Introduction to Atmosphere and Space Science
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Lecture – 20
Geopotential and Scale Height

Hello dear students. So, we have stopped our discussion at the virtual temperature. So, just in order to continue I will say the definition again. So, the virtual temperature is the temperature to which the dry air must be heated, so that it acquires the density of moist air right. So, there are two important aspects here. So, when you when you heat a gas, its density decreases right, that means, in order to reach moist air, you are decreasing the density and you are increasing the temperature, so that means, that the virtual temperature will always be larger than the actual temperature. So, but it gives the provision to accommodate the information of how much amount of moisture is present in the gas without changing the gas constant; that is a beauty of using the virtual temperature right.

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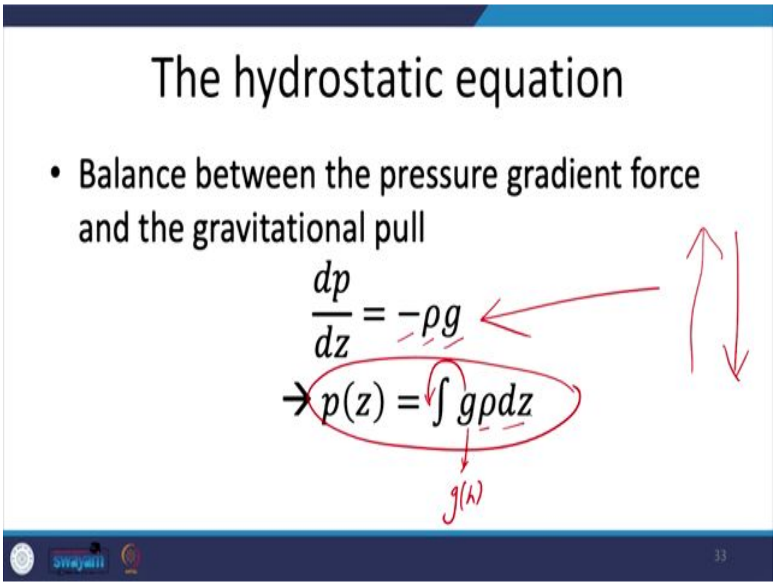

Now, so we already seen how the hydrostatic equation is derived right. So, I hydrostatic equation just before I introduce what is geopotential is very important for us to be able to use this expression for the pressure or use this expression as it is right. Now, the hydrostatic equation or the hydrostatic balance is the simple force balance between the pressure gradient force which allows which pulls the air to move away from the earth and the gravity which is holding the atmosphere to the earth. So, these two forces are balanced, and then we get what is called as the hydrostatic equation.

So, the hydrostatic equation is written as dp by dz, z is a vertical coordinate, rho is the density, g is the gravity and p is the pressure. So, the rate at which the pressure changes with respect to height is equated with a negative sign between the product of density versus the gravity right. So, as a consequence, the p, the pressure at any given point z is equal to integral of 0 dz right. Now, g is a function of actually a function of height that means, g varies as a as the height increases, but we can pull the g out of this integral and integrate rho versus dz right. So, this is a basic hydrostatic equation which we will use.

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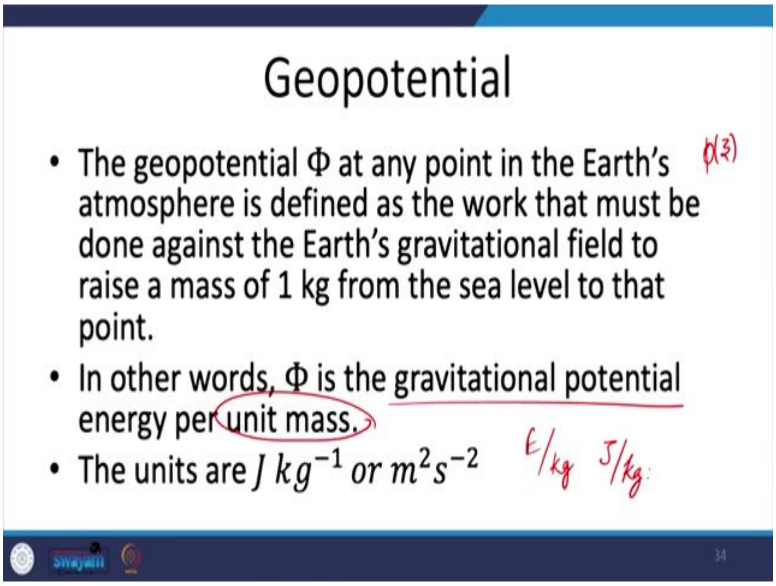

So, here we talk about what is called as the geo potential. Geo potential is indicated with a phi. So, geo potential at any point in the earth's atmosphere at any height, at any point is defined as the amount of work that must be done against the earth's gravitational field to raise a mass of 1 kg from the sea level to that point. So, you are defining geo potential at any given point in the earth's atmosphere is the amount of work that is to be done in rising an object of unit mass from the surface of the earth to that particular point.

So, geopotential, now obviously, becomes a function of the height. So, at different points, obviously, the amount of work that is to be done by for rising the object will be different, that means, the geo potential is obviously, a function of the height right. Or in simple words phi is a gravitational potential energy per unit mass, simple. You take, you suspend the particular mass at a particular height, the gravitational potential that is held by this object per unit mass is the geo potential. So, units are energy per joule per kg; right gravitational potential energy per unit mass. So, energy per kg right, joule per kg is other unit right.

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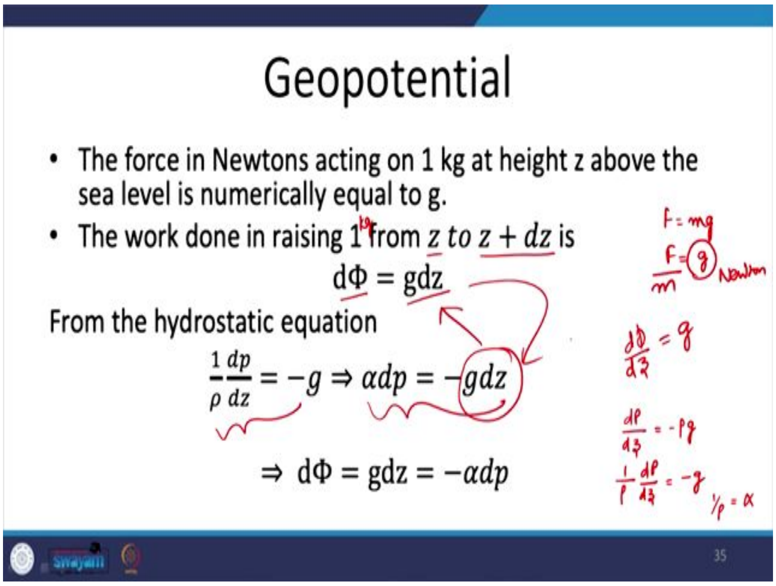

So, most importantly, so the force in Newton's let us say if you dig in more the force in Newton's acting on 1 kg of mass at a height z above the sea level is numerically equal to g right. We write the force as F is equals to mg. Now, if I want to calculate force per unit mass F is simply g right in Newton's. So, now, you have gotten rid of the mass, so which is by mass, force per unit mass right. Force in Newton's on a 1 kg object and height z is numerically equal to small g which is the acceleration due to gravity.

Now, the work that is done in raising 1 kg of mass from 1 kg of mass from z to z plus dz work done when rising 1 kg of mass from one portal height z to z plus dz right. Now, what we have defined is at any given point this phi from 0 to z; all right. So, between two points, between z and z plus dz, the change in the geo potential is gdz, that means, that the rate at which the geo potential changes with respect to height is equal to the gravity right. So, we know that the from the hydrostatic equation, what is the hydrostatic equation dp over dz is minus rho g or 1 by rho dp over dz is equal to minus g right, this is what we have written here.

So, from the hydrostatic equation 1 by rho dp by dz is equals to minus g, that means, alpha dp is equals to minus g dz because 1 by rho is written as alpha, the specific volume right. So, if you say, so one this is the hydrostatic equation and a consequence, this is rearranged to be this is still the hydrostatic equation, but we know that g dz is equal to d phi from this equation right. So, we use this equation here.

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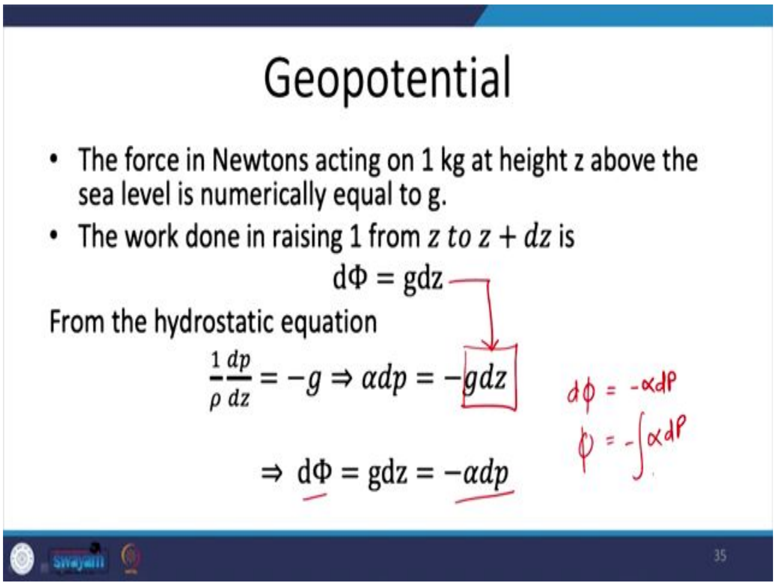

So, this is the hydrostatic equation. In the hydrostatic equation, you are making a substitution for dz from d phi. So, d phi is now equals to minus alpha dp right. So, d phi is equals to; minus, so the change in the geopotential is equals to minus alpha dp.

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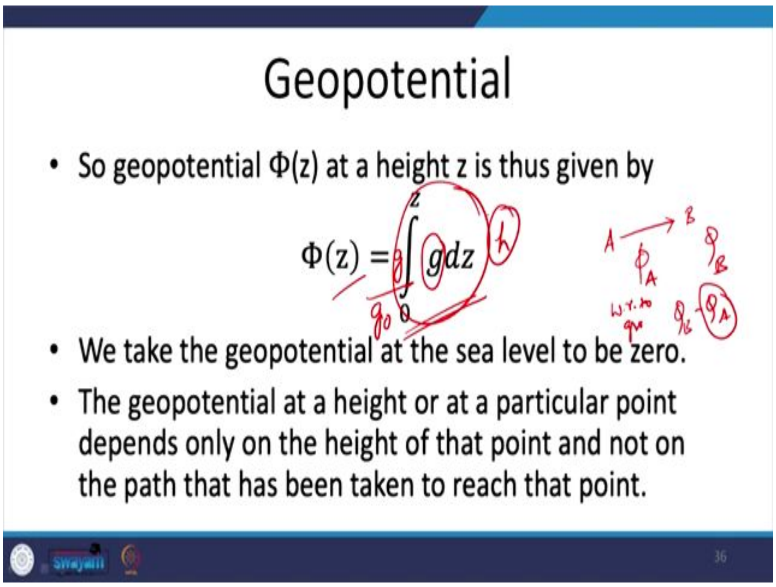

So, geopotential phi of z at a height z is thus given here. So, what we have done if we just integrated this equation. So, d phi is equals to, so phi is integral minus alpha dp; is just an integration. So, so the geopotential phi of z at any height z is can be given as phi of z is equal to 0 to z g dz. So, we take the geopotential at the sea level to be 0 that means, because we have taken the reference from the sea level, the amount of energy that is spent to raise an object from the sea level to a particular point is the geopotential.

So, if you take the reference to be the surface the amount of energy that is that should be spent on the surface is 0, we take the geo potential at the surface or the sea level to be 0. So, the geo potential at a height or at a particular point, depends only on the height with reference to the surface, but not on the path that has been taken to raise or to reach that particular point. Again, it is very simple. So, geopotential is the amount of energy from 0 to h. So, it does not matter in which direction or in which path, you reach this point; the energy is always independent of the path, but dependent only on the height difference between the surface on that particular point.

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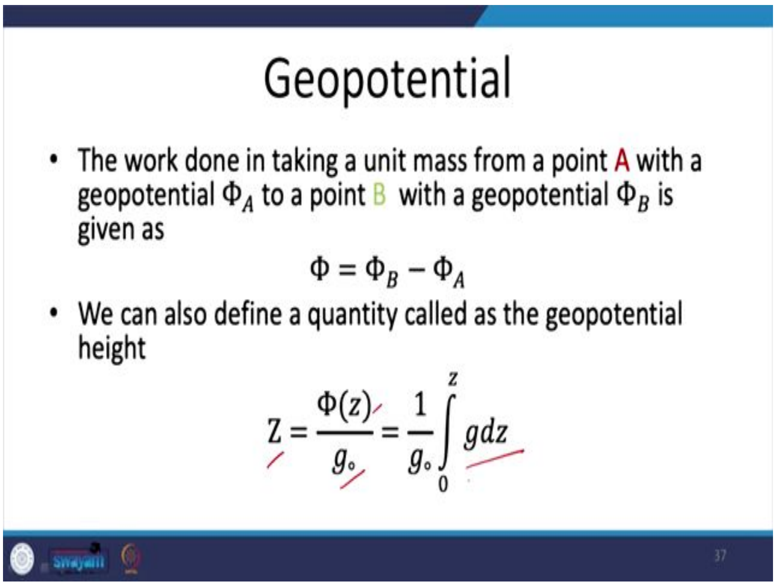

The work done in taking a unit mass, unit mass of object from a point A with a geopotential phi A to a point B with a potential phi B, is the difference in the geo potentials between phi B and phi A. So, we are taking mass from point A to B, point A to B, you are taking it, but basic thing is phi at a point is defined as z. So, at a point A, the geo potential is already defined to be phi A, the point B is already defined with phi B. So, if you want to take an object from here to here the amount of energy is simply phi B minus phi A, because phi A is already defined with respect to the ground. So, this information is already there right.

So, we can also define a quantity. So, based on this we can also define a quantity which is called as the geopotential height, not geopotential, but the geopotential height. So, geopotential height is when you normalize the energy phi z with respect to the gravity at the surface at the sea surface. So, z the geopotential height is defined as the geopotential of A at a particular point in comparison to let us say g naught or 1 by g naught phi of z is defined to be 0 integral of g dz. So, from the hydrostatic balance, so z is equals to 1 by z naught integral 0 to z g dz fine.

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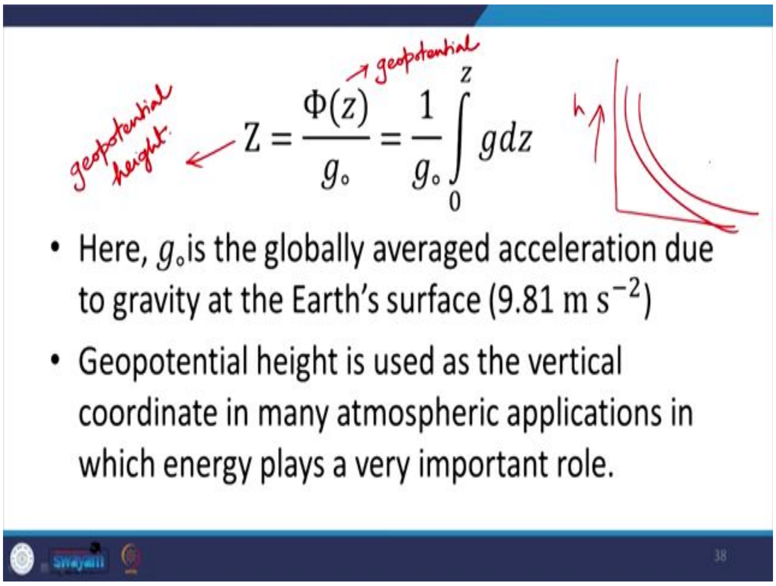

So, phi is the geo potential, and Z, capital Z is the geo potential height right. So, here g naught is the globally averaged acceleration due to gravity at the surface; 9.81 meter per second square. So, this is a standard value which we take for calculations right. So, geopotential height is used as the vertical coordinate right. So, here what is a vertical coordinate, vertical coordinate is with respect to which you measure the variation of pressure or you measure the variation of density, vertical coordinate is generally height. So, we take height to be the vertical coordinate against which you draw the pressure like this, you are you draw the density like this an exponential decay that is it.

So, what has been realized for specific purposes height may not serve as a good vertical coordinate, we replace height with density. That means, we define pressure surfaces in term instead of instead of heights, or so geopotential in that geopotential can also be used as the vertical coordinate. Because every core, every point in the vertical direction is unique in the sense that it requires a certain amount of energy to raise a unit mass from the surface to that particular point. And more importantly this energy does not change if you take a different path, if you take a curved path to reach from the ground to that particular point, so there is no this, this dependence of phi over z a single valued, that means, we can replace the height with the geopotential right.

So, geopotential height can be used as the vertical coordinate in many atmospheric applications in which energy plays a very important role. So, we take those cases where energy input or output plays a very important role. In those cases, we take geopotential height rather than using the height itself right.

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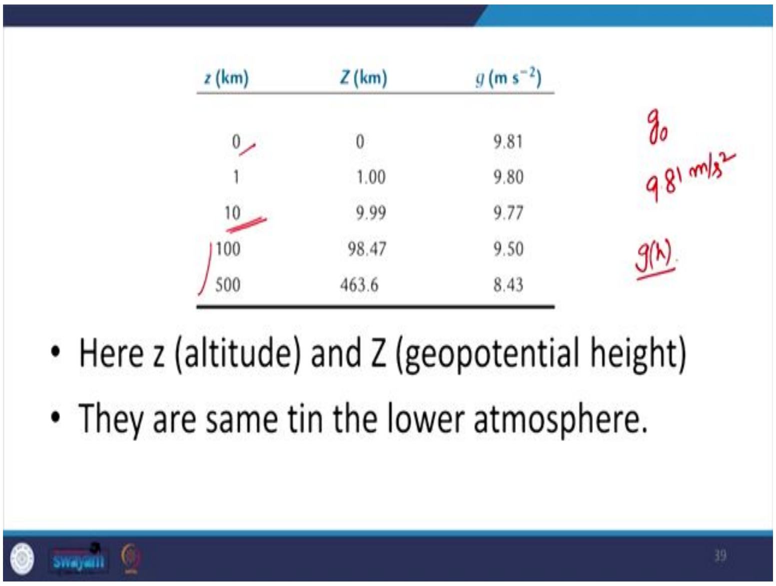

So, let us say, so here a table is given, so where z is the height in kilometres in the in the base units, let us say in meters or kilometres. So, phi of z yeah phi of z is this right. So, if gravity is constant, you take gravity outside. So, this has the dimensions of gravity times the distance right. Now, you divide by g naught you have information of height alone right, so that has been used in defining the geopotential height like this.

And if you use it for defining the vertical coordinate, so z, small z is the distance in kilometres, and capital Z is again distance in kilometres, but this time this has been normalized with g naught. What is g naught? g naught is 9.81 meters per second square.

Now, most importantly you see; as you increase the height, the geopotential height which is normalized with respect to surface, acceleration due to gravity is almost the same up till 10 kilometres, I mean both the height and the geopotential height are almost the same. But they start to deviate after that this is because how the gravity varies with respect to height right. So, this small z is the altitude in kilometres and capital Z is the geopotential height right.

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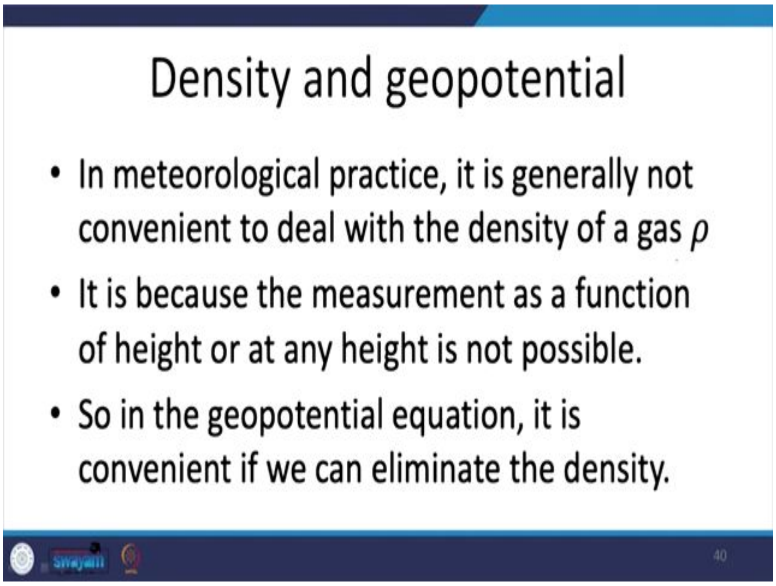

So, they are same till in the lower atmosphere up to 100 up to 10 kilometres, after that they start to deviate right. So, in meteorological applications or calculations on theory, we do not is generally not convenient to deal with the density of a gas. Let us say if you want to perform some calculations at 100 kilometres, or let us say at 15 kilometres in meteorological perspective. So, here generally we always have an empirical relation for calculating the density how density varies with respect to height.

So, we do not know how what is the actual value of density at a particular point, but rather we use a crude equation for calculating the density in terms of its scale height and in terms of the density at the surface. So, if you want to have an actual measurement, it is not possible. So, in the geopotential equation is generally very convenient if we eliminate the density. So, geopotential has a density term appearing in it. So, if we eliminate it, it will be more convenient right.

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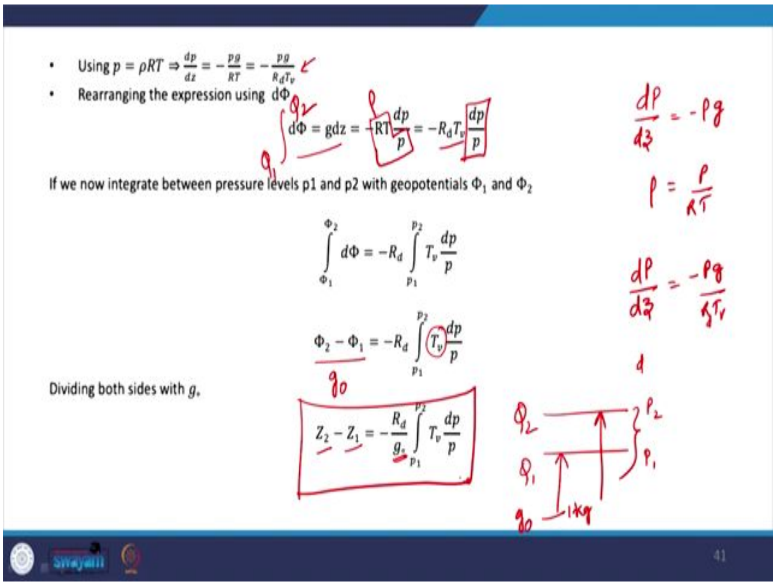

So, what we do here is, we use p is equal rho RT – the gas equation. And from this we can say that dp by dz is minus rho yeah. So, here, so the hydrostatic equation is taken as it is minus rho g, and rho is equals to p by RT right. Now, we use this in the hydrostatic equation dP by dz is equals to minus p g divided by RT right. Now, again we use not the universal gas constant, but use a use a dry air's a specific gas constant, immediately you should allow the provision of moisture by using virtual temperatures right. So, dP by dz is equal to minus pg divided by r dt v; this is what we have here right. So, rearranging this expression, so we know that d phi is d gz g dz right.

So, we are using by using this; d phi is equals to g dz that is minus RT times dp over p right. So, R d, so this is RT by p, RT by p is the density that is it right. So, instead of density we are using RT by p, so that gives a logarithmic term d p over p R d times T v into d p over p. Now, if we integrate now this expression between two geopotential surfaces let us say phi 1 to phi 2; which so we have gotten rid off density by the way because we did not wanted the density, density term appearing in the geopotential, we gotten rid off density.

So, now the density is now written in terms of the virtual temperature, the specific gas constant of dry air and the pressure itself right. So, here, so if you assume two geopotential surfaces to be existing let us say phi 1 and phi 2, what does it mean? It takes different amounts of energy to raise an object of unit mass 1 kg unit mass from the surface 0 or g 0 level to these heights. So, the difference of these two energies or four I mean these two surfaces should obviously, be defined in terms of two pressures, two unique pressures.

So, if you integrate from phi 1 to phi 2 is equals to minus R d times p 1 to p 2 t v dp over p all right, then we have phi 2 minus phi 1 is equals to simple integration. So, I have not pulled temperature T v out of the integral; that means temperature is now assuming temperature will have some functional dependence in terms of p, I kept it inside. So, z 2 is phi 2 minus phi 1 is equals to minus R d times p 1 to p 2 T v dp over p. So, it is simple. Now, if I divided this with g naught phi 2, phi 2 by g naught is Z 2, phi 1 by g naught is Z 1 is equals to minus R d by g naught times integral p 1 to p 2 T v dp power p fine right. So, we take this equation forward all right. So, the change or the difference of geo potentials between two pressure surfaces is defined with p 1 and p 2 is Z 2 minus Z 1 is equals to minus R d by g naught integral p 1 to p 2 T v times dp over p right.

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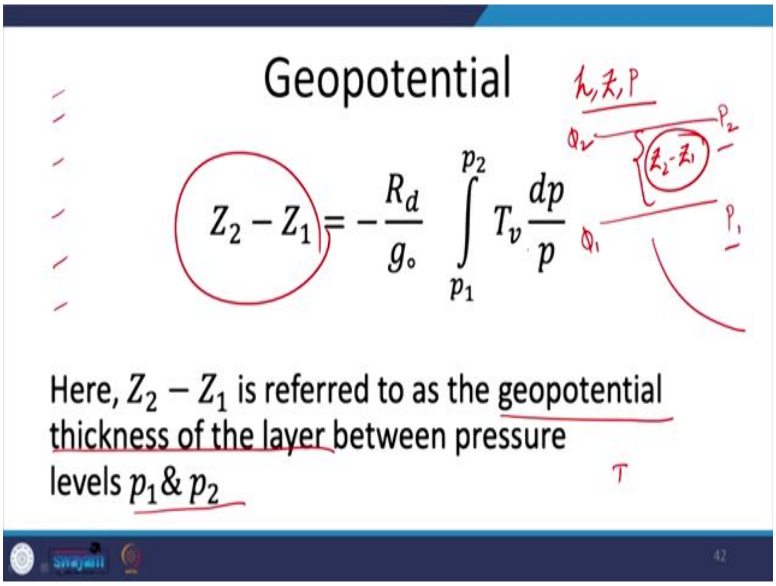

So, Z 2 minus Z 1, here Z 2 minus Z 1 is defined as the geopotential thickness of a layer between pressure surfaces p 1 and p 2. So, if you take p 1 and p 2; now see here most importantly p 1 can p 1, p 2 can be used as pressure coordinates, see height varies linearly I mean we have 1 kilometre, you have 2 kilometres, you have 3 kilometres it like this, right; but pressure does not vary linearly pressure varies exponentially. So, it gives an additional advantage, an additional information when you use pressure as a vertical coordinate in instead of instead of height because some certain physical phenomena are more dependent on pressure. So, they will their a mechanism can be understood by using pressure as a coordinate the variation with respect to pressure is more easy to understand. So, pressure can if the pressure is used as a coordinate the pressure must be tagged with a certain geopotential.

So, the height difference is Z 2 minus Z 1. So, this is Z 2 minus Z 1 is called as the geopotential thickness of the atmospheric layer which is existing between p 1 and p 2 right. So, here we have the we have the information of height, we have the information of geopotential height and we have the information of pressure. All these things they are they all go hand in hand we can use any of these as a as a vertical coordinate right.

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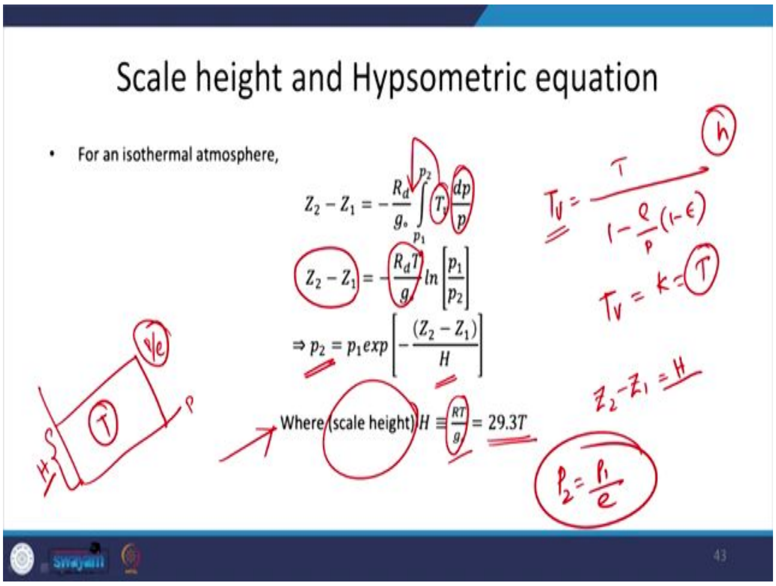

So, for an isothermal atmosphere, that means, isothermal atmosphere when I say temperature is constant. So, here previously temperature had a pressure term inside. As the pressure changes the density changes, the mean molecular weight changes and ultimately the virtual temperature also changes right. So, it is I mean we have taken the virtual temperature equation sometimes. So, T v is equals to T by 1 by 1 minus epsilon into one minus e by p into 1 minus epsilon, but this is what we had derived the expression; to be if we just recall that expression it will be helpful why to define an isothermal atmosphere rather not to work with the temperature variation along with the height right.

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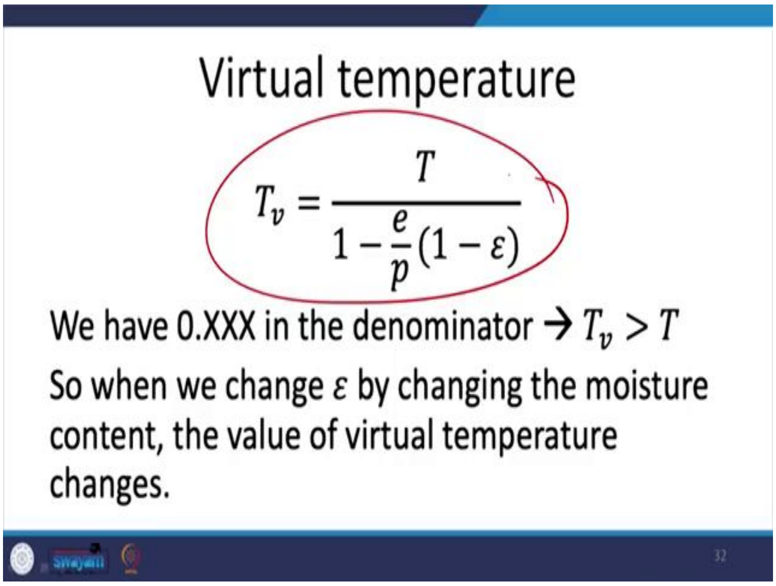

So, here you see T v is equals to T the actual temperature divided by 1 by 1 minus e by p into 1 minus epsilon right. So, here we have defined T v is equal to T divided by 1 minus e by p into 1 minus epsilon right. So, here it makes sense to have T v inside the integral. Now, if you say that isothermal atmosphere; you know longer you know longer variate T v with respect to height right. So, we will say that T v is a constant which we are free to call as T. So, here, so T v comes out of the integral R d times T divided by g naught ln. So, you evaluate this integral ln p 1 to p 2.

So, the geo geopotential thickness of two pressure surfaces p 1 and p 2 is equal to Z 2 minus Z 1 is equals to minus R dt divided by g naught ln of p 1 by p 2, that means, now you are you are able to write p 2 with this with this with this ln term p 2 is equal to p 1 times exponential minus Z 2 minus Z 1 divided by capital H. What is capital H, capital H is R T by g naught which is appearing outside the integral, R d T by g naught right. So, this is the scale height.

So, what have we done? we have We have taken the geopotential height at a particular pressure surface, and we have taken the geopotential height at a different pressure surface, and we have calculated the difference in the geopotential height between these two pressure surfaces. What we realize is that this expression gives us the variation of pressure with respect to height, but interestingly now we have a we almost have a constant R dt divided by g naught, so 29.3 times T. So, you take the value of temperature at a particular height, you substitute it inside you will get the value.

The more importantly here we define a very important physical parameter which is called as the scale height. So, what is scale height? So, if you see p 2 is equals to p 1 times exponential minus Z 2 minus Z 1 by H. So, if Z 2 minus Z 1 which is which is in the units of length or the height; right if it is equals to capital H, you will realize p 2 is equals to p 1 divided by e. So, scale height is defined as that distance over which the pressure drops to 1 by eth of its original value right.

So, here R is a constant, T is the temperature right. So, within a region you define the average temperature to be T. If you know that value of T, you g naught is again a again a constant. So, you will realize by travelling over a certain distance capital H, the pressure will drop to 1 by eth of the pressure; this dimension of length is called as the scale height right.

So, I think we will stop here. So, we will continue our discussions about how we can use the hypsometric equation, we have already derived the hypsometric equation in a different class. We will we will try to see how we can deduce meaningful conclusions based on the understanding of hypsometric equation and this scale height; ok.