Introduction to Atmospheric and Space Sciences
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Lecture – 25
Pseudo- Adiabatic Processes

Hello dear students, we will continue our discussions on atmospheric thermodynamics. So, far we have seen what is the idea of latent heat? And when do we have to accommodate latent heat in the discussions of saturation and evaporation right.

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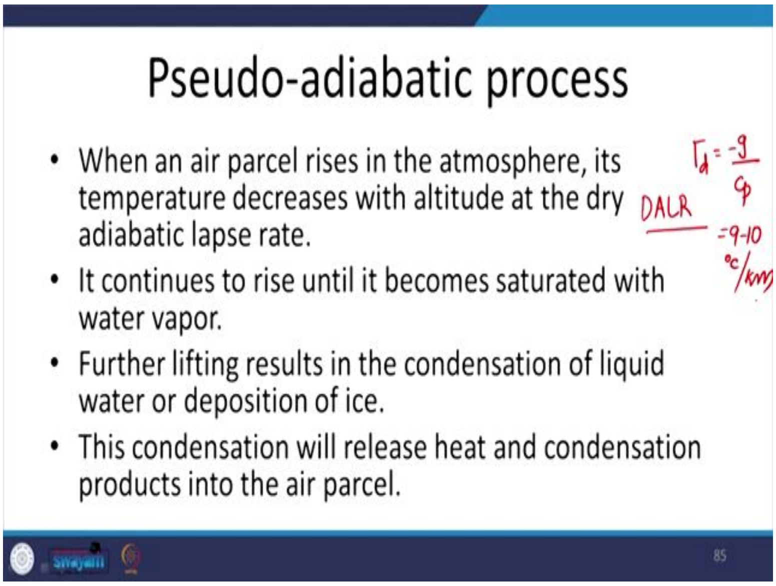

So, what are the pseudo adiabatic processes? When let us say an air parcel rises in the atmosphere its temperature decreases with altitude at the dry adiabatic lapse rate. So, when an air parcel rises in the atmosphere its temperature decreases with altitude at the dry adiabatic lapse rate. So, dry adiabatic lapse rate we will always refer as DALR. So, the dry adiabatic lapse rate is minus g over C_p which is 9 to 10 degree Celsius for every kilometer right, so this is what we have derived already right.

So, when the air parcel rises what happens? Its temperature will decrease with respect to height. As it continues to rise until it becomes saturated with water vapor right. So, here the inside the air parcel there is no physical addition of moisture, but its saturation mixing ratio is lowered with respect to the temperature. So, it reaches the mix existing mixing ratio on the parcel then can be called as saturated right.

When you lift it beyond the lifting condensation level or the point of reaching w_s; for the lifting results in the condensation of liquid water or deposition of ice. If it is very cold it may result in a phase transition between vapor to solid or generally when it is reached the lifting condensation level the moisture which is now saturated.

So, generally what will happen if you reach saturation you cannot add any more amount of let us say water vapor then the water vapor will condense. The condensation will give out two things; one it will release some heat into the air parcel, and two it will release or it will make the droplets tiny droplets to form.

So, these two can be called as the condensation products. So, this condensation will release heat and condensation products into the air parcel. Now, what is your air parcel? Idea of air parcel is such that it is an adiabatic entity. I mean it does not allow any amount of heat to be added or any amount of mass to be added into it this is a basic idea right.

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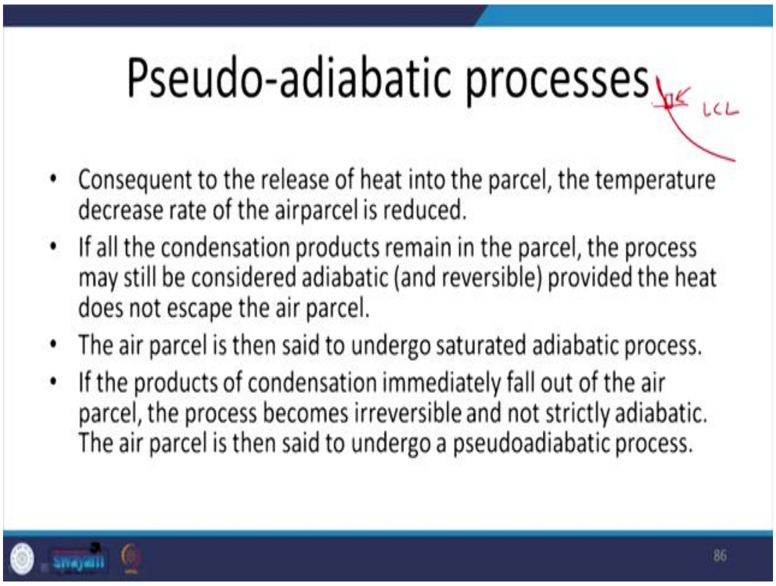

So, consequent to the release of heat into the air parcel the temperature decreases the rate of the air parcel is reduced right, so the temperature decrease. So, the idea is, if the temperature is decreasing with respect to height along this like this let us say suddenly at this point you identify LCL. Suddenly, when it reaches this point, so the temperature should decrease at this rate ideally.

But, since there is addition of heat into the air parcel at this point the rate at which the temperature decrease will not be this, will not be so fast, it will be lesser fast right. So, the idea is as the air parcel reaches saturation it will leave out the products of condensation which is heat and water droplets. As a result of this additional heat that is being added inside the air parcel the temperature of the air parcel will not decrease as it used to decrease according to DALR.

If all the condensation products remain inside the air parcel; that means, if the walls are not allowing the heat to be given out or if the walls are not allowing the mass the liquid mass to be given out. The process may still be considered as adiabatic right and reversible. Reversible in the sense that if the air parcel is now heated it can come to the same pressure or it can come to the same amount of moisture content. That means, if the air parcel is raised right, so its temperature T has decreased to this point.

So, this is the point of beginning where the temperature is T. So, T prime is the temperature when the air parcel rises to a particular height. And if this height happens to be the LCL then it will result in the formation of droplets and its temperature will again go up right. Now, when do you call this process as a reversible? At this point if the walls are not adiabatic if the moisture is lost or if the temperature that has been raised is also lost. Then if you bring down this air parcel the temperature should increase right.

Even the temperature increases at this point they should be moisture because now if it is not adiabatic completely adiabatic where will you bring the moisture inside the air parcel, so this process is not reversible right. So, if the condensation products remain inside the parcel the process may still be considered as adiabatic or irreversible provided the heat does not escape the air parcel.

In this process the air parcel is then said to undergo saturated adiabatic process because now the things are saturated. And the rate at which the temperature decreases is ideally not equal to the dry adiabatic lapse rate right. If the products of condensation immediately fall out of the air parcel the process becomes irreversible like I said right.

And not strictly adiabatic because, it has lost; the parcel is then said to undergo a pseudo adiabatic process not completely adiabatic, but a pseudo adiabatic process right. Now, with the idea that the temperature of the air parcel suddenly increases as it reaches saturation or as it reaches lifting condensation level. It is evident that we cannot use the same value of dry adiabatic lapse rate throughout the atmospheric column.

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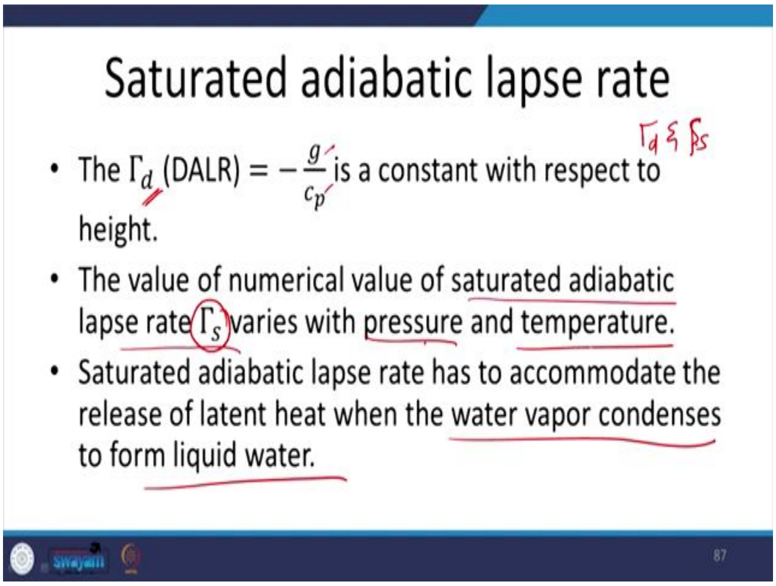

Let say 10 kilometers generally what we say is it will be a 10 degree Celsius decrease in the temperature for every 1 kilometer that you travel upwards right. So, this dry adiabatic lapse rate is a constant, constant in the sense g over $C_p$right.

So, is a constant with respect to height there is no variation in gamma d with respect to height. But, generally what happens when it reaches saturation the parcel reaches saturation suddenly heat is added and temperature increases. So, the rate at which the temperature decreases after reaches saturation will be the different in comparison to the rate at which temperature before it reaches saturation.

The numerical value saturated adiabatic lapse rate varies with pressure and temperature. So, now, we are talking about another lapse rate gamma s. So, far this is gamma d which is the dry adiabatic lapse rate. Now, we talked about what is called as gamma s which varies with pressure and temperature. The saturation adiabatic lapse rate should now accommodate the release of latent heat when the water vapor condenses to form liquid water this is the basic idea. So, gamma d and gamma s, are different right let us see how?

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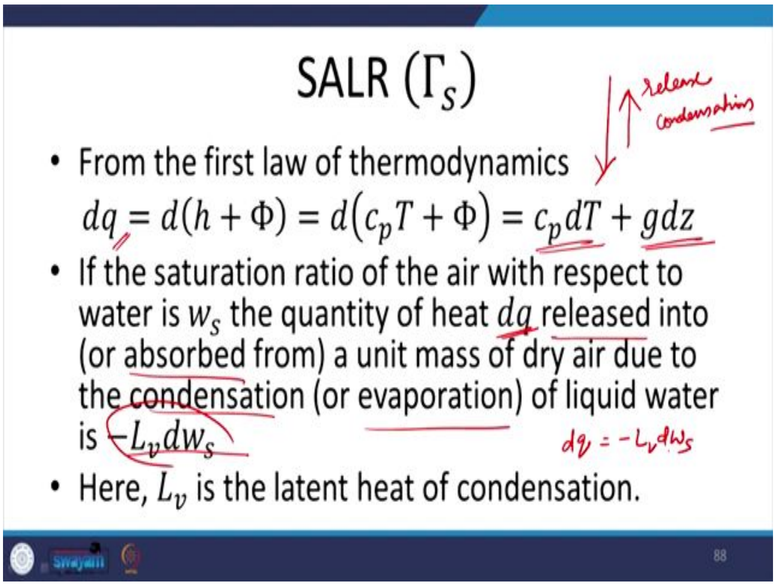

We will try to derive an expression for the saturated adiabatic lapse rate by starting from the first off thermodynamics. Where h plus phi is defined as the dry static energy d q is equals to that the total heat content of the system plus the geo-potential right.

So, , so we can simply write g d q the heat that is supplied is equal to C_p dT, C_p dT is coming from the enthalpy gdz is coming from the geo-potential right. If the saturation ratio of air with respect to water is w_s, the saturation ratio of air with respect to water not dry air here with respect to water is w_s. The quantity heat the quantity of heat that is d q that is released into the air parcel by a unit mass of dry air due to the condensation right of liquid water becomes minus L_v dw_s minus is because the heat is released right.

Now, here I have accommodated the provision to release of heat as well as absorbed condensation or evaporation. So, release is when the parcel rises right heat is released and release is also accompanied with condensation.

Now, when the parcel is sinking let us say exactly opposite will happen instead of release it will be absorbed and it will be evaporation right. So, L_v is the latent heat of condensation, so like we said we need a minus sign to accommodate the release of heat right. So, we will simply substitute d q, so d q is minus L_v dw_s (Refer Time: 09:36) we will put this into d q it is C_p dT plus gdz.

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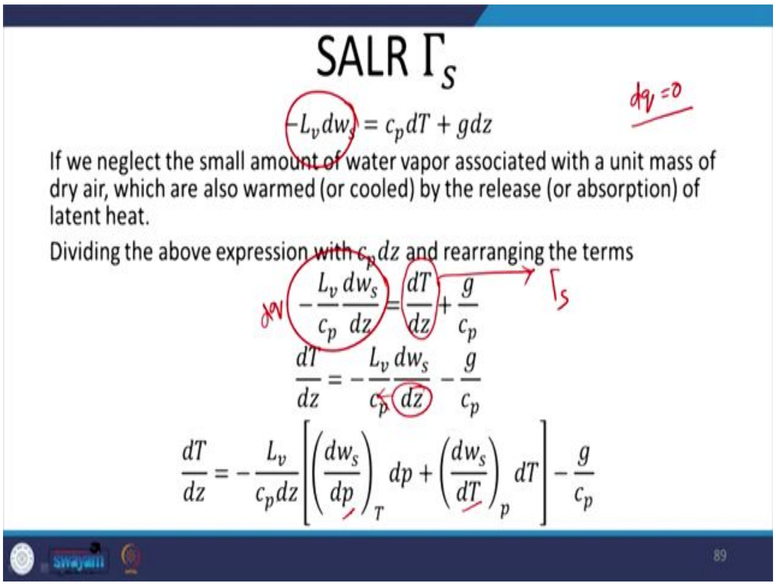

If we know at this point if we neglect the small amount of water vapor associated with a unit mass of dry air which are also warmed or cooled by the release of latent heat let us say ok. So, then let us say we divide this expression with dz we will divide this expression with d z we get this minus L_v over C_p, So, I bought C_p over here, so, for forgetting this in this form g by C_p is conveniently replaced with gamma d can be replaced right.

So, L_v v by C_p dw_s by dz is equal to dT by dz same thing. So, I divided this expression with C_p and divide this expression with rather simply put C_p dz right. So, here dT by dz is now the change of temperature with respect to height. But, how is it different? Because now earlier case we have simply equated this to be 0 right.

So, earlierwhen we were discussing the dry adiabatic lapse rate. So, simply we have made dq is equal to 0, because it is perfectly adiabatic there is no addition of heat there is no release of heat. So, dq is equal to 0, but this term now dq is this right. So, dq upon simple algebra, so this term is still dq you make this term to be 0 you will still get dT is dt by dz is equals to minus g over C_p which is the dry adiabatic lapse rate. Now, we have to deal with this nonzero term dq right.

So, dT by dz since now this is saturated adiabatic lapse rate, so this gamma can be called as gamma s. So, dT by dz simple rearrangement dT by dz is equal to minus L_v dw_s by C_pd z minus g by C_p right. Now, what I did is I brought dz out and variation of w_s with respect to pressure and temperature. Like I said in the slides before they lay the saturation with it will be varying with respect to. So, the numerical value of saturated adiabatic lapse rate varies with pressure and temperature right.

So, we will see what is the variation ofsaturation mixing ratio With respect to pressure plus with respect to temperature alright. So, this is a simple algebraic rearrangement. So, dT by dz can now be written as, so dw_s by d s by dp plus dw_s by dT times dT at a constant temperature and at a constant pressure right.

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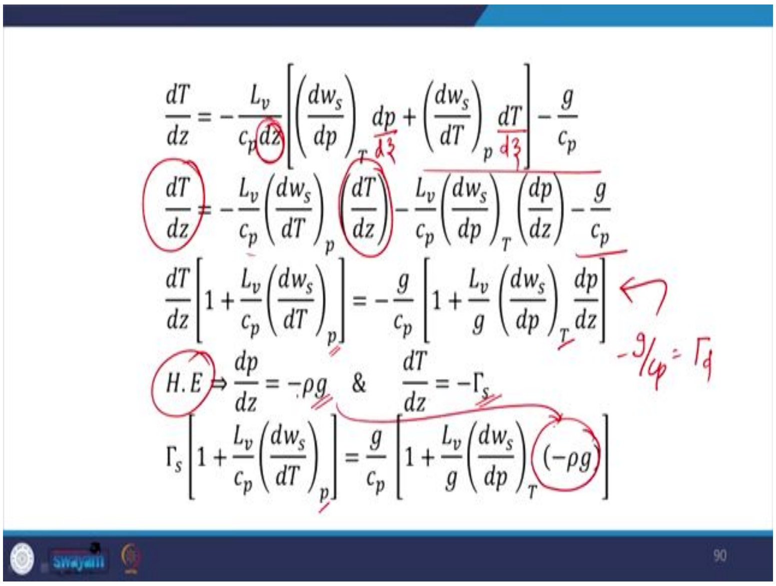

So, we will make this algebraic rearrangement again. So, dT by dz have been pulled out from here right; dT by dz is equal to L_v by C_p dz, L_v by C_p dz this as it is minus g by C_p. So, here dw_s by dT is this part this part is pull here L by C_p dz, dz is taken now here dt by dz minus L_v by C_p dw_s by dp into dp by dz.

So, d this is dz goes here and here minus g by C_p remains right. Now, what I have done is I have taken dT by dz common from this part and this part dT by dz is equal to 1 plus this becomes plus when you pull it to the left side. 1 plus L_v by C_p into dw_s by dT at a constant pressure right is equals to minus g by C_p 1 plus L_v by g dw_s by dp at a constant temperature times dp by dz right. So, the hydrostatic equation says the rate of change of pressure with respect to height is equals to minus rho g right.

And the rate of change of temperature with respect to height is gamma s and g over C_p is gamma d right. So, you substitute this, this, and this into this equation I will get gamma s is dT by dz into 1 plus L v by c p dw s by dT at a constant pressure is equal to g by c p into. So, I have substituted this hydrostatic equation here right.

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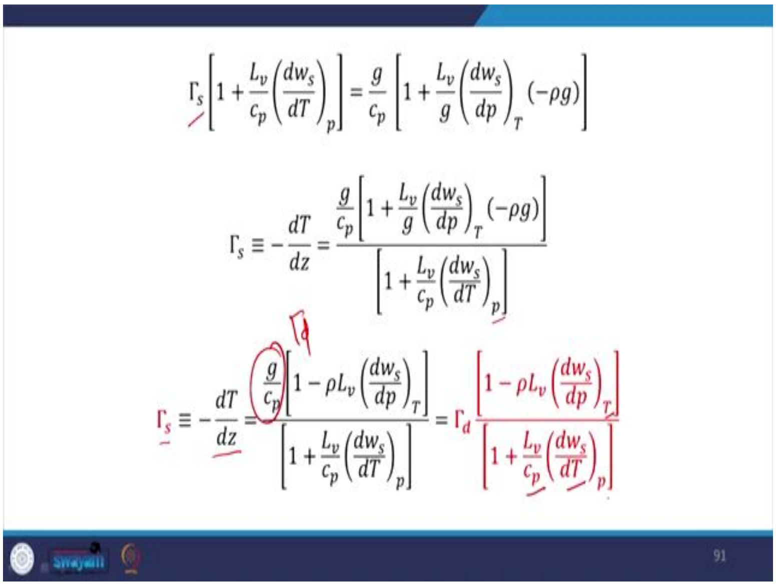

Then gamma s the saturated adiabatic lapse rate simply becomes g by c p times 1 plus L v by g dw s by dp at constant temperature times minus rho g divided by 1 plus L v by c p dw s by dT at a constant pressure right. So, gamma s is minus dT by dz that to signify the decrease of temperature is g by c p.

g by c p now called as gamma d gamma d times 1 minus rho is the density which came into the into this mathematics from the hydrostatic equilibrium is rho L v dw s by dp at a constant temperature. And 1 plus L v by c p dw s by dT at a constant pressure right.

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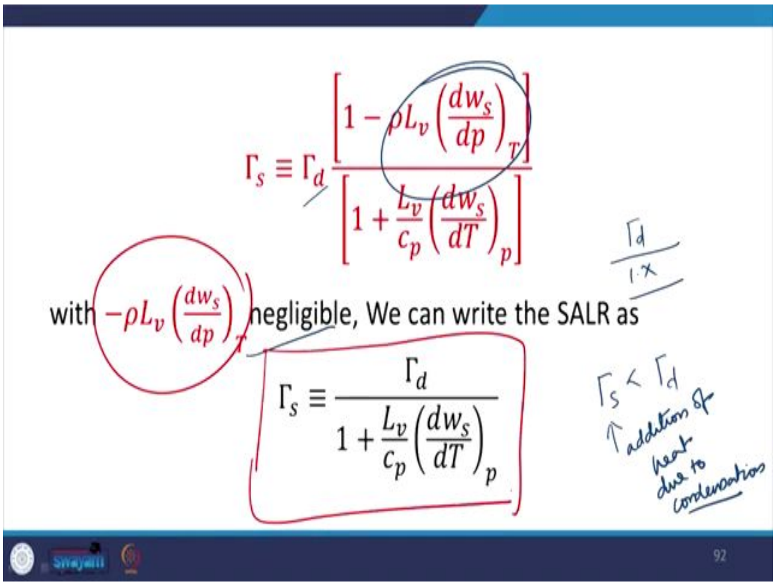

So, generally for atmospheric conditions we can say that this term which appears here this term is negligible right. So, here, I can say that gamma s is gamma d times 1 by 1 plus L v by c p into dw s by dT constantly right. So, here in the denominator you have a 1 plus term right.

So, whatever is gamma d when you divide it with 1 point let us say x gamma s will always be less than gamma d. That means, the saturated adiabatic lapse rate will always be less than the dry adiabatic lapse rate. So, we will realize that the saturated adiabatic lapse rate is nearly 6 to 7 degrees Celsius per kilometer.

And the dry adiabatic lapse rate is we have seen that it is 10 degree Celsius per kilometer right. So, but, so why is it less? Why is it less because saturated adiabatic lapse rate has to accommodate an addition of heat addition of heat due to condensation that is it as simple as that right.

So, one, so this is something about the saturated adiabatic lapse rate we start from the thermodynamic the first law of thermodynamics. So, the first law of thermodynamic was also the starting point for deriving the dry adiabatic lapse rate, but then we simply made dq is equal to 0. So, here is very simple, so we just that dq being written in terms of the latent heat has given us this expression right. So, this is a very important expression in the sense that, so this expression is for saturated adiabatic lapse rate right.

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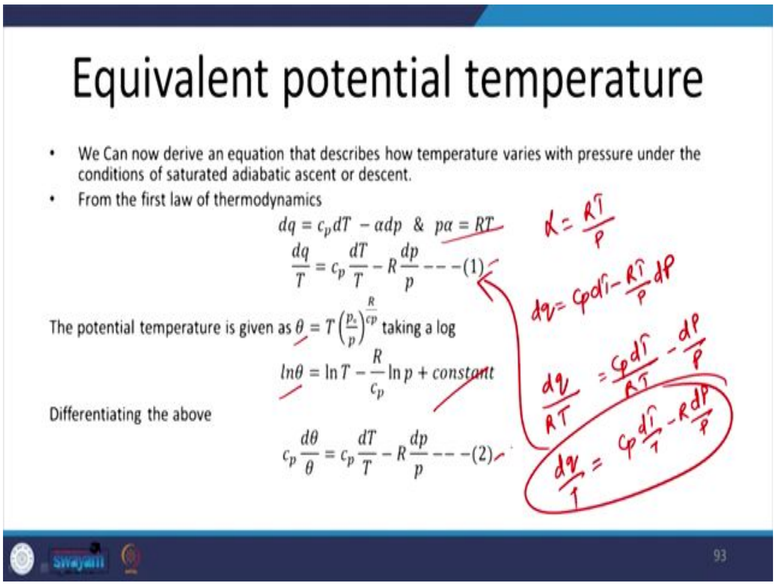

Now, we define what is also called as we have seen what is the potential temperature. The idea of potential temperature was simply, you take an air parcel it can exist at any particular time any pressure it can be at a pressure which is less than the atmospheric pressure standard atmospheric pressure at sea level or more than right.

When you bring it to the level of atmospheric pressure the temperature then is called as the potential temperature. We can now define what is equivalent potential temperature? When let us say how equivalent potential temperature describes? How temperature varies with pressure under the conditions of saturated adiabatic ascent or descent.

So, how is this different? Let us say, so earlier when we compressed the air parcel to reach atmospheric pressure or we expanded the air parcel. When we expanded air parcel to reach the atmospheric pressure the resulting temperature was known to be the potential temperature. But, in this process if saturation happens or if addition of heat happens then the resulting temperature will slightly be different.

Because the temperature should also simply speaking if you allow condensation to happen the resulting temperature will be larger in comparison the potential temperature right. So, basically the process is still the same where you reach the standard pressure, but here you define the equivalent potential temperature to accommodate saturation right.

So, we start from the first law of thermodynamics dq is equal to du plus dw where we have used the enthalpy and the geopotential or specific volume dq is equal to c p dT minus alpha dp. So, p alpha is written as RT, so R alpha can be replaced with RT by p. So, I write dq is equal to c p dT minus alpha is RT by p RT by p dp or dq will divide by RT dq by RT is equal to c p dT by RT minus dp by p.

Or in this convenient form it is dq by d is equal to cp dT by d minus R dp over p all right, so this is the same equation that we have here right. So, potential temperature was defined as this theta is equal to temperature times p naught by p rise and R by cp rise it to the power.

Now, take a logarithm lon of theta is equal to lon of T minus R by c p lon p plus constant. So, here, when we differentiate this expression let us say lon theta its c p d theta by theta is equal to c p dT dT by T I take an log and then we have differentiated this is really get rid of this constant. So, let us call this equation as 1 and this equation as 2. So, what have we done? We have done two things I mean we have taken the first law of thermodynamics replaced this specific volume.

So, that we have gradients with respect to pressure and temperature we call that equation as 1, then we took the potential temperature taken a logarithm of it then differentiate it. So, that we have gradients with respect to temperature and pressure right. Now, these two this two terms these two equations are identical, identical in the sense that, so on the right hand side of both the equations c p dT by T minus R dp by p both the equations are the same. That means, there should be an equivalence between dq by T and c p d theta by theta.

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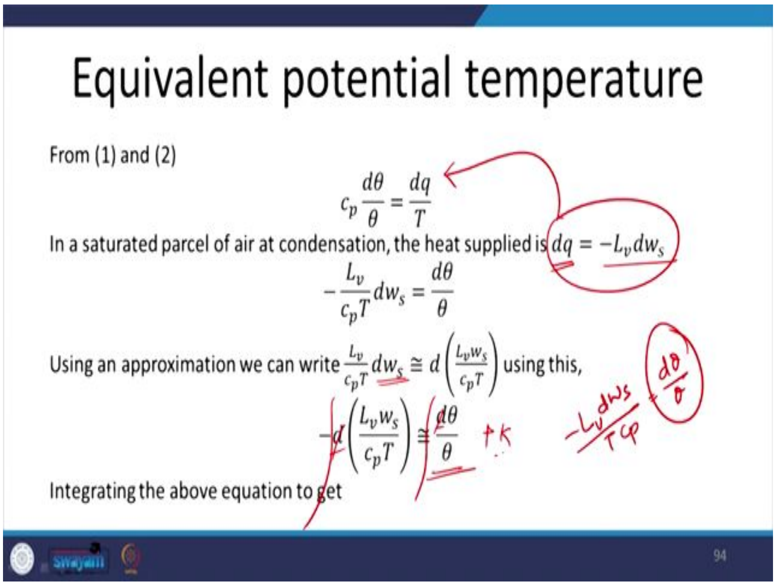

So, from 2, so c p d theta by theta is equal to dq by T this is equal to dq by T, so this one is equal to this one. So, now we bring in the idea of latent heat just like we did in the case of getting the expression for the saturated adiabatic lapse rate now we bring in the idea of latent heat. So, the latent heat is dq which is minus L_v dw_s release of heat.

So, dq we substituted this dq into this right. So, minus L v c p T dw s is equal d theta by t right minus L v dw s by T c p is d theta over theta. Now, this change I mean this temperature theta to d theta should be the equivalent potential temperature. So, using an approximation, so what we are doing is, so the this is derivative with respect to the mixing ratio only on the mixing ratio.

Under special conditions we can say that the we can pull out this derivative and say that d is not just operating on w_s is rather operating on the product of L_v and w_s. So, in that case and, so d theta by theta remains as the same on the right hand side, so this is d of L v ws by c p T. So, we have d and d, so now, we can integrate this equation right now we can integrate this equation to give you.

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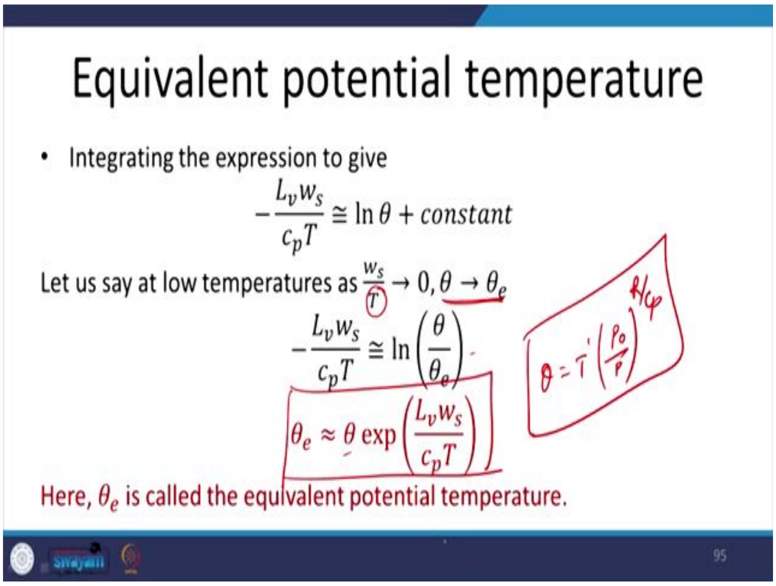

So, it is just if you integrate this equation you will get rid of these and we will get a constant. So, minus L_v w_s by $_p$ T is lon theta plus constant. So, at let us start at low temperatures at very small temperatures generally when w_s by T the saturation mixing ratio over temperature when it tends to 0 that means temperature is very low right.

Then theta the potential temperature can be equated to the equivalent potential temperature. That means, lon, so this is a minus L_v w_s by C_p T becomes approximately equal to lon of theta by theta e. So, theta e is, so from this expression theta e is equals to theta times exponential L $_v$ w_s by Cp T, this is called as the equivalent potential temperatures. So, just in contrast, so, theta is equals to T prime times p naught by p over R by c p.

So, this is the potential temperature and this is the equivalent potential temperature right. So, this is these are some details about equivalent potential temperature right. Now, the important message is that, so when you allow saturation to happen. So, as long as you are discussing mixing ratio when, but when you discuss saturation mixing ratio from that point you should accommodate the addition of heat.

So, all the adiabatic equations that we have seen should now be complemented with a non zero dq term. And then by doing, so we have derived an expression for gamma s. And by doing, so we have also derived an expression for a potential temperature then we are calling it as an equivalent potential temperature not just potential temperature right.

So, we will stop here, so we will continue our discussion, so on deriving lapse rate for the parcel. So, far we have derived lapse rate for the atmosphere we will try to derive what will be the rate at which the temperature decreases with height inside the parcel will it be different right. When will it be equal to the dry adiabatic lapse rate things like that.