Introduction to Atmospheric and Space Sciences
Prof. M. V. Sunil Krishna
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 23
Moisture Parameters

Hello dear students, so far in our discussions of atmospheric thermodynamics we have been able to understand what is in air parcel and what are the basic laws of thermodynamics written in terms of atmospheric variables and how the changes of pressure and volume can be attributed to something called as dry adiabatic lapse rate, what is called as the potential temperature and so on right.

So, we have seen that by adding a little moisture in the atmosphere which is otherwise nitrogen oxygen and argon and carbon dioxide, if we add a little amount of water vapour, its main molecular weight changes and its density parameters will also change. So, in today's class we will try to understand what will be the role of moisture, how we quantify the amount of moisture that is present in the atmosphere or how do we make modifications in the dry adiabatic lapse rates to be able to suit the real atmosphere which also contain some amount of moisture right.

So, the amount of water vapor in a certain volume of mass of air let us say may be defined as the ratio of mass of water vapor to the mass of dry air.

So, let us say how do you quantify the amount of water vapor in a certain volume of air may be defined as the ratio of mass of water vapor to the mass of dry air. So, dry air is considered to be the 1 and you are adding moisture into it. So, this mixing ratio let us say w, so the way of quantifying the amount of moisture in dry air is by defining the mixing ratio.

So the mixing ratio is defined or is generally indicated with a w; w is m_v by m_d . So, m_v is the amount of water vapor in its vapor form let us say amount of water vapor and this is dry air ok. So, the mixing ratio is generally expressed in grams of water vapor per kilogram of dry air. I mean this is a way to express it. We cannot; obviously, use it in the calculations in the same units. But, we express it in terms of the number of grams of water vapor per 1 kilogram of dry air is very important.

But in; however, in calculations we use it as a number which is kg per kg, kg of water vapor. The amount of water vapor in kilograms per 1 kilogram of dry air, it is just a number ok. So, in earth's atmosphere most importantly in earth's atmosphere the magnitude of mixing ratio w typically ranges from few grams per kilogram generally. Or it may go as high as 20 gram per kilogram in the tropics. So, why tropics; they are hot, so there is more amount of evaporations. So, naturally you expect more amount of water vapor to be present in the dry air right.

So, generally the highest number that you may want to remember is at an given situation is 20 gram of water vapor per 1 kilogram of dry air right. If there is no condensation or if there is

no condensation that is trying to take away this vapor and can I mean make it into liquid water. Or there is no evaporation which can add more amount of water vapor into the air parcel, the mixing ratio of the air parcel more or less remains a constant or it remains to be a conserved quantity right. So, then now we will see how we get from the idea of mixing ratio to specific humidity right.

(Refer Slide Time: 04:21)

So, the mass of water vapor m_v in unit mass of air not dry air. So, here most important the mixing ratio was defined to be the mass of water vapor divided by mass of dry air right. Now, we define another physical quantity which is called as the specific humidity not a relative humidity we will get there. This is specific humidity; the specific humidity is defined as the amount of water vapor in a unit mass of air not dry air.

What is the air? Air is dry air plus water vapor which has both of it; I mean, so this is called as the specific humidity. So, this is m_v water vapor divided by m_v plus m_d. So, which is w by 1 plus w; we divide by m_d on numerator and denominator you will get this right. So, this quantity q is generally called as the specific humidity. So, we define w to be the ratio between water to dry air, we define q to be the ratio between water to the air that is it. So, it is to be noted that the magnitude of w is only a few percent it is always a very small percent.

It follows that the numerical values of w and q are nearly equivalent I mean they do not differ much right. So, here when you take a mixture of gases generally you talk about partial pressure. What is the partial pressure? Partial pressure is the pressure that is exerted by a

constituent of the mixture as if the constituent itself is occupying the entire volume to itself at the same temperature right, so that is the idea of partial pressure.

So, the partial pressures that is exerted by any constituent in the mixture of the gases is proportional to the number of moles of that constituent in the mixture, what does it mean? So, see here a partial pressure was defined as the amount of pressure that the constituent gas exerts as if it alone occupies the entire volume.

And the magnitude of pressure is proportional to the number of moles; obviously, the number of molecules is written how, in terms of number of moles. So, more the number of moles the partial pressure will be larger right. So, then it will be the number of moles of that particular constituent in the mixture right.

(Refer Slide Time: 06:54)

Hence the partial pressure, so we always define I mean from here onwards we always define the partial pressure due to water vapor is defined as e small e. The pressure e due to the water vapor is given by this is a partial pressure which is proportional to the number of moles of water vapor in comparison to the total number of moles of the dry air itself right. And this is the total pressure, p is the front pressure and e is the partial pressure. Now, we know that if you want to write the partial pressure for the rest of the gas spare apart the moisture you will be able to write P is equal to n d divided by n v plus n d.

Such that your total pressure P is equal to partial pressure e plus the pressure due to the dry air right. So, if you rewrite the number of moles as weight by molecular weight. So, small m_v is the mass of water vapor and capital M_w is the molecular weight of water. And, so this is what is done m_d is the mass of dry air and capital M_d is the molecular weight of the dry air right.

So, now we use the mixing ratio the idea of mixing ratio w is equals to m_v by m_d. So, what is this? This is the expression for the vapor pressure is small e. We use w is equal to m_v by m_d we can rewrite this expression this particular expression as e is equals to w by (this w ,we already seen what is this w) w plus epsilon times the total pressure right. So, this epsilon is already defined this epsilon is equals to 0.622 as the ratio of specific gas constants right.

Now, , so we have seen what is the mixing ratio? How do you define the mixing ratio in terms of masses of water vapor and the dry air. Then we have seen how is specific humidity different from the mixing ratio? Then now we want to write the specific humidity also in terms of the virtual temperature right. The virtual temperature is defined to be T_v is equals to T by 1 minus e by p into 1 minus epsilon.

(Refer Slide Time: 09:10)

Where e by p is the ratio of partial pressure of the moisture to the total pressure, epsilon is the ratio of molecular weights right. So, from the earlier slide say e by p e by p is w by w plus epsilon right. If you use that, so the virtual temperature can be written as T_v is equals to T

times w plus epsilondivided by epsilon into 1 plus w all right. This is just simple algebra you can just substitute this into this and we will get that right.

So, now if you take this epsilon into 1 plus epsilon above and multiply it. Let us say, so what you have got is let us say T_v is equals to T times let us say w plus epsilon into this is you take with a minus you take this with a minus you take it up. If you dividing the denominator into the numerator and you multiply it and if you neglect all the higher order terms in terms of w square. Because, we have seen that the w the mixing ratio is very small in comparison to the 1 kg of dry air, so there is no point of having the terms w square.

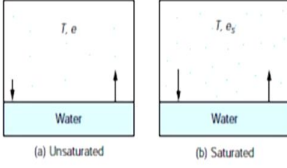So, we can write that T_v minus T is approximately equal to 1 minus epsilon divided by epsilon into w into T; where w is in the mixing ratio T is the temperature right. So, with epsilon is equals to 0.622 we can write the T v is equals to T into one plus 0.61w. So, this is a very important expression that we may use in our numerical calculations. So, with the given mixing ratio, so we already seen what should be the temperature that you get? If you heat an air parcel such that its density becomes equivalent to the density of moist air is called as the virtual temperature right.

So, you have to heat it, so that the temperature increases the density decreases, so that it reaches the density of moist air right. Now, moist air of course, indicates the idea of having a little amount of moisture in the dry air, so this that is signified with w. So, w speaks of the information of how much amount of moisture is there and by what magnitude the density has gone down right. So; obviously, with this information, so you can calculate the virtual temperature if you know the temperature and the mixing ratio right.

So, that that was something about the vapor pressure. So, simply put vapor pressure is the amount of pressure that is exerted by the vapor. So, if you have a small closed enclosure and if you keep water at the surface of this closed enclosure what will happen? Slowly evaporation will take place and it is evaporated molecules which are in the vapor phase cannot escape.
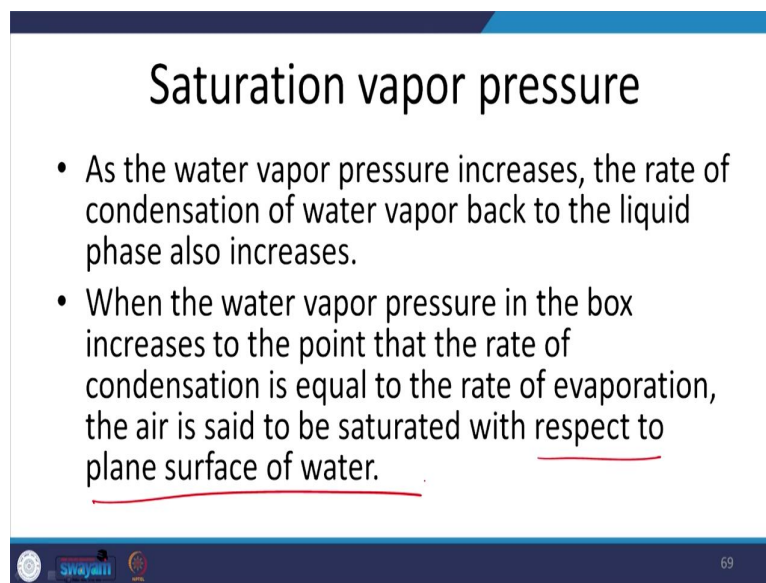
In addition to the air that is existing in this enclosure vapor molecules will also exert some pressure. So, that partial pressure this vapor molecules will exert on the surface is referred to as the vapor pressure right. Now, this vapor pressure is a partial pressure always remember that. And in addition to that, so far these are the variables I mean how do you define? How much amount of moisture is there? What is the relevant physical quantity which signifies this percentage to which moisture is present? What is specific humidity? How do you write specific humidity in terms of the virtual temperature? Things like that right.

Now, after that we will see what is called as the saturation vapor pressure right? So, consider a small closed box the floor of which is covered with pure water at a particular temperature T. Now, if this temperature is very small or large there is a different story this time there is a particular an ambient temperature T. Now, initially let us say when you a close the enclosure right, initially that there is the air is completely dry because once you create this enclosure the air is completely dry, so there is no moisture in it ok.

So, water will begin to evaporate and it does the I mean as it as the evaporation is happening the number of water molecules in the in the box and therefore, will increase. So, therefore, the vapor pressure also increases; obviously, right. So, here, so the diagram is unsaturated and saturated we will get there right. So, here the temperature is T and e is the vapor pressure. Now, as the vapor pressure increases the rate of condensation of water vapor back to the liquid phase also increases.

There is an evaporation which is happening and as the time progresses some molecules may condense and return to the liquid phase. But, also remember that the dry air the air that was in the enclosure was dry to begin with. So, when the water vapor pressure in the box increases to the point that the rate of condensation is equal to the rate of evaporation the air is said to be saturated with respect to plane surface of water.

(Refer Slide Time: 14:48)

This is always is always remember this, the air is said to be saturated with respect to plane surface of water not you should not leave the statement the statement is complete when you say it is saturated with respect to the plane surface of water right.

Now, the pressure exerted by the vapor at the saturation at this point where the evaporation is equal to the condensation. At this point the partial pressure that is exerted by the vapor water vapor is called as the saturation vapor pressure e_s ok. So, this is over a plane surface of water. So, so we can also extend this argument saying that rather than having water on the surface in the closed enclosure we can also have pure ice I mean justice not water right.

So, then also some point molecules will vaporize and also this vaporized molecules will condense and form liquid water on the surface of this ice. So, ultimately if you leave it long enough it will saturate and that vapor pressure which exists at equilibrium or at saturation is called as vapor pressure with respect to plane surface of ice.

So, in this in our discussions we will write e_s as the saturation vapor pressure with respect to plane surface of water, and e_{si} as a saturation vapor pressure with respect to plane surface of ice. Now, which process do you think is the easiest to create moisture in the dry air? Because, let us say if you want to create moisture out of the ice then you will have to go by two different phases. You will have the first the first ice has to melt and then this melted water should evaporate and create water vapor in the dry air right.
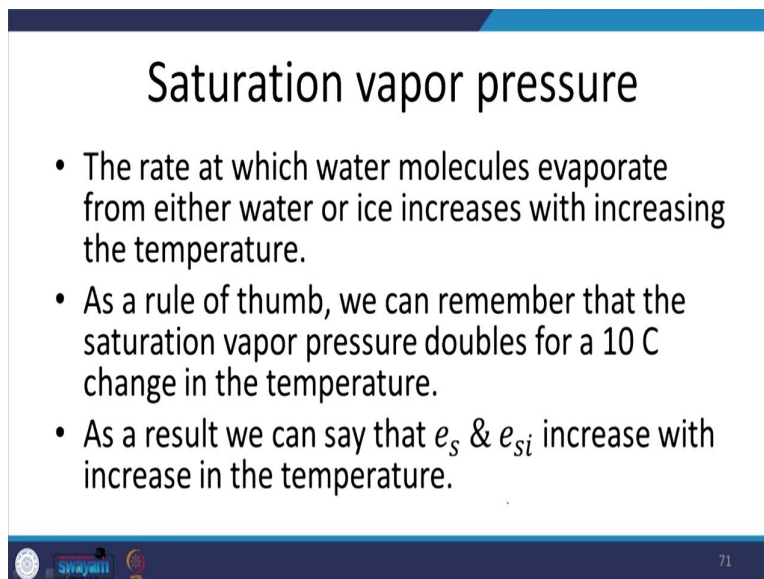
So, because at any given temperature I mean now comes the idea of temperature. So, at any given temperature evaporation rate over a plane surface of ice is always lesser than the evaporation rate over a plane surface of water. That means, it takes more amount of energy

for water vapor to form from a plane surface of ice in comparison to the plane surface of water.

That means if it is taking more time or if it is taking more energy. So, saturation at the saturation there will always be more number of molecules due to the plane surface of water in comparison of the plane surface of ice. So, more number of molecules means more amount of pressure. So, the saturation vapor pressure at a given temperature over a plane surface of water will always be greater than the saturation vapor pressure over a plane surface of ice.

So, this is what is given here e_s of T is always greater than e_{si} of T; that means, but always remember at a given temperature right. Now, what will happen if you change the temperature is the most important thing. So, the rate at which water molecules evaporate from either water or ice increases with increase in the temperature.

(Refer Slide Time: 18:06)

So, this idea is the one which explains all the clouds or all the different types of clouds the fog that you see, the condensation, the rain, the height at which the clouds form. All these concepts are all this concepts can be understood from the simple idea that the saturation vapor pressure will depend on the temperature right

So, just before we get there, so one simple foundation that you should have is that vapor pressure is simply a partial pressure. So, you see the total pressure you have if you take a gas

which also has some moisture. The total pressures that this gas exerts on the surface is equal to the sum of different partial pressures right.

You can go to the extent that you can divide the constituents of the gas what and all oxygen is there, nitrogen is there, carbon dioxide is there. You can go to the extent that what is the partial pressure that is exerted by oxygen alone? What is the partial pressure is exerted by the nitrogen alone? What is the partial pressure is exerted by the carbon dioxide alone? Like that you go and say that the amount of pressure that is exerted by the H_2O molecules is called as the vapor pressure
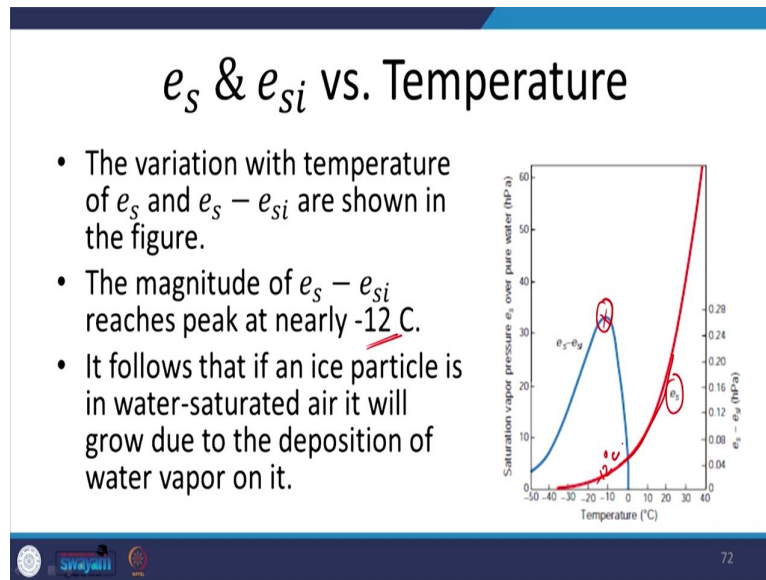
When saturation is achieved; that means, equal amount of condensation and equal amount of evaporation is happening. You define this vapor pressure due to water molecules as the saturation vapor pressure. Now, if you have water surface or ice surface the saturation vapor pressure will be more due to plain surface of water in comparison to plain surface of ice.

Now, most importantly this values the saturation vapor pressure; that means, the pressures that can be exerted by the water vapor will depend on the temperature of that gas. If you have more temperature the saturation vapor pressure will be larger, and if you have less temperature the saturation vapor pressure will be smaller, how? If the temperature is more; obviously, the evaporation will be more right.

So, the more number of molecules can naturally be expected to be present in the given closed enclosure. If there are more number of molecules; obviously, the pressure will also be more right that is why the rate at which water molecules evaporate from either water or ice increases with increase in the temperature. So, always remember as a rule of thumb we can always take it for granted that the saturation vapor pressure doubles for a ten degree Celsius change in the temperature.

So,; that means, every 10 degrees that you increase the temperature of the gas the saturation vapor pressure nearly doubles ok. So, as a result we can simply say that the saturation vapor pressure due to plain surface of water or saturation vapor pressure due to plain surface of ice increase with increase in the temperature right.

So, this is e_s and e_{si} versus temperature. So, if you see this is the red curve that you see to the right is the saturation vapor pressure due to a plain surface of water. So, what do you see? You see that as the temperature is increased it increases very fast right. That means, that this increase is what I am referring to right. So, if you increase the temperature I mean it will increase to a very large values.

So, the magnitude of e_s minus e_{si} , so this difference that comes just because thepresence of ice. The magnitude of saturation vapor pressure minus the saturation vapor pressure over plain surface of ice at a given temperature varies something like this(figure in the upper slide) that you see varies something like this right. So, this is e_s minus e_{si}, so on the x axis you have the temperature on the y one axis you have the saturation vapor pressure over pure water.

And on the y 2 axis you have the saturation vapor pressure difference between pure water and the plain surface of ice right.

So, the magnitude of this difference reaches a peak nearly at minus 12 degrees Celsius this is where it is minus 12 degrees Celsius. So, what does it mean is that if an ice particle is water saturated then it will grow due to the deposition of water vapor onto it. If a small ice particle is saturated with respect to the ambience then it will grow due to the deposition of water vapor on it right.

Now,. So, we have seen what are the mixing ratios? What are the specific humidities? Right. We extend this to this to the saturation mixing ratios, simply we have a mixing ratio which is the ratio of mass of water vapour divided by the mass of dry air. If you define this mixing ratio, so that it is explained at the saturation you call it as saturation mixing ratio.

(Refer Slide Time: 23:34)

The saturation mixing ratio w_s with respect to the pure surface of water is defined as the ratio of mass of water vapor in a given volume of air that is saturated with respect to plane surface of pure water to the mass m_d of the dry air right. So, m_v is the mass of water vapor at saturation this suffix e_s is to indicate saturation with respect to dry air that is it.

So, water vapor and dry air both obey the gas law right. So, what I have written here is just the pressure p alpha is equals to RT is what is being written here right. So, in terms of, so to accommodate mass in terms of density not in terms of specific volume. So, mass is written as a ratio of the densities.

So, if I write density from the ideal gas equation, so this is this is, e_s by R_v T divided by p minus e_s , what is p minus e_s? P is the total pressure, e_s is the saturation vapor pressure due to the moisture right. So; that means, that what I am accounting here is the pressure due to p due to dry air and I have in the denominator density of the dry air right. So, the saturation mixing ratio is defined as rho v s prime divided by rho d prime which is equals to e_s by R_v T, R_v is the specific gas constant of the humidity or water vapour divided by p minus e_s or p_d,

p_d is the partial pressure due to dry air divided by $R_d dt$ R_d is the specific gas constant of the dry air.

So, rho v s is the partial density of water vapor required to saturate air with respect to water at a temperature T right. Now, so now, the most important point that that the simple take home message is that this rho v s or you define e_s right. You define e_s the saturation vapor pressure is always defined at a particular value of temperature not everywhere. So, the saturation vapor, so you define e the vapor pressure and e_s is defined always at a particular temperature.

So, so e_s is not to be taken as the same number at any value of temperature, the value of e_s changes whenever you change the temperature. That means, within a closed enclosure itself it is not necessary that only if the air contains only this much amount of water you call it as saturation, no. At any given value of temperature there is always a saturation vapor pressure.

So, if you want with a very small amount of moisture content inside the closed enclosure if you still want to say that this is saturated you need not add more amount of moisture physically. What you can simply do is that? You can reduce the temperature to a very small value by reducing the temperature what you are doing is you are decreasing the ability of the air to hold the moisture. By decreasing its stability you are taking it to a point that whatever the moisture that is existing inside is saturated that is it right.
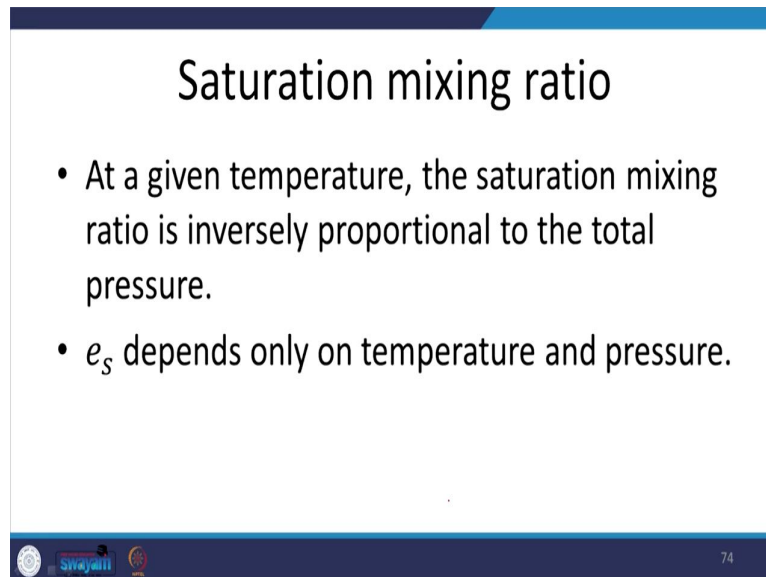
So, the most important message is that it is always the temperature which decides the saturation vapor pressure. Now, why is saturation important for us? All the weather that you see all the clouds rains everything that you see is a phenomena of saturation is something that happens at the saturation. Or when some when the air tries to surpass the saturation point that is when the clouds form, that is when the rains will result all these things.

So, saturation is a very important physical property of this air which results in the formation of the weather right. So, now the most important thing is this saturation depends on the temperature right. So, rho d prime is the density of the dry air, so this is just in this expression. Now, substituting for epsilon we have seen an epsilon relation for epsilon.

So, we can use epsilon in this and we can say that the saturation mixing ratio is 0.622 times e_s saturation vapor pressure divided by total pressure minus the saturation vapor pressure. So, for the range of temperatures that we come across in atmosphere, I mean generally the

saturation mixing ratio can simply be approximated with this right. So, you always need this expression sometime right.

(Refer Slide Time: 28:27)

So, at a given temperature the saturation mixing ratio is inversely proportional to the total pressure at a given temperature this is what I mean to say w_s is inversely proportional to the total pressure. At a given temperature the saturation mixing ratio is inversely proportional to the total pressure and the saturation mixing ratio depends only on temperature and pressure nothing else right.

So, this is some information about moisture variables I mean adiabatic lapse rate was defined such that it is valid for completely dry air in the in the absence of any moisture right. Now,at this point we started from dry air, we added some moisture, we define various parameters, various physical quantities which could quantify the amount of moisture.

And amount of moisture as well as the amount of moisture to call this air as saturated right. Now, we will try to see how we can derive expression for the lapse rates which is valid for the realistic atmosphere; that means, atmosphere which also has some moisture right. So, I will stop here, so I will continue this discussion on atmospheric thermodynamics and lapse rates in the next lectures.