequency. So, this is a characteristic frequency of an atmosphere under a particular condition.

So, when the parcel is displaced away from the mean position or equilibrium position; the parcel will oscillate about the mean position, it is not going anywhere. So, by the means of these oscillations, the parcel is actually trying to come back to it is original position. So, its oscillations will decrease in amplitude with respect to time, but eventually it will come back to its mean position. So, we can simply say that, the parcel is in a stable situation; that is we can say that, the parcel is displaced would not go away from the mean position.

Let us say if you want to understand this physically how what does it mean? So, you have an atmosphere which is kind of stable. So, generally, I mean thermodynamically what is happening. So, we have to understand this situation, thermodynamically. Let us say, we draw a diagram to understand this.

So, let us say this is temperature across the x axis and it is the height across the y axis right. So, here let us say this solid line you have the lapse rate for the environment and you have a dashed line which is for the parcel. So, it is gamma v prime and this is gamma v. So, let us say here, now you imagine an parcel here. So, when this air parcel is displaced, what will happen? So, since the temperature inside the air parcel is decreases faster in comparison to the environment, at any given height to maintain the same pressure the parcel will be colder in comparison to the environment.

So, this is the part where environment is warm and the parcel is cooler. So, if you just put these two things together. So, you will realize that the cold entity should have a natural tendency to sink, right. So, you can put it the other way, I mean if you sink it, I mean if it is in equilibrium condition; then let us say if you put from the equilibrium at z is equal to 0, if you push it downwards let us say, what is the tendency?

If you push it downwards; then you have to think exactly opposite to the lapse rate, the rate at which the temperature will increase with respect to height is going to be in the negative z direction. So, here as the parcel is made to sink physically by applying the force, the parcel will be at a warmer temperature in comparison to the surroundings; so that means that, the parcel will now have a tendency to rise, right.

So that means, that if it is just the rate at which temperature changes inside the air parcel, which decides what should be the parcels trajectory or position with respect to time, right. Now, let us look at the second case; the second case is when the lapse rate of the environment is greater than the lapse rate of the parcel.

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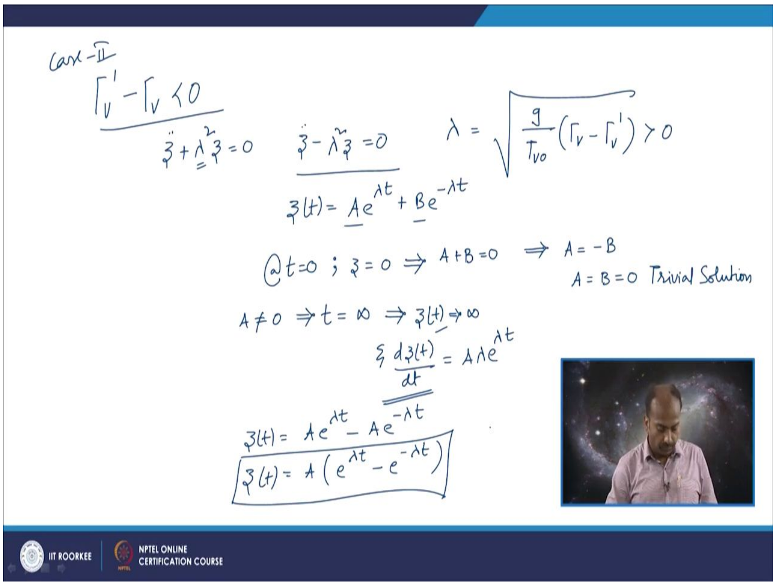

So, this is case II. So, here gamma v prime minus gamma v is less than 0, so that parcel lapse rate is less than the environment lapse rate, right.

So, in this case what happens is, we can use the simple same equation again. So, the same equation is z double prime plus lambda square z is equals to 0. Now, since this is within this, right. So, we should write the equation as z double prime minus lambda square z is equals to 0, where lambda is equals to square root of g by T v 0 times gamma v minus gamma v prime which is greater than 0.

So, you should observe the order in which I have written gamma v and gamma v prime. So, here it is gamma v prime minus gamma v; since gamma v is now greater than gamma v prime, I have written gamma v minus gamma v prime, right. So, a natural, solution of this equation, a simple solution is z of t is A e to the power of lambda t plus B e to the power of minus lambda t, right.

So, now what is the natural process; we should find out the values of these constants A and B. Let us say at t is equal to 0 at the initial condition, the displacement or the position of the air parcel is referenced to be z is equals to 0. So, if you substitute this into this equation, you will realize A plus B is equals to 0 or you can say that A is equals to minus B. Can we say that it has to be A equals to B equals to 0?

We can actually say that, mathematically it is true, but this will lead to what is called as a trivial solution; a trivial solution would not signify or would not tell you any information about the nature of motion, right. So, we should always discard the trivial solution. So, let us sayA is of course, not equals to 0, if you take it. Then when t tends to infinity, but if you go on increasing the time; you will realize that, z of t the position also tends to infinite, right. And the change of position with respect to time, the velocity is A lambda e to the power of lambda t, right.

So, now this is non-zero, right. So, the velocity is non-zero, right. So, we simply take A is equals to minus B; that means, we do not take two constant rather, we take B also equals to minus A that is it. So, now, we can write the solution z of t as A e to the power of lambda t minus A e to the power of minus lambda t orA times e to the power of lambda t minus e to the power of minus lambda t.

So, what does it means? The parcel is experiencing an accelerated motion, so z of t. So, what do you mean by accelerated motion; you calculate the velocity, you calculate the acceleration which is non-zero. So, the parcels motion is accelerated, and the parcel once displaced will never return to the original position. So, you do not see if you substitute any value of z, any value of t; you will get the z of t is equals to infinity, you will never get the value z of t is equals to zero again, right.

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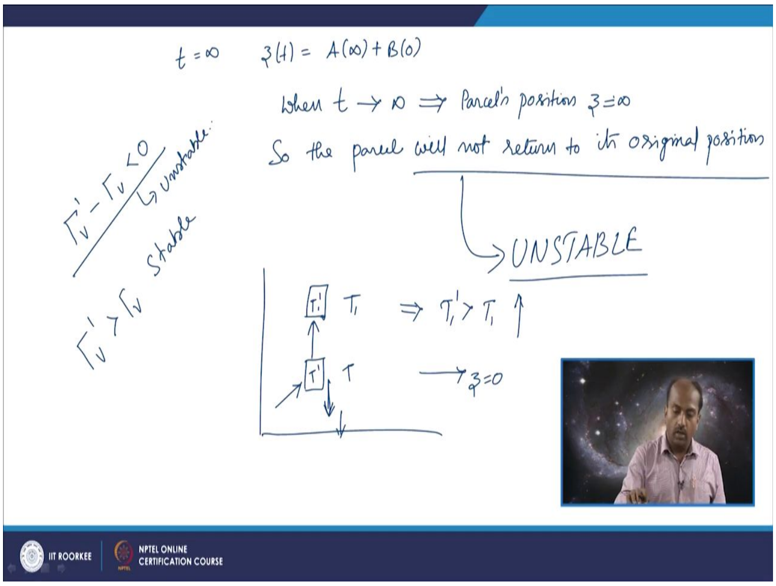

So, let us say at t is equals to infinity, z of t is A in to infinity plus B into 0. So, when time t tends to infinite, the parcels displacement, the parcels position z is also infinite. So, the parcel will not return to its original position. So, when does it happen; if the parcel lapse rate is smaller than the environment lapse rate, then the parcel will be displaced and it will never come back to its original position.

So, you see that, the displacement is growing with respect to time that is a simple thing, right. Now, how do you interpret this physically, how do you understand the interpretation? So, previously we have seen that, if the parcel is made to rise with respect to height by giving an impulse or by giving a small external force; the parcels temperature will be decreasing faster, so that it is colder in comparison to the environment. So, it has to naturally sink.

Let us put the same argument again; if you rise a parcel in an unstable environment. So, before that let us say if the parcel is not coming back, it will not return to it is original position, this situation is unstable. So, it goes with with the basic definition of stability. So, if the parcel is not coming back to the original position, it is the situation is simply unstable or you call the atmosphere unstable or you call the atmosphere is unstable for the vertical movement of air parcel, so to be precise, right.

Now, the argument is, if you consider, if you just take an air parcel; now that parcels temperature is T prime and the environment temperature is T, right. Now, it goes to a particular height, so this is your reference, this is your z is equals to 0; if it goes to a height which is at this height let us say, non-equilibrium height. Here the temperature let us say we will take it as T 1 prime and the temperature at this height is T, right.

Now, the temperature inside the air parcel is decreasing slower in comparison to the temperature outside the air parcel, right. So, naturally, let us say if you these two are the same temperature for beginning, ok. Now, let us saybecause the temperature inside the air parcel is decreasing slower, that it naturally means T 1 prime is greater than T 1. So, what does it mean? That means that, the air parcel is warmer in comparison to the environment.

So, anything which is warm has a natural tendency to rise, right. So, you have created an impulse, you have given an impulse here and made the parcel to rise at this particular height. Because of the parcel lapse rate decreasing slower in comparison to the environment; at any given height the parcel is warmer in compassion to the environment, so the parcel will naturally rise so, right. So, now the point is the parcel is not coming back. So, this is the simple situation in which the parcel, the atmosphere is unstable.

Now, let us look at it the other way; if the parcel is made to sink, let us say, right. So, the parcels lapse rate is again decreasing slowly, slowly when compared in the environment. So, if you apply the same argument you will realize that, the parcel if it is displaced, since the rate of decrease of temperature is smaller, it will go and it will sink further into the towards the lower altitudes, thereby never returning back to the original position. So, this is the situation which is called as the unstable equilibrium or unstable atmosphere, right.

Now, for the third case, so we have two cases, so we simply say that, gamma v prime is greater than gamma v, we say stable; gamma v prime less than gamma v, we say unstable, right.

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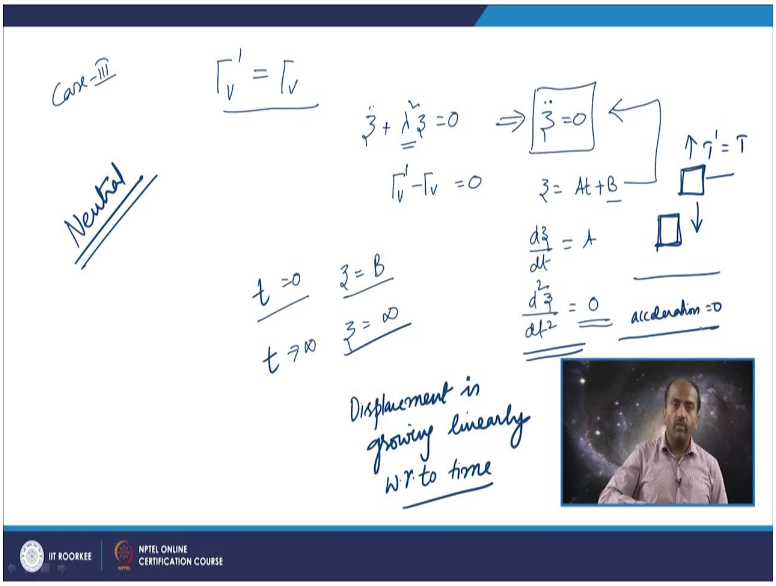

Now, the third case, case III; what will happen if gamma v prime is equal to gamma v? That means, the rate at which temperature is decreasing inside the air parcel is equal to the rate at which temperature is decreasing in the environment.

So, what will happen in this case? So, eventually, so this is the condition how fast the temperature decreases inside the parcel in comparison to the environment. So, in this case the differential equation simply reduces. So, we have z double prime plus lambda square is z is equals to 0, since I have what is called as gamma v prime minus gamma v. Now this becomes equal, your second order differential equation is simply this.

Now, what kind of solution can you put it? So, the second derivative of z should be 0; you take a function whose second derivative is 0. So,I will simply take z is equals to A t plus B; I mean this B is anyway irrelevant, right. So, this first derivative is d z by d t which is A and d square z by d t square the second derivative is 0, all right. So, this means that, this simple solution does satisfy the differential equation.

What is this differential equation? we got this differential equation by managing the force imbalance between the preservation force and the gravity, and we managed to write this equation in terms of virtual temperature, right. So, what does this mean? I mean if you see z is equals to 0, the position is of course, something which is B, right.

So, at t is equals to 0, z is equals to some value at a particular position; at t is equals to infinity, z is equals to infinity. That means, the displacement is growing linearly, so the displacement is growing linearly with respect to time. So, in the unstable situation what did we see? We saw that, the displacement was growing exponentially with respect to time; but here you see that the displacement is growing linearly with respect to time.

So, do you think this is a stable situation or do you think it is an unstable situation, what is it? This is an unstable situation or this is this should be called as the neutral situation as such. So, indicating the displacement grows linearly, an acceleration is anyway zero. So, acceleration is zero. So, in the unstable situation the acceleration was nonzero. So, the parts parcel will be accelerated with respect to time. So, how is this different, why do not you call this as unstable situation? We call it as this situation is called as a neutral state neutral.

So, displacement is not changing; I mean displacement is changing linearly with respect to time. So, you put a value of t, you will get which is z which is away from the mean position, right. So, in this case the parcel once displaced will leave to their mean level and never returns to the original level. So, what it means that? If you put, if you displace the air parcel from here to here; the parcel will stay here, because the temperature is T prime is equals to T, right.

Now, the parcel; of course, see in the original stable situation when you displace the air parcel, the parcel was coming back; in the unstable situation when the air parcel was displaced, the parcel was going away, it never used to come back, right. But here when you displace an air parcel from its mean position to the new position, where the temperature inside the air parcel is equal to the temperature outside the air parcel.

Now in this case the parcel has no reason to rise or no reason to sink. So, the position the equilibrium position is away from the original equilibrium position. So, this situation is called as the neutral equilibrium or neutral state, right. And most importantly this also has the acceleration of the air parcel is 0. So, this is the difference; I mean you have a stable situation, you have an unstable situation and you have a neutral situation. So, neutrally is when the parcel once displaced, it will stay there.

So, whatever I mean with respect to time, it is changing linearly. So, there is no acceleration. So, the parcel is not moving away without any effort; the parcel just takes the impulse and whatever height it reaches, it is at the same height as the surroundings. So, if it is at the same height; what does it mean? It has no reason to rise or it has no reason to sink, right. So, this is something about a mathematical treatment for understanding the stability.

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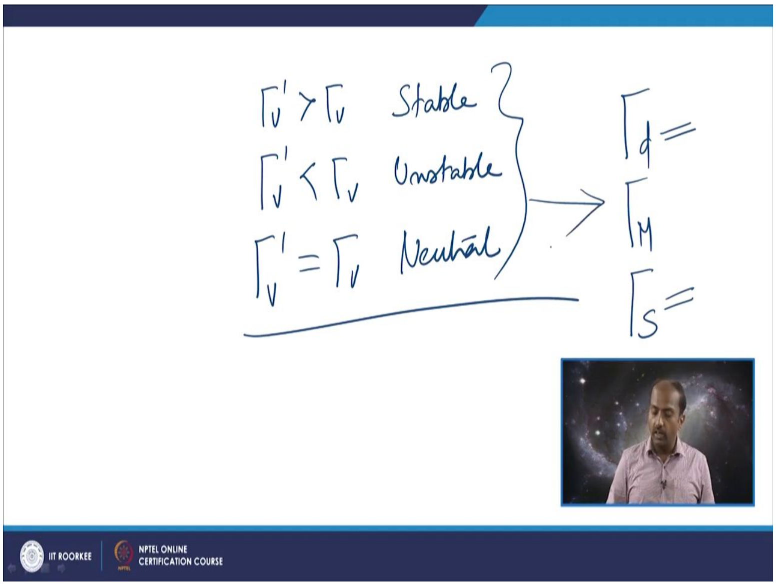

So, if I simply write it let us say for. So, if gamma v prime is greater than gamma v, we have stable atmosphere; if gamma v prime is less than gamma v, you have unstable atmosphere; and stable atmosphere for the vertical movement of the air parcel. If gamma v prime is equal to gamma v, we have a neutral atmosphere, right. So, this is some discussion about the atmospheric stability, a simple mathematical treatment, right.

So, in the next class what we will do is; we will try to reframe this stability condition in terms of other lapse rates that we have already derived. For example, we have seen what is a dry adiabatic lapse rate, what is the moist adiabatic lapse rate, how are these two different. We have also seen what is a saturated adiabatic lapse rate and why is gamma d greater than gamma S things like that.

Now we will try to see how we can reframe these conditions of stability in terms of lapse rates that we are very well acquainted with. So, we will stop here.