Introduction to Atmospheric and Space Sciences
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Lecture - 22
Air Parcel and Potential Temperature

Hello dear students. So, yesterday we have seen what is the idea of enthalpy, few basic laws of thermodynamics, how we can apply those laws in addition to the hydrostatic equilibrium condition. So, we have stopped the lecture at the discussions of enthalpy.

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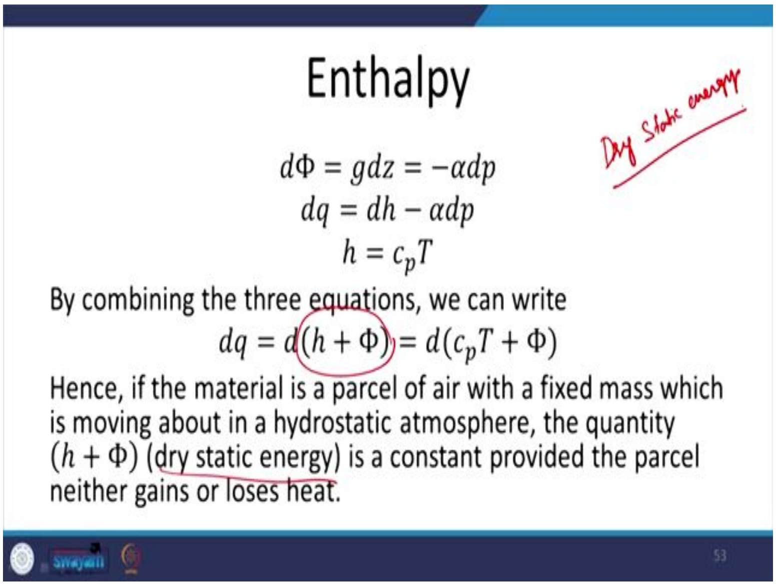

Enthalpy

Dry Static energy

$$d\Phi = gdz = -\alpha dp$$
$$dq = dh - \alpha dp$$
$$h = c_pT$$

By combining the three equations, we can write

$$dq = d(h + \Phi) = d(c_pT + \Phi)$$

Hence, if the material is a parcel of air with a fixed mass which is moving about in a hydrostatic atmosphere, the quantity $(h + \Phi)$ (dry static energy) is a constant provided the parcel neither gains or loses heat.

So, where we have we have identified a specific quantity which is called as h plus phi, where h is the enthalpy, enthalpy is also defined as the total heat content of the system and h plus phi, phi is the geopotential. So, we have managed to combine the enthalpy that is Cp T and the total and the geo potential to identify the heat that is given to the system. So, this h plus phi is known as the dry static energy right.

So, this dry static energy is relevant for the discussions of air parcel or for the discussions about the movement of air parcel in an adiabatic environment ok, dry static energy right. So, here we will take home this particular physical quantity h plus phi and we will try to understand how we can use this to understand, how we can use this particular dry static energy to understand the movement of an air parcel right. Before that, we will try to see what is an air parcel.

The most important thing is if an air parcel, if a volume of air, which is considered under certain conditions moves in the atmosphere with respect to height let us say; such that it is not allowing any kind of energy transferred between the surroundings and the environment that is inside the air parcel. So, in that case it is naturally expected that this dry static energy will remain conserved, I mean it will be a constant in an adiabatic picture right.

So, we will try to see what is the consequence of this conservation right. So, before let us see what is an adiabatic process. So, in majority of the discussions that we have in atmospheric physics, we always take the process to be adiabatic, the movement or the process from one state of equilibrium to another state of equilibrium to be an adiabatic process.

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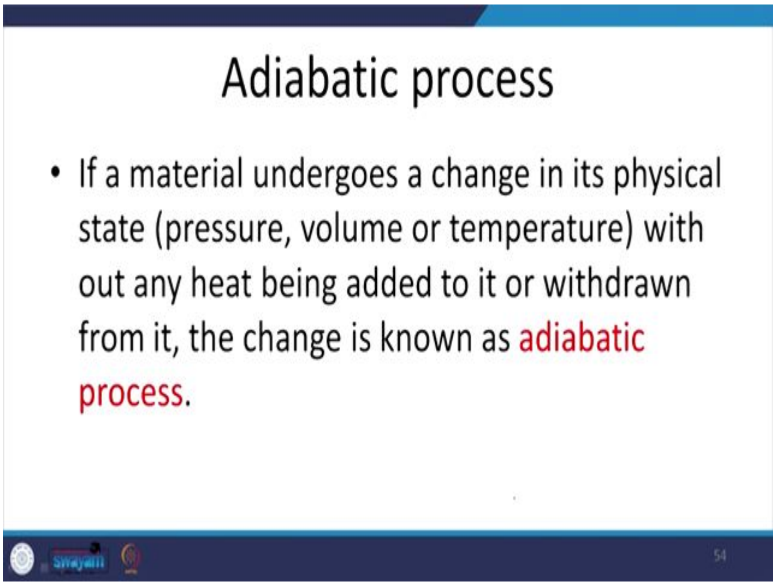

Adiabatic process

- If a material undergoes a change in its physical state (pressure, volume or temperature) with out any heat being added to it or withdrawn from it, the change is known as adiabatic process.

So, what is an adiabatic process? If a material or a system undergoes a change in its physical state in terms of change in the pressure, volume or temperature; without any heat being added to it or withdrawn from it, then that particular change is called as adiabatic process. That particular process by which a system undergoes a transition from one state of equilibrium to another state of equilibrium without accepting any heat from the surroundings or without giving away any heat to the surroundings, this process is called as adiabatic process.

Now, most importantly in many of our atmospheric physics discussions we will take the process to be adiabatic in nature right.

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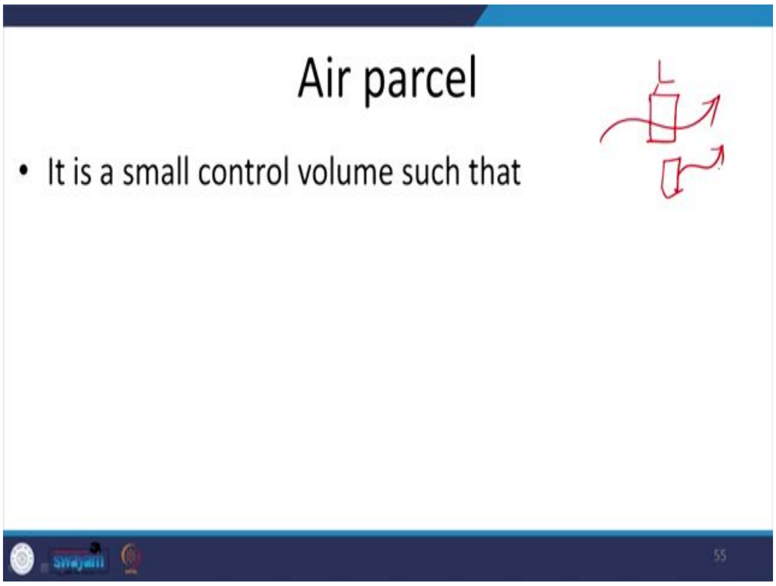

Air parcel

- It is a small control volume such that

So, what is an air parcel? Why do we need to consider the control volume to be an air parcel? Air parcel is a small control volume, its like in fluid mechanics we take the concept of control volume to describe the motion of fluid from one point to another point or to describe physical processes that happen within the fluid, we take the idea of air parcels.

So, rather than considering the entire volume of the fluid and to be able to write an equation of motion for the fluid is tedious because, fluid system is completely different in comparison to a rigid body or in comparison to a mass of a point mass right. So, in fluid mechanics what we do is we consider an infinitesimally small volume of the fluid and we apply the laws of physics or the laws of conservation of mass, momentum or energy onto this small volume of fluid. And, we extrapolate the equations of motion that we get out of this small control volume to the entire fluid. So, the idea is let us say we want to apply the conservation of mass for the fluid.

Then what do we do is, we take a small control volume then we equate the amount of fluid that is going into this control volume and the amount of amount of fluid that is going out of the control volume. Then if both are equal then, there is no accumulation of fluid inside the control volume naturally right. So, like that we always conserve energy, momentum and mass on this small control volumes.

And, these control volumes can be of two types: a Lagrangian control volume and Eulerian control volume. So, the so without going into much details I can simply say that an Eulerian control volume is the one which is fixed with respect to the movement of the fluid; that means, the volume will stay in the fluid as it is at the same point. But, the fluid will flow or will transfer from one point in the control volume to another point.

So, the control volume is fixed in space, but in a Lagrangian control volume type you have the control volume which is kind of moving with the fluid right. So, this is the basic difference. So, why are we talking about control volumes here? Air parcel is also a control volume under certain conditions, we take certain conditions over which the air parcel approximation or the theory is valid. Let us see what are those conditions ok.

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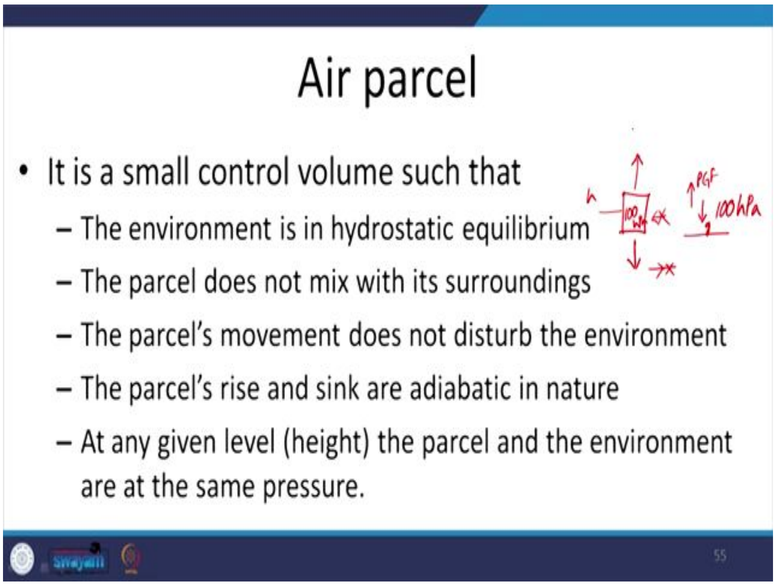

Air parcel

- It is a small control volume such that
  - The environment is in hydrostatic equilibrium
  - The parcel does not mix with its surroundings
  - The parcel's movement does not disturb the environment
  - The parcel's rise and sink are adiabatic in nature
  - At any given level (height) the parcel and the environment are at the same pressure.

So, the environment we take a smallest possible volume of air in the atmosphere we call it as air parcel. Now, if this is a volume of air, is the volume of air allowing mass transfer? Is the volume of air allowing energy to be transferred inside and to the outside? Is the volume moving with respect to the height? Is the volume having different pressures at different heights or is the volume having the same pressure at any different height? These are the questions that we should answer before we take up the approximation of air parcel right.

So, air parcel is a small control volume such that the environment; where the air parcel is situated is in hydrostatic equilibrium. So, what is hydrostatic equilibrium? Hydrostatic equilibrium is the condition where, a force balance occurs between the vertical pressure gradient force and the gravitational pull due to the earth right. So, you take control volume in a case where hydrostatic equilibrium is valid right. So, air parcel is not allowed to randomly escape right; so, that is the basic idea right. Then what else? The parcel does not mix with its surroundings.

So, it means that the constituents or the contents of the air parcel are not going away or the constituents of the environment are not mixing well with the air parcel, the basic idea right. The parcels movement does not disturb the environment, (Note-see if this is an assumption it has to be like this); the parcels movement does not disturb the environment. So, environment stays as it is, the thermo dynamical variables describing the nature or the state of environment are not going to be influenced by the movement of the parcel and the parcels rise and sink are adiabatic in nature.

So, parcel is of course, in the atmosphere, in the environment; now this environment is in hydrostatic equilibrium and the parcel is not rising or sinking at the expense of energy rather it is rising and sinking in an adiabatic nature. So, its rise is not due to the heat that is taken up from the system from the environment, its sink does not give away energy into the system right. So, the parcel is completely adiabatic in nature. So, it is a closed parcel which is not allowing energy transfer by the walls of the volume right; most importantly.

This is the most important assumption that is very important for us to understand the stability of atmosphere. So, we will use this assumption or we will talk about this assumption in many different ways where while we discuss the stability, the cloud formation and these things right. So, at any given level, at any given height in the atmosphere the parcel and the environment are at the same pressure. So, the point is if the parcel is at a particular pressure of let us say 100 hPa , now this atmosphere is in hydrostatic equilibrium right. This atmosphere is in hydrostatic equilibrium and this parcel that you take at this particular height is also having the same pressure of 100 hPa. Now, if it rises it is not taking energy, its not taking energy inside or if it sinks it is not giving away energy as it sinks.

So, it is perfectly adiabatic right, but as it rises it is still at the same pressure as the surroundings. So, the natural question that should come into your thought is that what will happen to the volume of the air parcel because, the pressure if the pressure decreases the volume cannot be the same right. So, to compensate the decrease in the volume, the pressure has to increase right. So, these two things always work together right.

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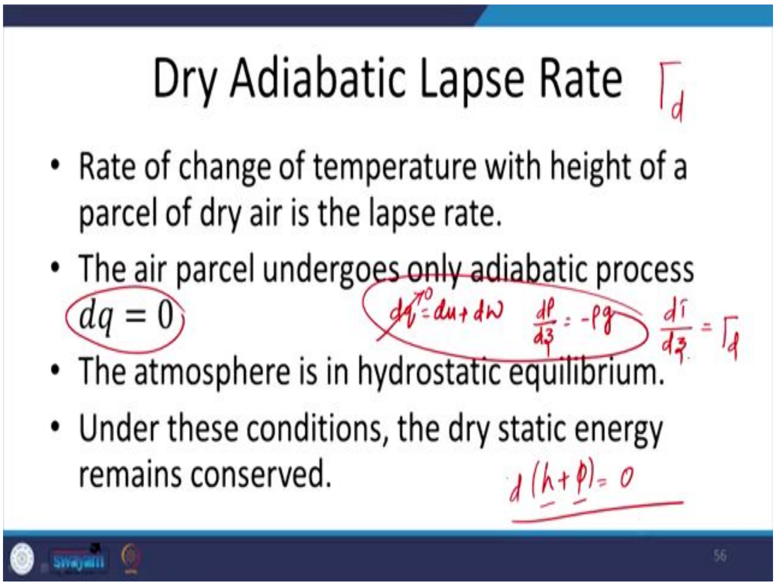

Dry Adiabatic Lapse Rate Γ_d

- Rate of change of temperature with height of a parcel of dry air is the lapse rate.
- The air parcel undergoes only adiabatic process $dq = 0$ $\cancelto{0}{dq} = du + dw$ $\frac{dP}{dz} = -\rho g$ $\frac{dT}{dz} = \Gamma_d$
- The atmosphere is in hydrostatic equilibrium.
- Under these conditions, the dry static energy remains conserved. $d(h+\phi) = 0$

So, based on our understanding of air parcel and the dry static energy, we will try to see what is the dry adiabatic lapse rate. What is lapse rate? So, dry adiabatic lapse rate is generally given as gamma d with a suffix d indicating the dry adiabatic lapse rate. So, what is dry adiabatic lapse rate? The rate of change of temperature with respect to the height of a parcel of dry air is called as the lapse rate right. So now, why do we call it as adiabatic? because we have assumed that the air parcel rise or sink is perfectly going to be adiabatic in nature right.

And, why should the temperature decrease? The temperature inside the air parcel if at all if it is not accepting any energy from the outside, the rise should be fuelled by the decrease in the internal energy of the air parcel. If the internal energy decreases, the temperature of the air parcel will also decrease. So, as a natural consequence as the air parcel rises its temperature will decrease inside. So, but that is why we call it as a lapse rate because, it signifies the decrease in the temperature and we also call it as the adiabatic because it is not accepting any heat from the system from the surroundings right.

So, let us say so, to be able to force adiabatic nature the air parcel undergoes a change such that dq is equal to 0. So, what is dq? dq is the heat that is taken or given out right; so, the atmosphere is in hydrostatic equilibrium. So, now we just have to combine dq is equals to du plus dw where, dq is already taken to be 0 and it is under hydrostatic equilibrium so, dp by dz is equals to minus rho g. So, these two equations are the ones. So, by combining these two equations we can probably get the rate of change of temperature with respect to height that is it; so, this is a basic idea.

So, this rate of change of temperature with respect to height is called as the dry adiabatic lapse rate ok. So, under these conditions so, we all says we already seen that under these conditions, the dry static energy which is; what is the dry static energy? Dry static energy is h plus what phi, the dry static energy is the is the sum of enthalpy and the geopotential, What is geopotential? h is enthalpy is the total heat content of the system and the geopotential at any given height is defined as the amount of work that should be done to raise the air parcel to that particular height from the surface right.

So, under these conditions, what are the conditions? If it is a hydrostatic, if the atmosphere is hydrostatic in nature and if the air parcel is adiabatic in nature then under these conditions we have already seen that the dry static energy will remain conserved. So, the rate of change of this dry static energy will be 0 right. So, these this will come handy when we derive the relation for the adiabatic lapse rate. So, the dry static energy is h plus phi, dq is equals to d of h plus phi; because dq is 0, change in the dry static energy h plus phi is equals to 0. So, C_p T plus phi is equal to 0.

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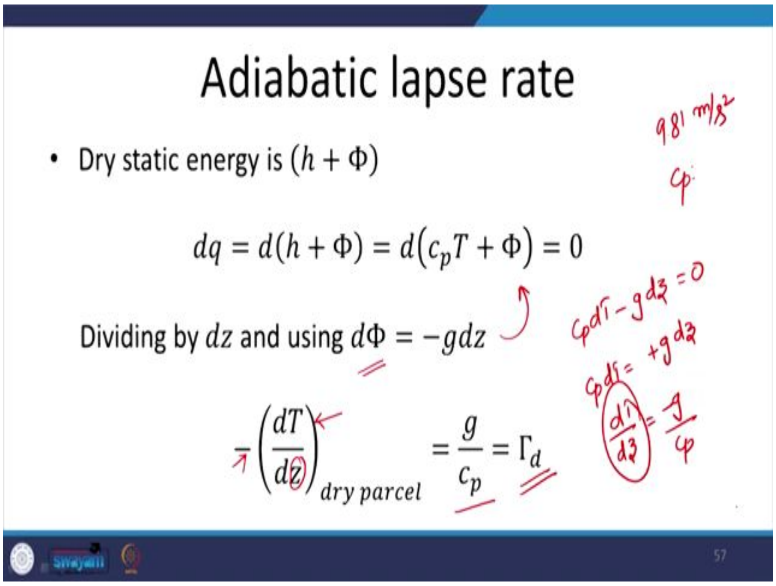

Adiabatic lapse rate

- Dry static energy is $(h + \Phi)$

$$dq = d(h + \Phi) = d\left(c_p T + \Phi\right) = 0$$

Dividing by dz and using $d\Phi = -g dz$

$$-\left(\frac{dT}{dz}\right)_{dry\ parcel} = \frac{g}{c_p} = \Gamma_d$$

9.81 m/s²

C_p

$C_p dT - g dz = 0$

$C_p dT = +g dz$

$\frac{dT}{dz} = \frac{g}{C_p}$

So, we also know from the geopotential that d phi is equal to minus d z right. So, we substitute this into this. So, basically C_p d T minus g dz is equal to 0, this is the basic idea right. So, C_p d T is equals to g dz. So, d T over dz is because this is the minus g by C_p or d T by dz we put a minus to signify the decrease in the temperature. So, the rate of change of temperature with respect to z is altitude here right. So, d T the rate of change of temperature with respect to the altitude is equal to g over C_p

So, g is known to be what g is known to be 9.81 meters per second square and C_p is the specific heat at constant pressure. So, we have already discussed what our specific heats at constant pressure and at under constant volumes right. So, the rate of change of temperature as it raises as the air parcel raises maintaining a constant value of dry static energy will be equal to g over C_p ; which is also called as the gamma d, gamma d is the dry adiabatic lapse rate.

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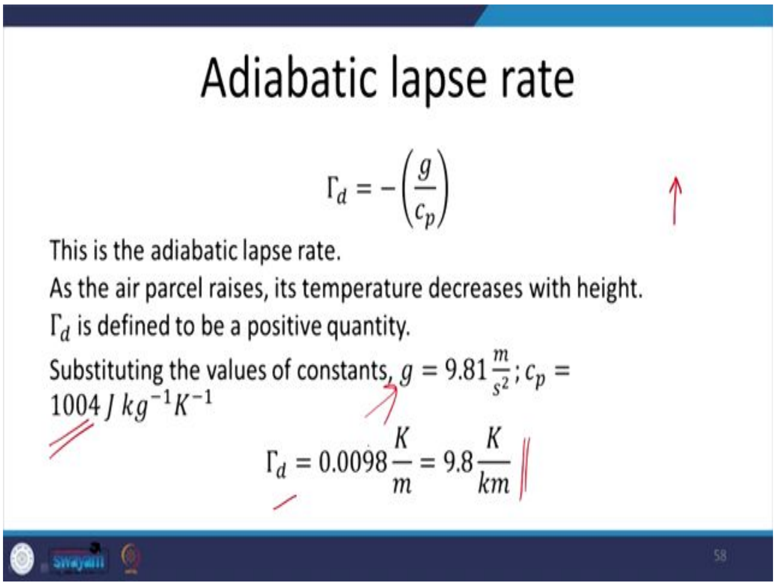

Adiabatic lapse rate

$$\Gamma_d = -\left(\frac{g}{c_p}\right)$$

This is the adiabatic lapse rate.

As the air parcel raises, its temperature decreases with height.

Γ_d is defined to be a positive quantity.

Substituting the values of constants, $g = 9.81\frac{m}{s^2}; c_p = 1004\,J\,kg^{-1}K^{-1}$

$$\Gamma_d = 0.0098\frac{K}{m} = 9.8\frac{K}{km}$$

So, as the air parcel rises its temperature decreases with height. So, gamma d, the gamma d is defined to be a positive quantity, understand I mean we keep it minus to signify the decrease in the temperature right. So, substituting the values of g is equal to 9.81 meters per second square and C_p is equals to 1004 Joule per Kelvin per kg. If we substitute this we will get gamma d as the dry adiabatic lapse rate as 0.0098 Kelvin per meter, I mean for every meter that you travel up the temperature will decrease by 0.0098 Kelvin right or 9.8 Kelvin per kilometer.

So, this is the basic lapse rate that we have discussed in the very beginning while we were discussing the layers of atmosphere, where I said the atmospheric that the troposphere that at the bottom most layer atmosphere exhibits the negative slope between temperature and the height; that means, temperature decreases as you go up. So, and the rate at which this temperature decreases is mentioned to be called as the adiabatic lapse rate. So,, if the atmosphere is a dry completely dry; that means, there is no moisture in it then this value of Cp that you have taken makes sense.

So, in that case it will be nearly 10 degrees Celsius or Kelvin for every kilometer. So, as you travel up for every kilometer that you travel the temperature in the atmosphere will decrease by 10 degree centigrade right. So, what is more specific is, I mean how is this value specific to earth's atmosphere let us say. If because the dry air yeah dry air could be could be taken on any other planet, but how is this 10 Kelvin per kilometer specific to the planet earth? Is in terms of its let us say gravity and the value of this C_p evaluated at a constant pressure. What is the pressure and what are the number of atoms or molecules in the dry air that you are talking about? Right.

So, this is something about the dry adiabatic lapse rate. So, always remember gamma d is equals to minus g over c p not C_v its C_v, the dry adiabatic lapse rate. So, this is this lapse rate is valid for completely dry atmosphere which never exists right, because there is always an amount of moisture right. So, most important is that gamma d is the rate of change of temperature following an air parcel of dry air that is being raised or lowered adiabatically in the atmosphere. That means, if you take an air parcel at let us say 10 kilometers; if you bring it down its temperature should increase right, if its temperature should increase.

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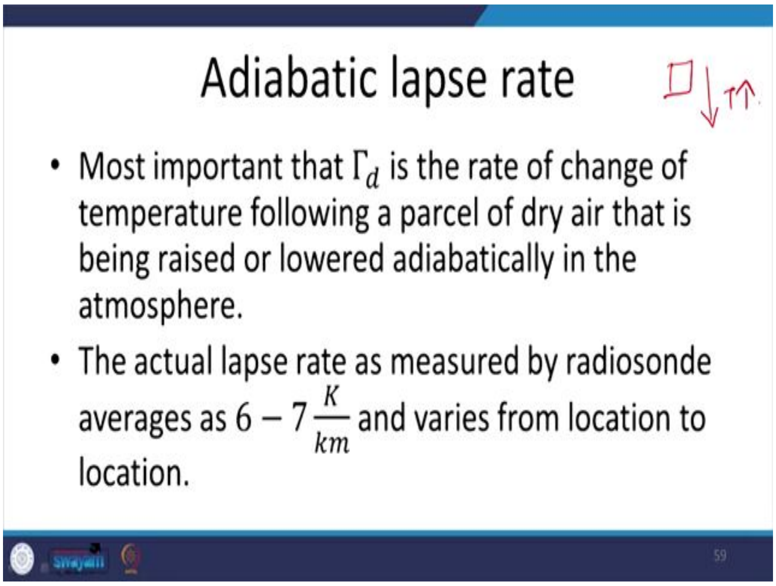

Adiabatic lapse rate

- Most important that Γ_d is the rate of change of temperature following a parcel of dry air that is being raised or lowered adiabatically in the atmosphere.
- The actual lapse rate as measured by radiosonde averages as $6-7\,\frac{K}{km}$ and varies from location to location.

So; that means, that it if you take an air parcel from the ground to the to a level or the other way the temperature should compensate in the same way right. So, the actual lapse rate as measured by different instruments such as radiosonde they do not converge at let us say 10 degree Celsius per kilometer rather it is 6 to 7 degree Celsius per kilometer. And, this dry adiabatic lapse rate varies from different from a location to another location. So, there are several processes which will contribute in the in deciding the average adiabatic lapse rate at a given location. Because let us say for example, if it is if this location is over a hilltop then you measure the dry adiabatic lapse rate, it will be different.

If you measure this dry adiabatic lapse rate on a location near the sea surface it will be different. So, like that if you measure it in the tropics it will be different, if you measure in the poles it will be different. So, like that , there are many factors which will decide the rate at which the temperature should decrease with respect to height in the in the atmosphere right. So now, we talk about a very important physical quantity which is called as the potential temperature right.

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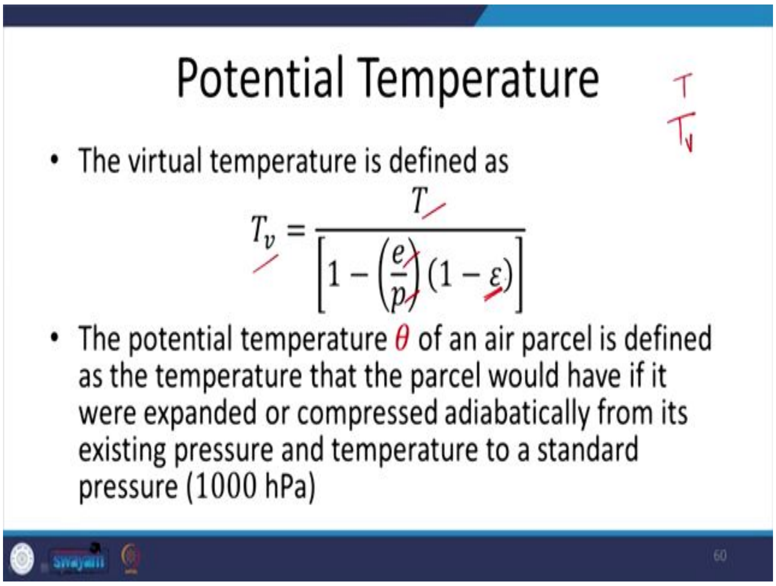

Potential Temperature

- The virtual temperature is defined as

$$T_v = \frac{T}{\left[1-\left(\frac{e}{p}\right)(1-\varepsilon)\right]}$$

- The potential temperature θ of an air parcel is defined as the temperature that the parcel would have if it were expanded or compressed adiabatically from its existing pressure and temperature to a standard pressure (1000 hPa)

There are few more temperatures that we are going to talk. So, first we have the temperature T which gives the measure of the energy that the system has or that the gas has in terms of cavity right. Then we have seen what is called as the virtual temperature. So, virtual temperature was defined as the temperature to which dry air has a density as moist air. So, at virtual temperature is always greater than the original temperature, because we have a denominator which is less than 1 right.

So, here so, T v is the virtual temperature, T is the temperature, e is the vapor pressure that is because of the moisture, p is the total pressure, epsilon is the ratio of the mean molecular weights or ratio of the gas constants . So, here we are going to define another temperature which is called as the potential temperature. So, the potential temperature is denoted by theta , potential temperature of an air parcel is defined as the temperature that the air parcel would have if it were expanded or compressed adiabatically from its existing pressure and temperature to a standard pressure 1000 hPa, the surface pressure right.

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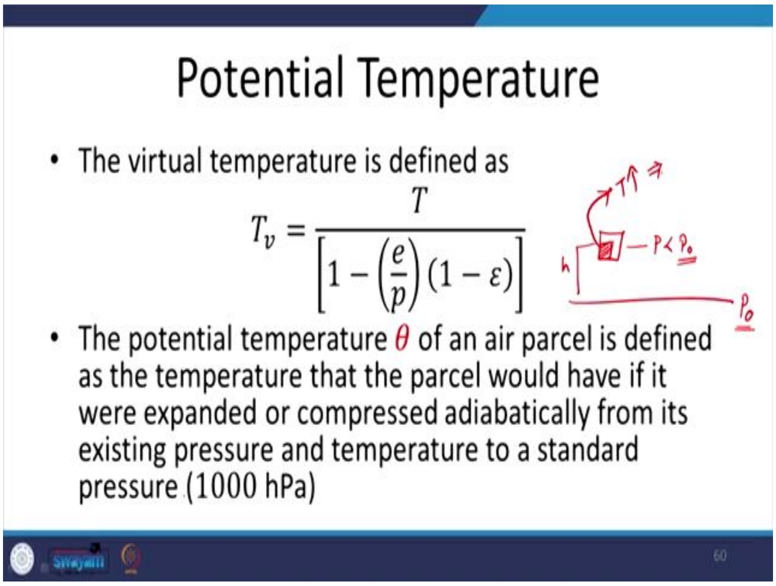

Potential Temperature

- The virtual temperature is defined as

$$T_v = \frac{T}{\left[1 - \left(\frac{e}{p}\right)(1 - \varepsilon)\right]}$$

- The potential temperature θ of an air parcel is defined as the temperature that the parcel would have if it were expanded or compressed adiabatically from its existing pressure and temperature to a standard pressure (1000 hPa)

So, what it means? I mean let us say if you take an air parcel ok, now this is the surface where the pressure is standard pressure p_0 , the surface pressure, the mean sea level let us say. If you take an air parcel, now if you want this air parcel to have this pressure what should you doI mean this is already at a particular height h ok. Now if you want this so; obviously, this the pressure p here is less than p_0. Now, if you want this air parcel to have this pressure what should we do? You should compress it. So, the pressure increases, when you compress it of course, you are bringing that pressure to this value p_0 .

But, at the same time due to the compression the temperature will be increased right. So, the new temperature that has in there has been increased I mean the new temperature that has been achieved just in an effort to reach the standard atmospheric pressure is called as the potential temperature. Now, look at it the other way, if this air parcel is already at a higher pressure, then if you want to reach this pressure I mean a lower pressure you have to expand the air parcel.

Now, when you expand the air parcel your what an adiabatic pauses, what happens when you expand it the automatically the pressure will decrease, the volume has increased; on pressure due to the pressure decrease the temperature will decrease correct. The new temperature that you are going to achieve; at the end of a process in which you are just trying to attain mean sea level pressure is called as the potential temperature ok. So, we can derive a simple expression for the potential temperature of an air parcel in terms of its pressure p.

What is this pressure? Pressure p is the initial pressure and p_0 means sea level pressure . The initial temperature is T and the final temperature when you achieve p_0 is theta which is the potential temperature right.

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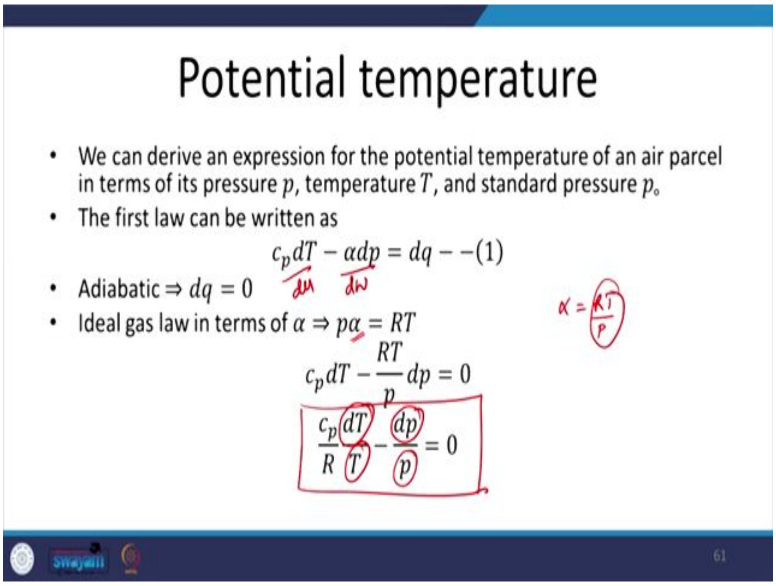

Potential temperature

- We can derive an expression for the potential temperature of an air parcel in terms of its pressure p, temperature T, and standard pressure p_0
- The first law can be written as

$$\underbrace{c_p dT}_{du} - \underbrace{\alpha dp}_{dw} = dq \; --(1)$$

- Adiabatic ⇒ $dq = 0$
- Ideal gas law in terms of α ⇒ $p\alpha = RT$

$\alpha = \frac{RT}{p}$

$$c_p dT - \frac{RT}{p} dp = 0$$

$$\boxed{\frac{c_p}{R}\frac{dT}{T} - \frac{dp}{p} = 0}$$

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So, the from the first law of thermodynamics we can say $C_p d T$ which is du and this is dw, dq is equal to du plus dw correct. So, that the heat that is given will increase the internal energy and also do some work right. So, if you take the picture to be adiabatic in nature you will simply write dq is equals to 0 right. So, from the ideal gas law which is p alpha is equal to RT which is written here, we take d of p alpha. So, we can write alpha is equals to RT by p.

So, alpha is RT over p. So, in this expression in terms of alpha we substitute this RT by p. So, C_p d T minus RT by p dp is equal to 0 or C_p by R d T over T. So, multiplying this expression with p by RT we will get C_p by R into d T by T minus dp by p is equal to 0 alright. Now, this expression looks like you want to address change in the pressure with respect to pressure and change in the temperature with respect to temperature, you want to address at this expression. We have gotten it into a convenient form right.

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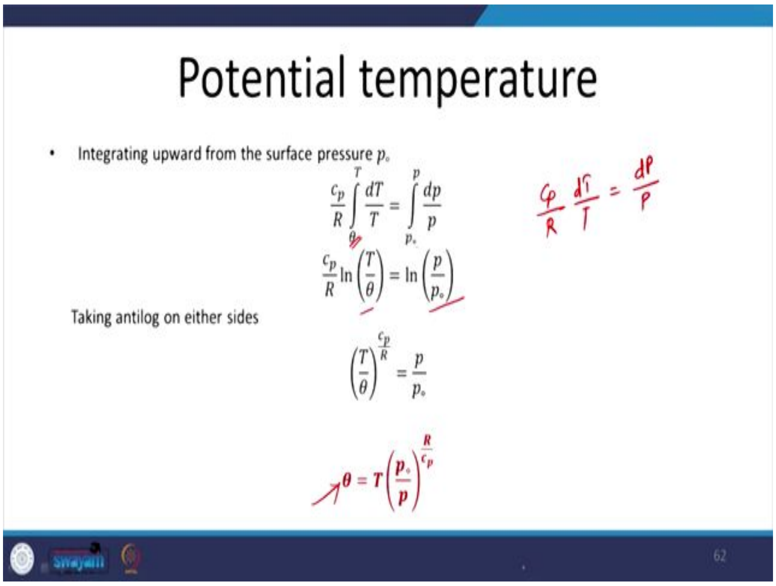

Potential temperature

- Integrating upward from the surface pressure $p_\circ$

$$\frac{c_p}{R}\int_{\theta}^{T}\frac{dT}{T} = \int_{p_\circ}^{p}\frac{dp}{p}$$

$$\frac{C_p}{R}\ln\left(\frac{T}{\theta}\right) = \ln\left(\frac{p}{p_\circ}\right)$$

Taking antilog on either sides

$$\left(\frac{T}{\theta}\right)^{\frac{C_p}{R}} = \frac{p}{p_\circ}$$

$$\theta = T\left(\frac{p_\circ}{p}\right)^{\frac{R}{c_p}}$$

$$\frac{C_p}{R}\frac{dT}{T} = \frac{dP}{P}$$

So, now integrating this expression for the same thing, we had this expression C_p by R into d T over T is equal to dp over p. So, this expression we integrate from let us say from upward from the pressure surface so, p_0 to p. So, if you get p_0 starting from a pressure p your initial temperature must have been T, but when you reach p_0 the temperature will become theta right. So, if you integrate this C_p by R into integral of theta to T, d T over T so, a simple integral. So, C_p by R lon of T by theta is equation lon of p by p_0 .

So, if you get rid off the lon terms on either sides so, T by theta raised it to the power of C_p by R is equals to p by p_0 . Then it means theta is given as temperature times p_0 naught by p into R by C_p . Where what is R? R is a universal gas constant, C_p is the specific heat at constant pressure, p_0 is the surface pressure or the standard pressure, p is the pressure at a temperature T and theta is the final temperature. So, potential temperature is defined as that value of temperature that you will get if you raise or sink an air parcel; so, that it reaches the standard pressure.

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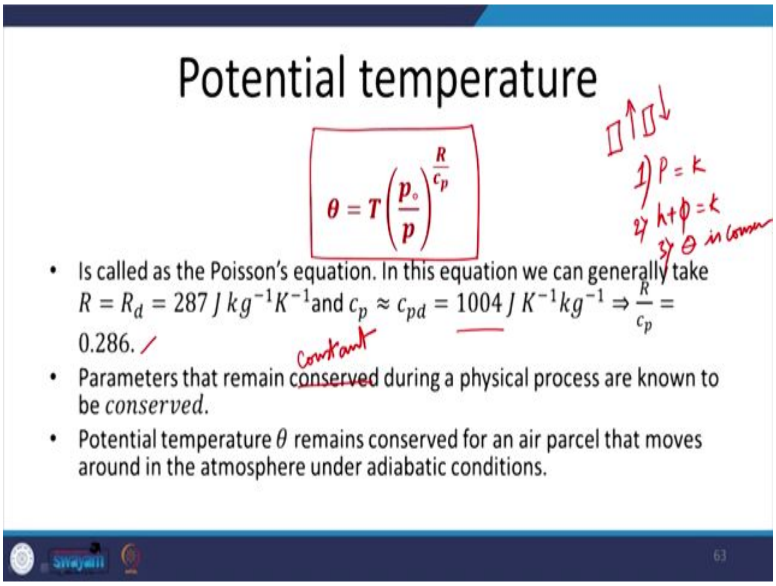

Potential temperature

$$\boxed{\theta = T\left(\frac{p_\circ}{p}\right)^{\frac{R}{c_p}}}$$

1) $P = k$

2) $h+\phi = k$

3) θ in conser

- Is called as the Poisson's equation. In this equation we can generally take $R = R_d = 287\, J\, kg^{-1}K^{-1}$ and $c_p \approx c_{pd} = 1004\, J\, K^{-1}kg^{-1} \Rightarrow \frac{R}{c_p} = 0.286.$
- Parameters that remain ~~conserved~~ Constant during a physical process are known to be *conserved.*
- Potential temperature θ remains conserved for an air parcel that moves around in the atmosphere under adiabatic conditions.

So, this equation is called as the Poisson's equation. So, this it is very important that we keep this equation handy, if you want analyze any weather maps or things like that. So, it is called as Poisson's equation, in this equation we can generally take R is equals to R_d is equal to 287 Joule per Kelvin per kg. The specific heat at constant pressure, the dry specific heat is 1004 Joule per Kelvin per kg. So, this ratio becomes 0.286 right. So, the parameters that remain conserved during the physical process are known to be conserved.

I mean parameters that remain constant, during the physical process are known to be conserved. So, potential temperature theta remains conserved for an air parcel that moves around in the atmosphere under adiabatic condition right. So,what else is conserved? So, if the air parcel is moving adiabatically any height to any height; so, according to the assumptions itself we took that pressure will be a constant right. Now, the pressure if it is not constant, I mean we also seen that h plus phi is also a constant.

Now, we are taking one more physical parameter which also remains conserved, this this parameter is called as the potential temperature, potential temperature theta is conserved right. So, I take change in these quantities, it will be 0 right. So, this was something about the basic thermodynamics extension of the things that we have been discussing right. So, we I will stop here. So, we will try to see how these parameters will change when we add some amount of moisture.