**Introduction to Atmospheric and Space Sciences**
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**Lecture – 32**
**Atmospheric Stability**

Hello dear students. So, in our earlier discussions we have discussed various aspects related to the formation of clouds. We have seen what is the idea of Atmospheric Stability and how it helps, in the formation of clouds and how it helps in the vertical growth of clouds several other things.

So, we carry out some mathematical treatment to understand the vertical stability of atmosphere. So, you know in our earliest understandings, we have seen that atmosphere to a very large extent over a large scale several 100s or 1000s of kilometers. We always assume that it is always under hydrostatic balance.

Hydrostatic balance is the one which results when the force due to the gravitational pull equals the pressure gradient force.

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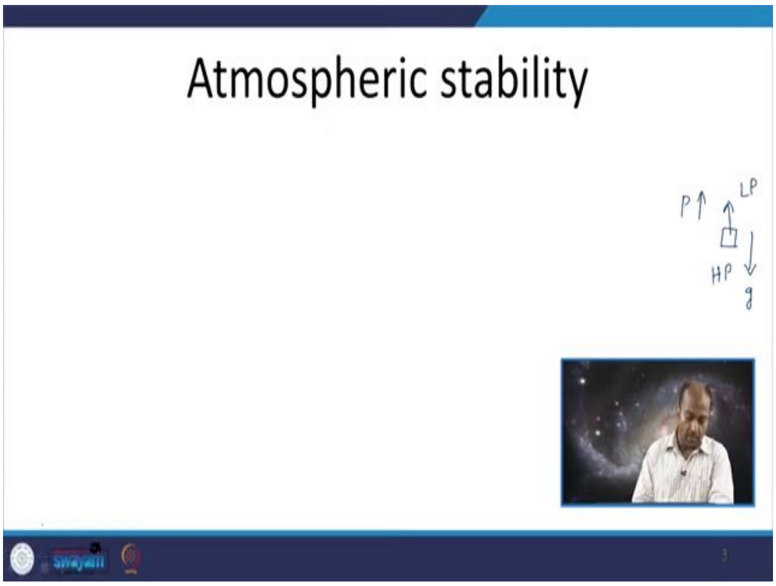


So, in the atmosphere starting from the surface the pressure decreases as you go up and so; that means, that every air parcel has a natural tendency to go from high pressure region to a low pressure region.

So, but the same air parcel is held to the gravity. So, when these two forces balance each other, the force due to the pressure gradient. And the force due to the gravity balance each other. We say that the atmospheric air is existing in hydrostatic balance fine.

So, now what we will do is. Let us say over small scales  if there is conviction or if there is a movement of air in the vertical directions, we say that there is a deviation from the hydrostatic balance. So, the hydrostatic balance is very well expressed with the help of the equation.

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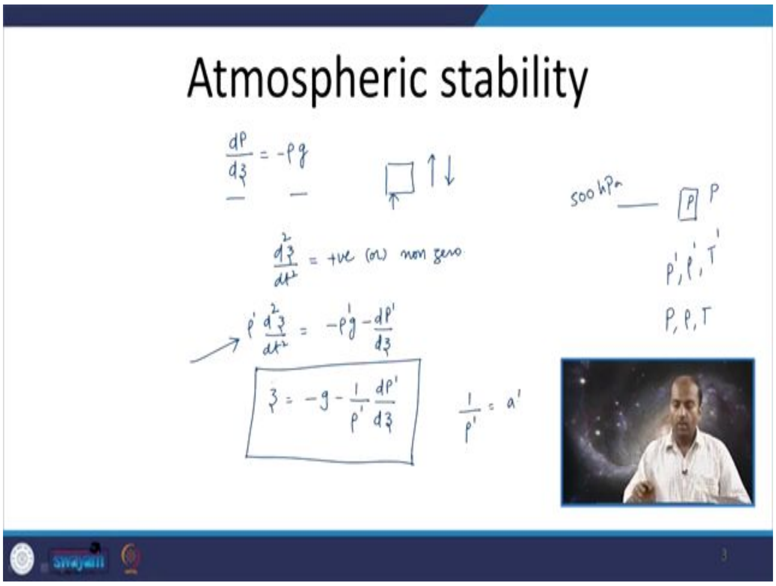


Let us say which is d P over dz is minus rho g. So, P is the pressure and z is the vertical dimension height; rho is the density and g is the gravity right. Now, according to the air parcels, approximation that we have just seen, we assume that any air parcel that is suspended in air is always existing at the same pressure in comparison to the environment.

So, here we treat the air parcel as an entity we treat the variables inside this air parcel. As let us say, pressure with the P pressure, density,  and let us say temperature these are the variables that are associated with the air parcels ok. And parameters are the physical parameters associated with the environment are treated differently without a prime.

Now, one approximation of the air parcel is such that pressure inside the air parcel is always equal to the pressure outside. So, if this air parcel is kept at let us say 500 hPa height then this air parcel inside this air parcel the pressure is also 500 hPa like that right. So, this is one very important approximation for us to carry forward this mathematical treatment ok. Now, let us say if the air parcel is absolutely balanced by these two forces the gravity and the pressure gradient. So, we say that the air parcel should remain as stationary .

So, you should not move either in the vertical or in the horizontal or in the any other direction. But if the air parcel is now given a small impulsive force such that it moves up. Then we say that the air parcels is moving non uniformly or due to the application of this force there is a net acceleration in the upward direction.

That means, there is a net acceleration such that d square z by d t square is positive or in this case let us say it is simply non zero. That means, the acceleration of the parcel can now be written as d square z by d t square is equals to minus rho prime g minus d P prime by d z. The rate at which its position changes with respect to time or the velocity changes with respect to time, is the imbalance between the gravity and the pressure gradient force.

So, this is the simple equation of motion. Now let us say if you write in the simple terms, z double dot is equals to minus g minus 1 by rho prime d P prime divided by d z.

So, if there is a small imbalance between the gravitational force and the pressure gradient force, this imbalance the net difference between these two forces will result in some acceleration given by this. So, we have written this equation in simple terms. So, that we know that the net acceleration is equals to this ok.

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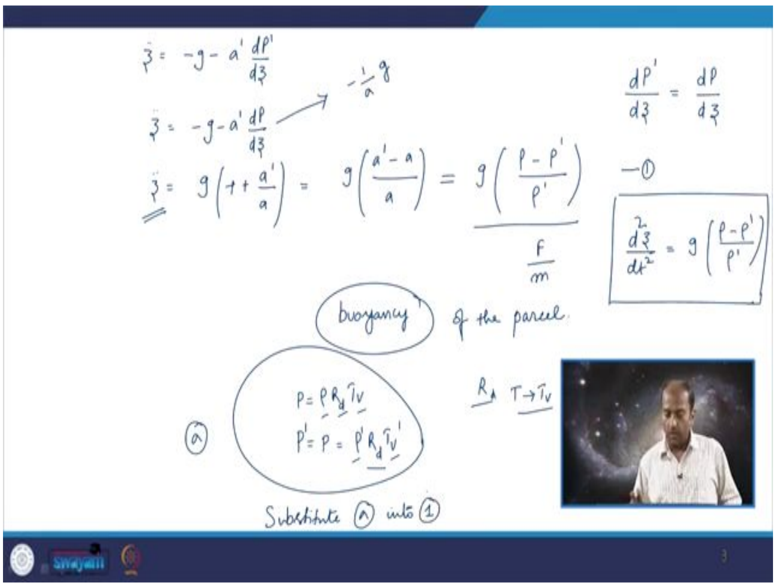

Now, we can write 1 by rho prime as a prime like we have seen in our earlier discussions. So, that we can say z double dot is equals to minus g minus a prime d v prime divided by d z. Now, as per the assumption number 5 of the air parcel we say that the pressure inside the air parcel is always equal to the pressure of the environment. So, if it is the case then I can simply write the rate at which the pressure changes within the air parcel, must also be equal to the rate at which the pressure changes in the environment.

So, if it is the case I will rewrite this equation as z double dot is equals to minus g minus a prime d P over dz ok. So, I have not changed anything I just changed P prime to P using the air parcel approximation. Then we say we rearrange this equation for mathematical simplicity. So, this is g times minus 1 plus a prime divided by a.

So, d P by d z is now minus 1 by a g. So, using this into this which we can write as g into a prime minus a divided by a, which is also equal to g into rho minus rho prime divided by rho prime.

So, what is this? So, this is the acceleration, the acceleration should be equal to force per unit mass. So, the right hand side of this equation is equal to the force per unit mass. Or let us say this force per unit mass is the difference between in the vertical pressure gradient force and the gravity; this is also called as buoyancy. So, the force that arises due to the difference between the pressure gradient force and the gravity is also called as the buoyancy ok. What is this? The buoyancy of the parcel right.

Now, we need to solve this equation. So, we simply have a second order differential equation, where the acceleration is equals to force per unit mass. We need to solve this equation and the solution of this equationwill represent how the parcel will move as a function of time. So, we have we simply have this. So, d square z by d t square is simply g times rho minus rho prime divided by rho prime. So, we need to solve this equation to understand the air parcels movement as a function of time.

Now, let us write this equation in terms of thermo dynamical variables. Let us say pressure is equals to rho R d T v where T v is a virtual temperature. So, instead of the temperature I am using the virtual temperature this is for the environment. And if I write for the parcel I should write P prime which is again equal to P is equals rho time rho prime times R d again and T v prime. So, R d is the universal gas constant of the dry air T v is the virtual temperature, T v prime is the virtual temperature inside the air parcel right.

So, here ideally if I want to write equation in terms of temperature then, depending on the temperature and the density of the gas inside the parcel and the outside the air parcel, the gas constant r should be different right. So, in order to use the same gas constant, I replace the temperature with the virtual temperature. So, that is a way that is why the basic definition of the virtual temperature comes into existence right.

So, we use the virtual temperature for this purpose only. So, the density is of course, assumed to be different. So, we have the same gas constant R d and the virtual temperature inside the air parcel is T v prime and the virtual temperature outside the air parcel or the environment is T v right. Now, let us substitute these two equations. Let us say let us call this set of equations as a substitute this set of equations into, let us call this equation as equation number 1. So, we say that substitute set of equations in a into let us say into 1.

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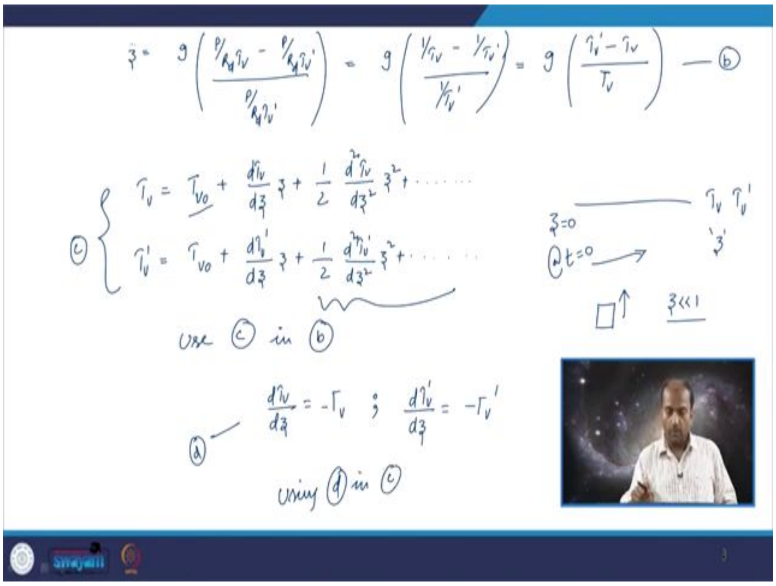

So, now we will write z double dot is equals to g times P by r d T v minus P by R d T v prime divided by P by R d T v prime. That is equals to g times 1 by T v minus 1 by T v prime divided by 1 by T v prime, which is equals to g times T v prime minus T v divided by T v. So, T v prime is the virtual temperature inside the air parcel T v is the virtual temperature of the environment right.

Now, let us say this equation is equation number b ok. So, now, what we are interested is now, earlier we had the equation written in terms of the densities of the air parcel and the environment. And this we have transformed that expression into expression involving double derivative of the position of the air parcel and temperatures of the air parcel and the environment right.

Now, let us assume the air parcel is now situated at z is equal to 0. So, z is equal to 0 is a position of the air parcel at t is equal to 0 right.

Now, the basic idea is if you displace it by applying an external force or if there is an imbalance between the horizontal vertical pressure gradient force and the gravity the parcel will move. The parcel will move such that there is an acceleration right. If the parcel is moving with the uniform velocity that is the velocity remains constant with respect to time then the acceleration will be 0. But if there is an external force that has been applied there should always be an acceleration right.

Now, what we are interested to find is, as time changes let us say from t is equal to 0 as the time changes linearly right. What will happen to the position of the air parcel what will this position depend on what parameters? Right. Now so we want to find out that. So, at any given height the temperature will change the rate at which the temperature changes with respect to the height will also change. So, what we want to do is, we want to see how the temperature T v and T v prime change with respect tothe height.

So, what we do is, we will make a Taylor expansion of T v and T v 0 about z is equal to 0. So, for very small changes in z is equals to 0. So, the displacement let us say the displacement is very small z is much less than 1. So, we want to see how the temperature changes with respect to z ok. So, we will expand the Taylor series expansion for virtual temperature. So, T v is T v 0 T v 0 is the temperature at z is equal to 0 right. So, T v 0 plus the rate at which T v changes plus half d square T v by d z square into z square right.

So, similarly this is for the environment and similarly for the air parcel. So, at the equilibrium position assuming the temperature is to be same. , if the temperature is not the same at equilibrium we cannot call that to be an equilibrium; because if the temperature is different.

For example, if the temperature of the air parcel is different, if it is larger than the environment then naturally the air parcel has a tendency to rise. If the air parcels temperature is smaller than the environment naturally the air parcel will sink lets. So, having said that at equilibrium when z is equal to 0, the air parcel and the environment are at the same temperature which is T v 0 right.

So, the Taylor expansion for the change of temperature with respect to height will look something like this right. So, what do you see? You see that the temperature T v is equilibrium temperature T v 0 plus something. And this could be the expanded terms generally,  they will not be as significant as the average term that has been written in the beginning right.

So, we neglect all the higher order terms right, higher order terms in z square or second order derivatives of the temperature. So, these terms can be neglected. So, let us call this set of equations as c. And now we will use equation number c in b ok.

So, let us say so, we will define this derivative. So, d T v over d z that is the rate at which the temperature, the virtual temperature changes with respect to height for the environment is defined as gamma v. The lapse rate minus gamma v; because it is decreasing the lapse rate of the environment.

Similarly d T v prime by d z can be written as minus gamma v prime right. So, we will use this into equation number b and we will see how it looks like ok. So, let us say this is equation number d using d in c.

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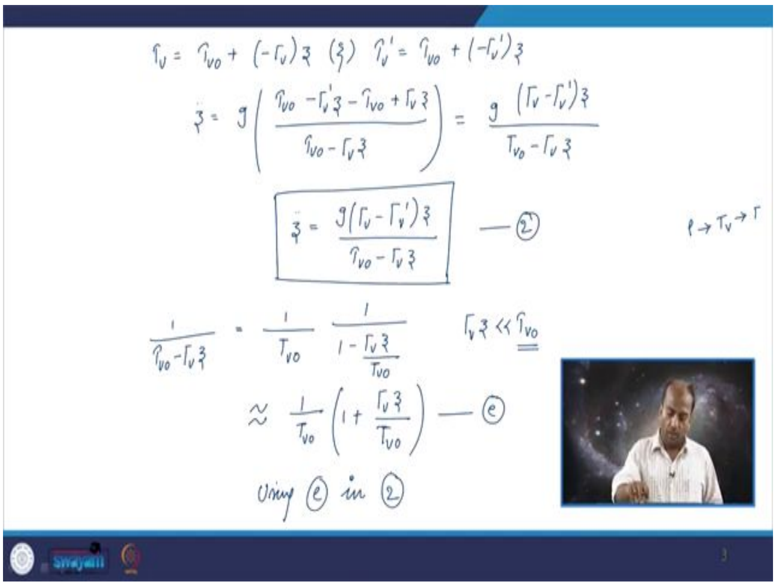

We can write. T v is T v 0 plus minus gamma v times z and T v prime is equal to T v 0 plus minus gamma v prime zok. Now using them, using the virtual temperature of the environment and virtual temperature of the air parcel in the second order equation of motion.

We can write z double dot is equals to g times T v 0 minus gamma v prime z minus T v 0. Plus gamma v z divided by T v 0, minus gamma v z which simplifies to g times gamma v minus gamma v prime times z divided by T v 0 minus gamma v z right.

So, in the beginning we had this expression in terms of the densities. Then we transformed this equation in terms of the virtual temperatures then we transformed this equation in terms of lapse rates right. So, gamma is the lapse rate gamma v is the lapse rate of the environment gamma v prime is the lapse rate of the air parcels. They simply say the rate at which temperatures are changing in the air parcel as well as in the environment right.

Now, let us do some mathematical simplification for the term in the denominator. So, let us say so, this is the original equation stands as it is. So, z double dot is now written as g times gamma v minus gamma v prime times z divided by T v 0 minus gamma v z. So, we need to solve this equation that is it; I mean we need to find out.

What will be the position z with respect to time subject to the changes on the right hand side? So, if gamma v is greater than gamma v prime what will be z what will be the what will be z like. I mean how do you write the analytical form of z will the analytical form of z change, if gamma v is greater than gamma v prime or if it is less than gamma v prime things like that right.

So, in order to do that we will make some more algebraic simplification. So, let us say 1 by let us say 1 by T v 0 minus gamma v z is can be written as T v 0 times 1 by 1 minus gamma v z by T v 0. Now so, according to the Taylor series we have seen that gamma v times z will be much less than T v 0; T v 0 is the mean value about which or around which you make a Taylor expansion right.

So, this is a very large value in comparison to this the rate at which the temperature. So, this can be approximated the expression of above convey approximated as T v 0 times 1 plus gamma v z divided by T v 0. So, the entire denominator 1 by T v 0 minus gamma v z is approximately equal to 1 by T v 0 times 1 plus gamma v z divided by T v 0 right.

Let us say we is called this as equation number e right. We use using equation number e, in let us say now this is equation number 2 . So, we have two system, I mean two different numberings going on parallely right; a b c d and 1, 2, 3, 4 right using e in 2.

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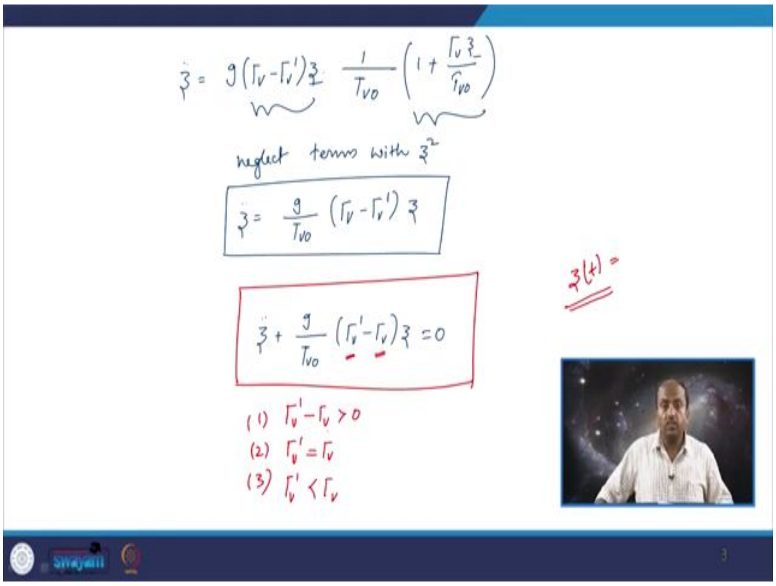


We will write z double dot is equals to g times gamma v minus gamma v prime time z into 1 by T v 0; times 1 plus gamma v z divided by T v 0.

So, this is you have to multiply this with this. So, g by T v 0 can stay outside and then if we neglect, terms because there is a z here and there is a z here terms with z square why do you want to neglect terms with z square? z itself is very small. So, any incremental displacement from z is equal to 0 is very small that there is the basis of your assumption. Now, if you want z square to be neglected you say that z square is even smaller than compared to z.

So, if you do that then the differential equation is z double dot is g by T v 0 times gamma v minus gamma v prime times z. So, what kind of differential equation is this? This is a second order differential equation by T v 0 times gamma v prime minus gamma v times z is equals to 0 right. So, this is the second order differential equation that you have you want to solve right.

Now, what does this equation signify? This equation gives you the position of the air parcel as a function of time, how it changes with respect to the with respect to time. If there is an imbalance between the pressure gradient force and the gravity right. So, if you solve this equation, the form in which z can be written as a function of time, we will tell you how the air parcel moves with respect to time right.

Now; obviously, the solution of this equation can accept will surely depend on the rate at which the temperature inside the air parcel changes in comparison to the rate at which the temperature of the environment changes with respect to the height right. So, there are three distinct possibilities. I mean  the solution z of t, which can satisfy this equation will depend on three unique possibilities. The 3 unique possibilities represent 3 different types of stabilities let us say.

So, ones possibility is gamma v prime minus gamma v greater than 0; gamma v prime is equals to gamma v and gamma v prime less than gamma v. Or gamma v prime minus gamma v is less than 0 right. So, these three possibilities will make the solution different, each time for this differential equation right now. So, we will discuss about these three possibilities what are these three possibilities how the solution will be different each time. And how the different solutions actually manifest different types of stability is going to be the topic of discussion for the next class ok.

So, we will stop here.