Introduction to Atmospheric and Space Sciences
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Lecture – 21
Specific Heats

Hello dear students. Today we will continue our discussions on Atmospheric Thermodynamics. In the last class we have seen the basic gas laws, how we define what is the universal gas constant and how we calculate the mean molecular weight of the atmosphere which is dry air, then we have seen how the things change if you add some water vapour into it.

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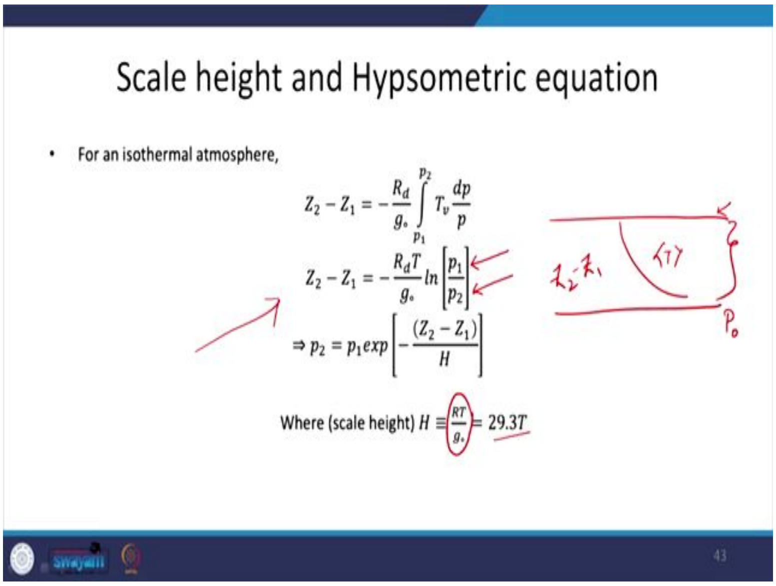

Scale height and Hypsometric equation

- For an isothermal atmosphere,

$$Z_2 - Z_1 = -\frac{R_d}{g_\circ}\int_{p_1}^{p_2} T_v \frac{dp}{p}$$

$$Z_2 - Z_1 = -\frac{R_d T}{g_\circ} ln\left[\frac{p_1}{p_2}\right]$$

$$\Rightarrow p_2 = p_1 exp\left[-\frac{(Z_2 - Z_1)}{H}\right]$$

Where (scale height) $H \equiv \frac{RT}{g_\circ} = 29.3T$

So, in the last class we concluded our discussion on this equation which is called as the hypsometric equation which tells you what is the height separation between two layers of atmosphere which are existing at two different pressures. Let us say, so the pressures were identified to be p1 and p2. So, you have a layer of atmosphere which is separated by these two pressure surfaces. The height difference between them in the units of length is given by this equation. So, the equation can also be converted to find out what will be the pressure at any given point with respect to the pressure at the surface, right.

So, we realized in this calculations and we came across something called as the scale height. Scale height is of the length dimension over which the pressure drops to 1 by eth (1/e) of its original value or the beginning value, right. So, scale height is generally written if you do not use a temperature, average temperature of a layer. So, generally a temperature structure within this layer varies or decreases with respect to height.

So, if you take an average temperature; that means, if you take; if you take the picture to be isothermal in nature you say that temperature or average temperature is this,then you will get the scale height to be this. So, today we will continue our discussion and we will try to explore some ideas of the basic laws of thermodynamics, how we can write these basic laws of thermodynamics in terms of the variables which are relevant for our discussions ahead. So, in that way, we will talk about specific heats first.

So, what is specific heat? These fundamental aspects must be very well known to you already, right. So, for example, specific heat. So, suppose a small quantity of heat dq is added to a unit mass material and as a consequence of adding this heat, adding this amount of energy to the system for the material if the temperature of the material changes from T to T+dT Kelvin (let us say) without any change in the phase of the material.

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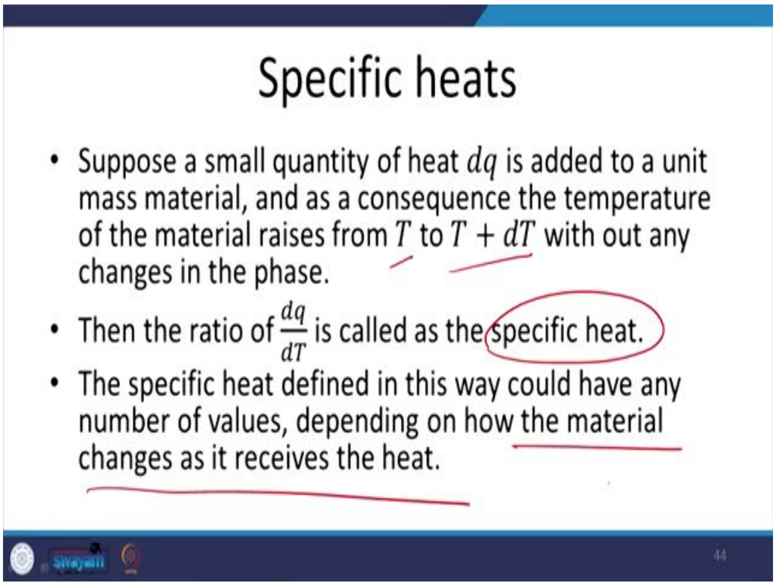

Specific heats

- Suppose a small quantity of heat dq is added to a unit mass material, and as a consequence the temperature of the material raises from T to $T + dT$ with out any changes in the phase.
- Then the ratio of $\frac{dq}{dT}$ is called as the specific heat.
- The specific heat defined in this way could have any number of values, depending on how the material changes as it receives the heat.

So, basically what you are doing is you are adding certain amount of heat to the system and you are expecting that this system does not go, does not allow or does not go under any phase transitions, and if this system's temperature increases from an initial value of capital T to

T+dT. So, the ratio of the heat supplied to the temperature change is called as the specific heat. So, the specific heat is defined in this way. I mean if you define the specific heat in this way it can have many values.

So, what do you mean by many values? So, of course, you have avoided the possibility of any phase transitions; that means, that if you are talking about vapor it will stay in the same phase, like that . So, if you do not allow any phase transition to happen, but still if you define your specific heat to be just a ratio of heat to the temperature change it can have any values, right any number of values depending on how the material changes or how the material behaves as it receives the heat, right.

So, if you keep a constant volume, then how does the temperature change? Because generally if you take a gas, if you add some heat generally, what will happen? The first thing that will happen is its internal energy will increase and if the gas is not bound by a constant volume its volume will change; that means, the external heat that is supplied will cause the internal energy of the gas to change and also some work being done by expanding the volume, right. So, I mean this is the general first law of thermodynamics, right.

So, in that case if you do not define what is pressure change in this particular process or what is the volume change in this particular process, then the simply the ratio of heat supplied to the ratio to the temperature change can have any number of values, right. So, it basically depends on how the material changes as it receives the heat, how the material materials volume or pressure change when it receives the heat.

Let us say for example, if you confine the volume to be a constant, then the gas cannot do any work on the system, right. So, the gas cannot do any work, the system cannot do any work, so gas cannot expand. So, in that case work done is 0; that means, the entire energy that you have given to the gas will simply make the internal energy of this gas to go up, right.

So, in that case whatever temperature change that you experience within the gas will be different. But, if you allow a small possibility that volume can also change then you are making the temperature change different from what the earlier case was, right. So, let us say if the materials volume is kept constant, so rather than having many number of values, if you keep the materials volume a constant the specific heat is defined at constant volume.

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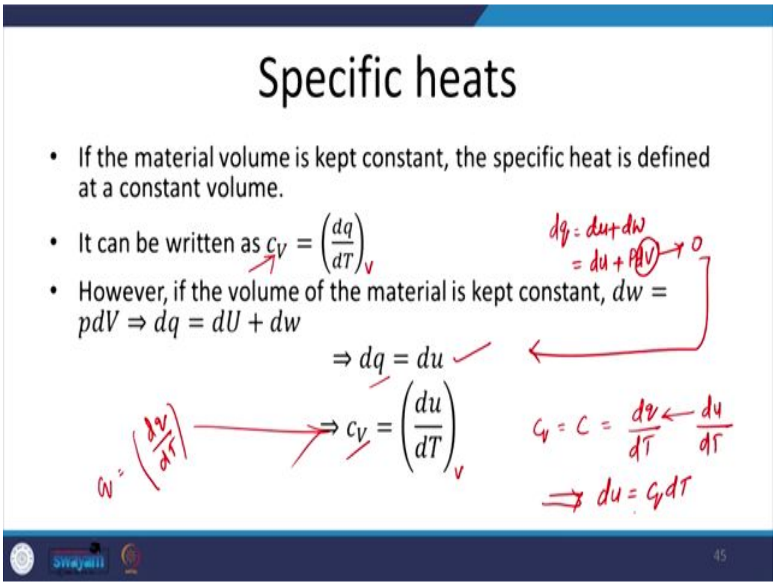

Specific heats

- If the material volume is kept constant, the specific heat is defined at a constant volume.
- It can be written as $c_V = \left(\frac{dq}{dT}\right)_V$
- However, if the volume of the material is kept constant, $dw = pdV \Rightarrow dq = dU + dw$

$$\Rightarrow dq = du$$

$$\Rightarrow c_V = \left(\frac{du}{dT}\right)_V$$

$dq = du + dw$

$= du + p\,\boxed{dV} \rightarrow 0$

$C_V = \left(\frac{du}{dT}\right)$

$C_V = C = \frac{dq}{dT} \leftarrow \frac{du}{dT}$

$\Rightarrow du = C_V dT$

The specific heat at constant volume is generally given by C_v. dq the same ratio again, ratio of heat supplied to the temperature change, but at constant volume. However, if the volume of the material is kept constant, one very important consequence is that, so from the first law dq is equals to du plus dw, du is the change in the internal energy we will write it as it is plus dw is p dV, right.

So, if the volume is kept constant this term dV becomes simply 0; that means, that dq is just du as a consequence of this. So, you have defined C_v or let us say c the specific heat as dq over dT. Now, your C_v has now become du over dT or du the change in the internal energy is Cv times dT. But for our discussion we define the specific heat as the ratio of change in the internal energy as a result of the additional input of heat to the change in the temperature at the constant volume, right.

So, C_v can as well be written in its own definition dq by dT, but since it is a process at constant volume, this is equivalent to this, right.

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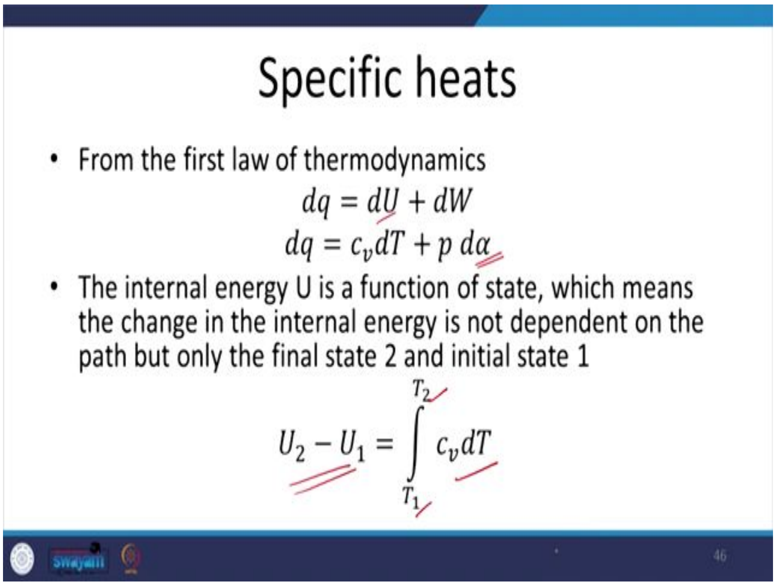

Specific heats

- From the first law of thermodynamics

$$dq = dU + dW$$
$$dq = c_v dT + p\, d\alpha$$

- The internal energy U is a function of state, which means the change in the internal energy is not dependent on the path but only the final state 2 and initial state 1

$$U_2 - U_1 = \int_{T_1}^{T_2} c_v dT$$

So, from the first law of thermodynamics again, we know that dq is equal to dU plus dW. Now, we have introduced alpha which is the specific volume, right, volume per unit mass, right. So, dq, dU is now written as C_v dT and dW is not p dv rather it is p d alpha, right, alpha is the specific volume. The internal energy U is a function of the state, right. Now, we have seen that internal energy capital U is a function of the state; that means, if you define internal energy for a particular system it has to be defined at a particular state.

Now, if the system changes from one state to another state or if the system undergoes a change then the change in the internal energy of the system will be just a difference between the final state minus the initial state, but it will not depend on the path through which this transition of state 1 to 2 has happened, right. So, it only depends on the initial state and the final state.

So, this is what it is. I mean the change in the internal energy U 2 minus U 1 in which the change in internal energy is not dependent on the path, but only the final state 2 and the initial state 1, so U 2 minus U 1. So, why is U 2 minus U 1 relevant here is that, if you keep the volume of a gas constant and if you supply some heat then the change in the temperature will depend on the change in the internal energy, but nothing else because du is equals to C_v dT, right.

So, change in the energy is from T 1 to T 2. So, the initial state and the final state characterized by temperatures T 1 and T 2 C_v dT. So, you can also write it like this, right.

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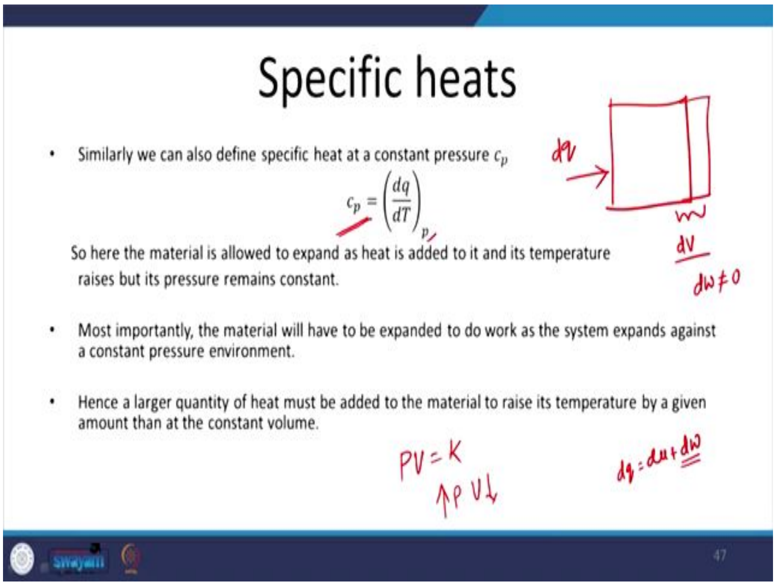

Specific heats

- Similarly we can also define specific heat at a constant pressure c_p

$$c_p = \left(\frac{dq}{dT}\right)_p$$

So here the material is allowed to expand as heat is added to it and its temperature raises but its pressure remains constant.

- Most importantly, the material will have to be expanded to do work as the system expands against a constant pressure environment.
- Hence a larger quantity of heat must be added to the material to raise its temperature by a given amount than at the constant volume.

$PV = K$

$\uparrow P \; V \downarrow$

$dq = du + \underline{\underline{dw}}$

dv

$dw \neq 0$

So, similarly we can also define specific heat at a constant pressure. So, we have defined specific heat at a constant volume. So, at the constant volume what is happening is you are giving heat, so the molecules or the atoms are vibrating faster. So, they build up more pressure. So, whatever the heat that you have given is accounted in the sense that it is able to increase the pressure, right.

But if you keep the pressure constant if you do not keep or if you do not allow the pressure to go up or down because of the additional heat that is being supplied, what happened? Then naturally you would expect that the gas will expand, then there will be a certainly change in the volume, right. So, the specific heat can also be defined at a constant pressure rather than at a constant volume. So, this specific heat is generally given by Cp, Cp is equal to dq by dT over at a constant pressure.

So, here the material is allowed to expand as the heat is added to the system and its temperature rises, but its pressure remains a constant. So, you have a gas chamber. Previously you kept the volume of the gas chamber a constant, now you allow a possibility that volume may change by dv when you supply an additional heat of dq. So, here how is the pressure constant? I mean the pressure is force per unit area the average amount of force that is exerted by the molecules per unit area is the pressure, right.

Now, if it is the case then you are not allowing molecules to exert more force on the surface; that means, the molecules are expanding into the additional volume, right. So, most importantly the material will have to be expanded to do the work. So, basically when dv is nonzero it simply turns out that dw becomes nonzero, the work done is 0 sorry the work done is nonzero. So, dq is now du of course, change in the internal energy plus dw.

So, earlier case dw was 0 dw in this case is not 0, right. So, the material will have to be expanded to do the work as the system expands against a constant pressure environment, right. So, this is the most important thing when you discuss the specific heat and when you want to understand why do the specific heats differ by the magnitude of the gas constant we will get there, right. So, the gas should expand such that the pressure is maintained constant generally, when you expand. What happens to the pressure?

So, we know that PV is equals to constant, right. So, we know that we know very well that PV is equals to constant that means, if you increase the pressure, the volume will decrease simple, right. Simple to understand even to comprehend because if you put the same amount of gas in a larger volume the force that is going to be experienced per unit area will obviously, decrease. Less number of molecules will be encountered within a unit surface area, so the force will be lesser, the area is still the same.

So, the mean that means, that the pressure will be smaller, right. But if you want to expand the gas, but still keep the pressure constant; that means, the energy that you are supplying is not only causing the gas to expand, but also a part of the energy is also utilized to increase the mean molecular velocity of the molecule, so that it exerts more pressure or equal pressure despite the volume changing, right.

So, hence we need a larger heat, large amount of heat to be given for the material to expect a given change in the temperature, right. So, in the specific heat at a constant volume what we have seen is that we when we supply external heat into the system the gas is not allowed to expand the volume is held constant. So, for whatever the heat that you are supplying is simply transferred to increase the internal energy du and you have defined your specific heat such that the change in the internal energy to the change in the temperature is the specific heat, so that means whatever the amount of heat is given is directly being used to get a required change in the temperature.

But here a certain amount of heat is already utilized for the change in the volume just to expand. Why should this be nonzero? Why should this change in the volume is nonzero? Because you know you are not kept the volume to be a constant. Whenever you heat a gas it is a natural tendency of the gas to expand and occupy more volume. If it is occupying more volume it is natural to expect that it will exert lesser pressure over the enclosure, but if you still want the same pressure to be maintained you will have to have, you will have to supply more amount of heat as simple as that, right.

So, hence larger quantity of heat must be added to the material to rise its temperature by a given amount, by a given magnitude, by a given know let us say by 1 Kelvin then what you supplied at constant volume. So, this is the basic reason why specific heats differ, specific heat at constant pressure is always larger than the specific heat at constant volume. So, this is the basic difference is.

The only simply put it is it comes because the effort to maintain the same pressure despite the change in the volume, right. So, for the case of an ideal gas the difference between the specific heats is always I mean the way, we use this particular quantity so many times in our calculator, right. So, for the case of ideal gas the difference between the specific heats can be simply calculated.

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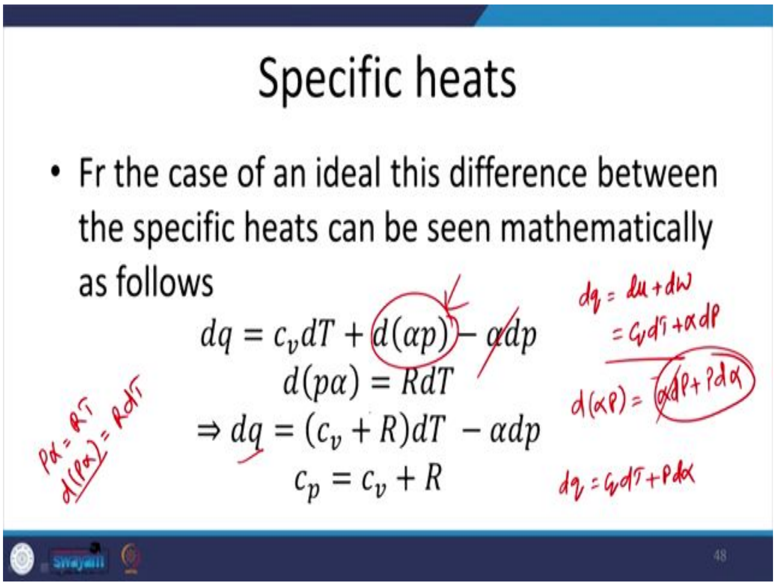

Specific heats

- Fr the case of an ideal this difference between the specific heats can be seen mathematically as follows

$$dq = c_v dT + \boxed{d(\alpha p)} - \cancel{\alpha} dp$$

$$d(p\alpha) = RdT$$

$$\Rightarrow dq = (c_v + R)dT - \alpha dp$$

$$c_p = c_v + R$$

$dq = du + dw$

$= c_v dT + \alpha dP$

$d(\alpha P) = \cancel{\alpha dP} + P d\alpha$

$dq = c_v dT + P d\alpha$

$P\alpha = RT$

$d(P\alpha) = RdT$

So, dq is equal to C_v dT plus. So, this is; so, you write dq as du plus dw, right or dq is because at the constant volume du is C_v dT plus, so dq is du plus dw, du is C_v dT plus alpha dp, right. So, if you just rearrange this you will have, so this is a derivative making sure it is a d alpha p is alpha dp plus pd alpha. So, if you put this here, so this alpha dp gets cancelled. So, you still have dq is equal to C_v dT plus p d alpha, right.

But we know that from the ideal gas law p alpha is equals to RT, right. So, d of p alpha is simply R is a constant, so R times dT. So, dq is equals to; so, you substitute pd of p alpha as R dT.

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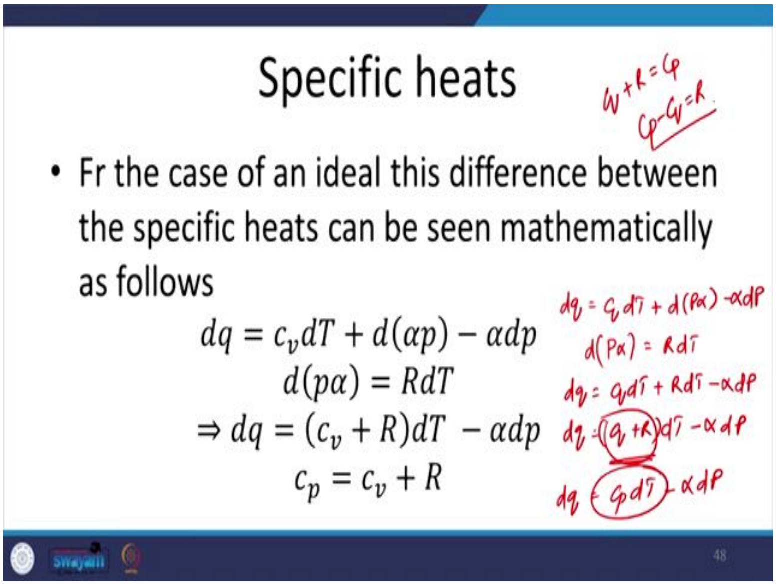

Specific heats

- Fr the case of an ideal this difference between the specific heats can be seen mathematically as follows

$$dq = c_v dT + d(\alpha p) - \alpha dp$$
$$d(p\alpha) = RdT$$
$$\Rightarrow dq = (c_v + R)dT - \alpha dp$$
$$c_p = c_v + R$$

$C_v + R = C_p$

$\underline{C_p - C_v = R}$

$dq = C_v dT + d(P\alpha) - \alpha dP$

$d(P\alpha) = R dT$

$dq = C_v dT + R dT - \alpha dP$

$dq = \boxed{(C_v + R)} dT - \alpha dP$

$dq = \boxed{C_p dT} - \alpha dP$

So, we take I will simply write between the steps dq is C_v dT plus d of p alpha minus alpha dp. I am not expanding this derivative and canceling out I am making use of d of p alpha is R times dT. So, dq is C_v dT plus R dT minus alpha dp. So, as a result I can write dq is C_v plus R times dT minus alpha dp, right. So, the point is C_v plus R, C_v plus R dq is nothing but c p dT minus alpha dp, right. So, this is the transition. C_v plus, R, so C_v plus R is C_p or C_p minus C_v is equals to R. What is R? The gas constant, right, right.

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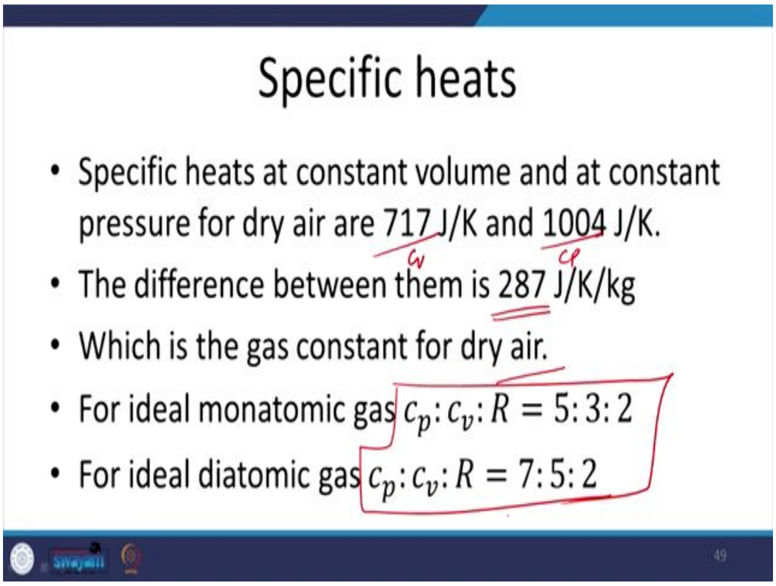

Specific heats

- Specific heats at constant volume and at constant pressure for dry air are $\underset{C_v}{717}$ J/K and $\underset{C_p}{1004}$ J/K.
- The difference between them is 287 J/K/kg
- Which is the gas constant for dry air.
- For ideal monatomic gas $\boxed{c_p : c_v : R = 5 : 3 : 2}$
- For ideal diatomic gas $\boxed{c_p : c_v : R = 7 : 5 : 2}$

So, the specific heat at constant volume and at constant pressure for dry air are 717 Joule per Kelvin heat energy per change in the temperature. And so specific heat at constant volume; so, this is smaller as we have seen just now C_v and this is C_p. And surprisingly as we know C_p minus C_v the difference between the specific heats at constant pressure and constant volume is 287 Joule per Kelvin per kg. That means, if you want to rise the temperature of 1 kg of let us say as a substance by 1 degree Kelvin you will have to supply 287 Joules of energy, right, so which is equal to the gas constant of dry air capital R.

So, we know the values of specific heats and at constant volume and constant pressures, if you take an ideal monatomic gas like oxygen O. The ratio of specific heat C_p, C_v and R, are 5 is to 3 is to 2 and the ratio of C_p to C_v to R for an ideal diatomic gas will be 7 is to 5 is to 2. So, these numbers are very important that you remember these numbers, so you will use these numbers many times in numericals and various exercises, ok, right.

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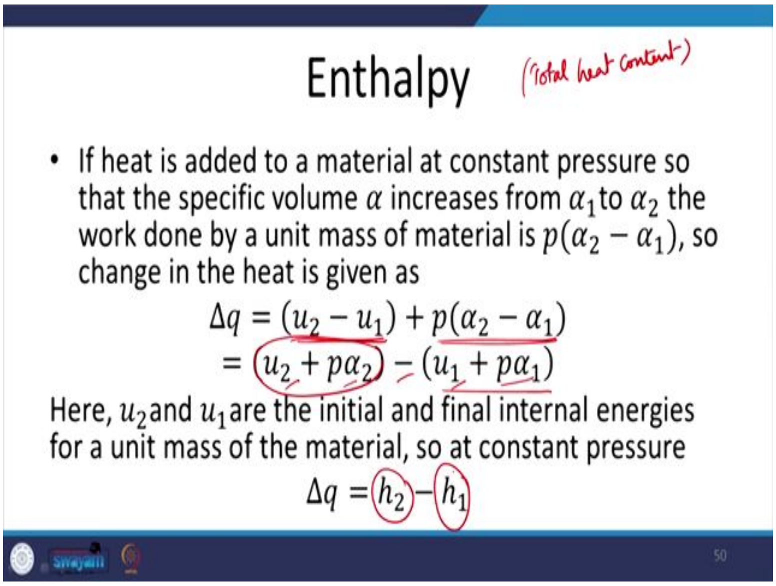

Enthalpy

(Total heat content)

- If heat is added to a material at constant pressure so that the specific volume α increases from α_1 to α_2 the work done by a unit mass of material is $p(\alpha_2 - \alpha_1)$, so change in the heat is given as

$$\Delta q = \underline{(u_2 - u_1)} + \underline{p(\alpha_2 - \alpha_1)}$$
$$= \boxed{(u_2 + p\alpha_2)} - (u_1 + p\alpha_1)$$

Here, u_2 and u_1 are the initial and final internal energies for a unit mass of the material, so at constant pressure

$$\Delta q = \boxed{h_2} - \boxed{h_1}$$

So, that was something about the specific heats. Why do you have to define specific heats is that when you discuss atmospheric thermodynamics or when you discuss the path or the movement of an air parcel you have to understand why does heat gets emitted or why does heat gets absorbed. So, all these things, the physical processes that result, as a result of exchange of heat will involve the understanding of specific heats, right.

So, once that is done we talk about what is called as enthalpy. Enthalpy is generally given in our earlier classes as the total heat content, heat content of a system or let us say gas. So, if the heat is added let us say; so, fundamentally what is enthalpy if the heat is added to a material at a constant pressure, so that the specific volume alpha increases from alpha 1 to alpha 2, the work done by a unit mass of material then is p times alpha 2 minus alpha 1. So, what is being done is at a constant pressure just like c p if we add heat such that its specific volume increases from alpha 1 to alpha 2.

The work done by this effort by a unit mass of material is then p times alpha 2 minus alpha 1. So, now, you have added heat and the volume has changed from one value to another value. So, if you write; so, from the first law of thermodynamics we always know that any amount of heat that is given to the system will first increase the internal energy of the system and will also do some work, right.

So, since the volume change he is nonzero the work then is also nonzero, right. So, the change the heat that is given is delta q can be written as the change in the internal energy u 2 minus u 1 and plus the change or the amount of work done. So, if you rearrange this equation delta q is u 2 plus p alpha 2 minus u 1 plus p alpha 1. So, if here u 2 and u 1 are the final and the initial internal energies of the system. So, this is kind of swapped you know, right. So, for a unit mass of material if you say all this u plus p alpha 2, can be said as h 2 and u 1 plus p alpha 1 can be said h 1, right.

So, for a unit mass of material at a constant pressure we say that the heat given to the system is just equal to the final enthalpy state of the system minus the initial enthalpy state of the system. So, then we define enthalpy as a total heat content of the system. So, in equation above h is equivalent to u plus p alpha.

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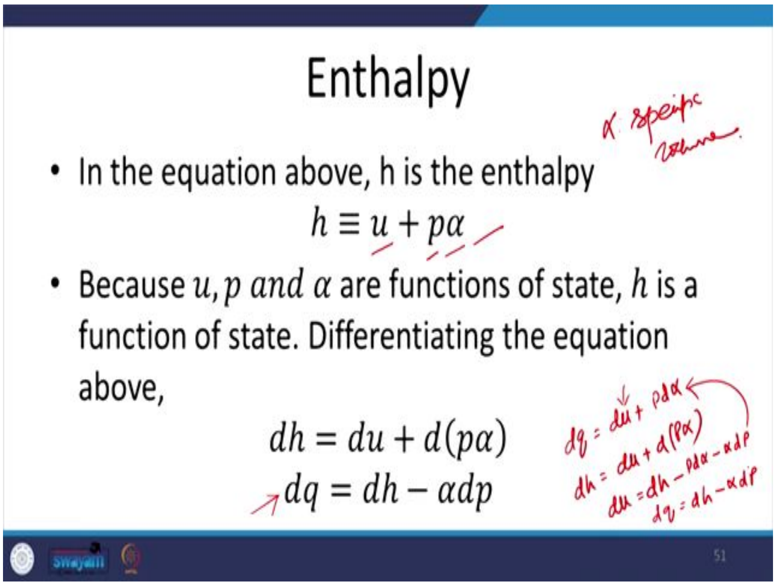

Enthalpy

- In the equation above, h is the enthalpy

$$h \equiv u + p\alpha$$

- Because u, p *and* α are functions of state, h is a function of state. Differentiating the equation above,

$$dh = du + d(p\alpha)$$

$$dq = dh - \alpha dp$$

α: specific volume

$dq = du + pd\alpha$

$dh = du + d(p\alpha)$

$du = dh - pd\alpha - \alpha dp$

$dq = dh - \alpha dp$

So, now, we know it very well that the internal energy, pressure and the specific volume are functions of state. So, functions of state means the change does not depend on the path that has been taken between the states, rather the change will only depend on the initial and the final configurations, right. So, because h is now equivalent to internal energy plus pressure times the specific volume, alpha is the specific volume, right because u and, u, p and alpha are the functions of this state, enthalpy h is a function of state.

So, if we simply differentiate the equation above is this equation h is equal to u plus p alpha if you differentiate this dh is equal to du plus d of p alpha, so dq is dh minus alpha dp. So, you have to bring in the specific heat equation. So, you can write that the heat that is a supplied. So, dq is du plus p d alpha and dh is du plus d of p alpha.

So, du is equal to dh minus pd alpha minus alpha dp, all right. So, this is du, we know dh. So, if you substitute this into this I will get dq is equal to dh. So, if you substitute du, I mean this equation goes into this part, so dq is equal to the dh minus alpha dp, right.

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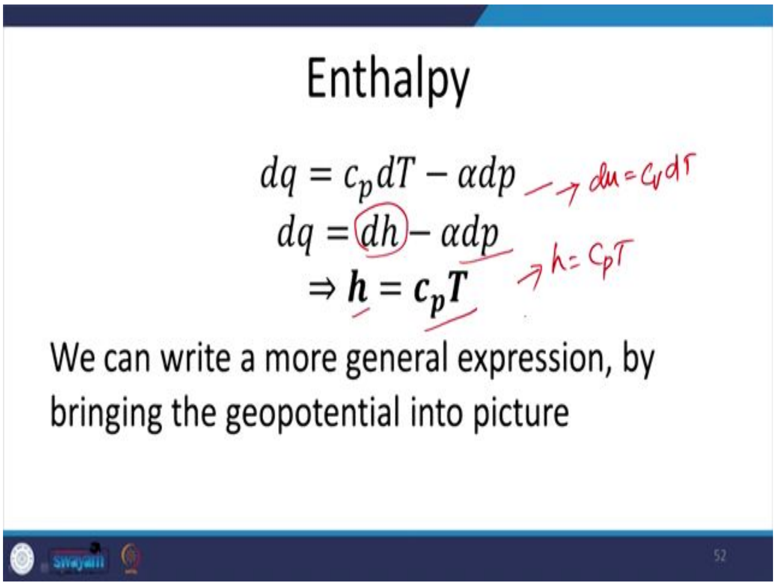

Enthalpy

$$dq = c_p dT - \alpha dp \rightarrow du = C_v dT$$

$$dq = \boxed{dh} - \alpha dp \rightarrow h = C_p T$$

$$\Rightarrow \mathbf{h = c_p T}$$

We can write a more general expression, by bringing the geopotential into picture

So, here, so then we say that dq is equal to C_p dT minus alpha dp. So, dq is equal to, dq is now equivalent to dh minus alpha dp. So, h is equal to C_p T. So, h is now C_p T and du is now equals to C_v dT. So, these two are very important thermodynamic relation. So, we can write a more general expression by bringing the geopotential into picture, right.

So, what is enthalpy? Enthalpy is just a state function which depends on the internal energy, the pressure and the specific volume, right, enthalpy depends on all these quantities. If you rewrite this equation if you just substitute the thermal and the first law of thermodynamics we can realize that the enthalpy is equal to C_v times T the temperature, right.

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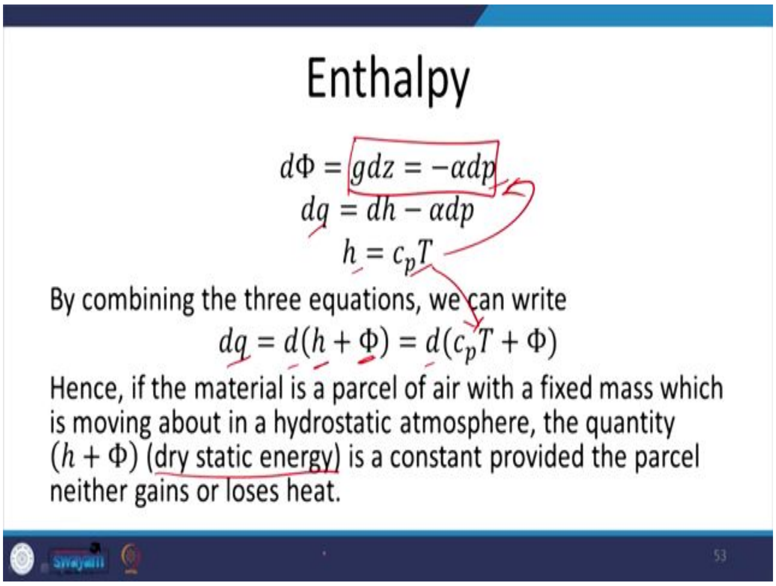

Enthalpy

$$d\Phi = \boxed{gdz = -\alpha dp}$$
$$dq = dh - \alpha dp$$
$$h = c_pT$$

By combining the three equations, we can write

$$dq = d(h + \Phi) = d(c_pT + \Phi)$$

Hence, if the material is a parcel of air with a fixed mass which is moving about in a hydrostatic atmosphere, the quantity $(h + \Phi)$ (dry static energy) is a constant provided the parcel neither gains or loses heat.

So, if we remember our last discussion we have seen what is the; what is the idea of geopotential or there is a work done, right. So, geopotential was defined as d phi is equals to gdz which is also equal to the from the hydrostatic equation, we also know that gdz is equal to alpha dp. So, dq is equals to dh minus alpha dp, there is the equation that we had and h is equals to C_p T. So, if you substitute this into this equation, right. So, by combining these 3 equations we can write dq is equal to d of h plus phi or d of C_p T plus phi. So, h is C_p T, this is being such a field here, so C_p T plus phi.

If the material is a parcel of a with a fixed mass, if the mass is not changing is moving in a hydrostatic, so hydrostatic equilibrium has allowed us to write this. So, dp by dz is equals to minus alpha g or dp by dz is equal to minus rho g. So, hence if the material is not is conserved the quantity h plus phi, so this plus this quantity h enthalpy plus the geopotential is generally referred to what is called as the dry static energy. This is called as the dry static energy.

So, this is a constant provided the parcel neither gains or loses heat, right. So, this is how you bring in enthalpy to write the geopotential and you define a physical quantity which is called as the dry static energy and this dry static energy remains conserved if the air parcel is in a picture in which it is neither gaining energy or losing energy, right.

So, I will conclude this class here. So, what we have learnt is we have learnt few details about specific heats what is the reason for them to differ by the gas constant and what is enthalpy and how we can manipulate or rewrite several different forms of the first law of thermodynamics to write specific heat or to write enthalpy or to define what is called as the dry static energy, right.

So, this is where I will stop. We will continue the discussion in the next lecture.