Introduction to Atmosphere and Space Science
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Lecture – 19
Thermodynamics - Moist Air

Hello dear students. So, in today's class we will continue our understanding of atmospheric Thermodynamics. So, in the last class we have been able to understand what is the apparent molecular weight, how is it calculated for a given chemical composition of an atmosphere. So, we have taken an example to see how the mean molecular weight of the earth's atmosphere below a certain height has been calculated.

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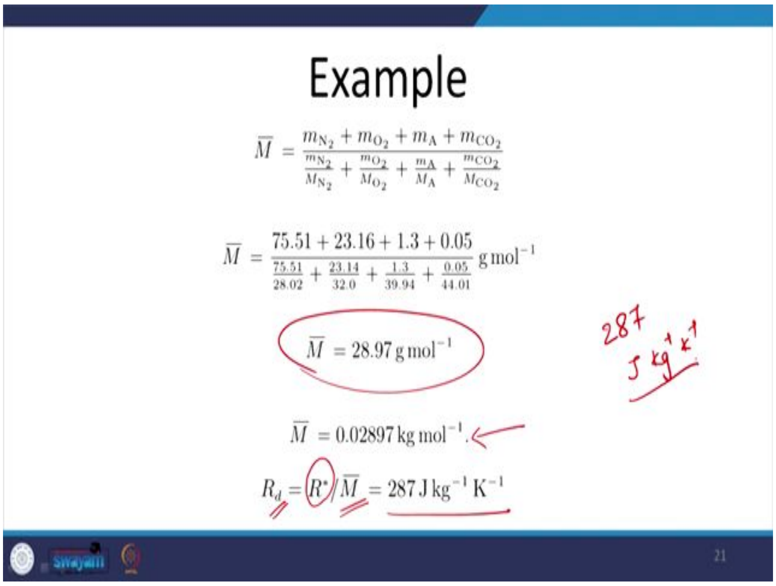

So, we realized that the mean molecular weight is 27, 28.97 gram per mole, this is for the dry air, right. So, we understood that the mean molecular weight has to be calculated every time when the composition of the air changes.

So, this is defined per 1 kg of a particular concentration or in our case for 1 kg of dry air, the mean molecular weight was calculated to be this. If you know the mean molecular weight, the gas constant for 1 kg of dry air can be calculated by knowing the value of universal gas constant. So, universal gas constant is a is a constant for any gas, but the specific gas constant was mentioned to be the gas constant for 1 kg of a particular chemical composition, right.

So, if it is the case, then the dry air's gas constant was 287 joule per kg per Kelvin right. So, this number 287 joule per kg per Kelvin will become hand will become handy for in many of our calculations and you know numericals, right. So, always remember the dry air's gas constant; the specific gas constant is 287 Joule per kg per Kelvin, right.

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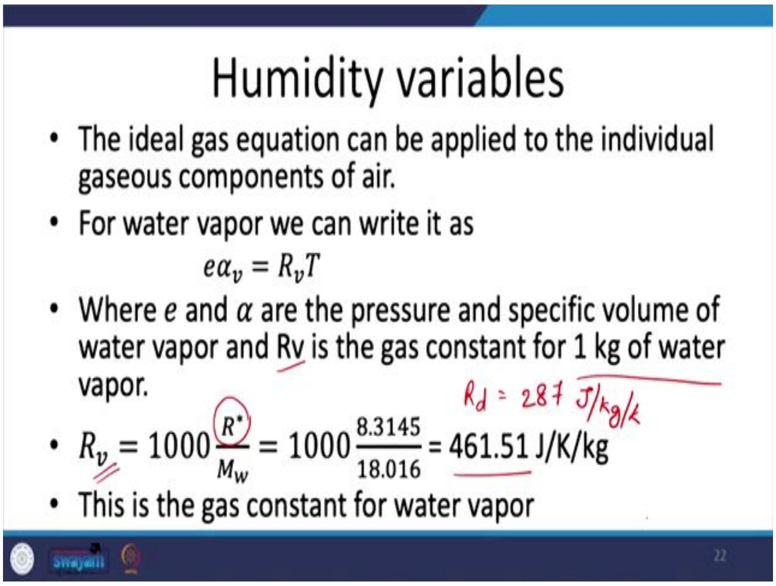

So, now so the standard atmosphere is made up of nitrogen, oxygen, carbon dioxide and argon. So, this is what we call as a dry air. The mixture of these gases in a specific concentration is generally referred to as dry air, right. Now, we will talk about adding some humidity or adding some water vapor into this into this dry air. These are this is this are called as the humidity variable.

So, ideal gas equation can be applied to individual gaseous components of air. So, ideal gas equation if you apply for the dry air, you take the specific gas constant of the dry air that we have taken you have. If you apply for individual gas components as if the individual gas itself is occupying the entire volume and exerting this pressure which is not the total pressure rather the partial pressure, then what you do is you apply you take the gas constant for a particular gas for 1 kg of a particular gas. So, ideal gas equation can be applied to individual gaseous components or air.

So, what for water vapor; let us say, if you take water molecules which are present in the in the gaseous form, so for water vapor we can write it as e alpha v is equals to R v T. So, e is the vapor pressure here. So, vapor pressure the idea of vapour pressure we have discussed many number of times. So, e alpha v alpha is the specific volume alpha v is equals to R v. R v is the gas constant of water vapor and T is the temperature, right. So, here one thing that you must notice this is the ideal gas equation for water vapor ok. So, here alpha and R alpha e and R the pressures and the specific volumes of water vapor, R v is the gas constant for 1 kg of water vapor; for 1 kg of water vapor alone nothing else, ok.

So, you can by knowing the value of R star which is a universal gas constant, we can calculate the specific gas constant.; the specific gas constant of water vapor in kgs as 1000 times; 8.3145 is the universal gas constant by 18.016 is a molecular weight of water. So, which gives you 461.51 Joule per Kelvin per kg. So, this is the gas constant for water vapor. So, if you put it to comparison, so what we know is the gas constant for dry air is 287.

So, let us say; let us write it here the gas constant for dry air is 287 Joule per kg per Kelvin, right. So, the gas constant of water vapor is 461.51 joule per kg per Kelvin right. Now, the most important thing as far as the atmosphere is concerned, atmosphere is neither completely dry nor completely moisture. That means it is not completely water vapor.

So, this gas constant that you have calculated here which is 461.51; is for completely water. You take complete water vapor, then you calculate the pressure that is exerted by this water vapor, then the suitable gas constant should be 461. If you take completely dry air devoid of any single molecule of water made up of entirely carbon dioxide, nitrogen, oxygen and argon, the gas constant should have been a 287.

So, these two are completely different, but in general the atmosphere that we see around us the atmosphere calculations that we do is generally a mixture of both dry air and water vapor. So, for a given pressure you always take it for granted that if you add some moisture into the air, if you if you take the dry air it is always at the expense of replacing some molecules of oxygen or some molecules of nitrogen by the water vapor right. So, the what it means is that if you keep the number of molecules if you take one mole of dry air, let us say where the number of molecules are constant, one mole of dry air or one mole of any gas should contain the same number of molecules.

So, given that if you take one mole of dry air and if you want to make it moist air, that means you by adding some molecules of water into it. So, you will have to conserve the total number of molecules to be equal to one Avagadro number. In that case this water vapor that is going to be added will replace some molecules of majority nitrogen or oxygen, but least the chances are for the carbon dioxide.

So, in that case what happens is the mean molecular weight of the moist air will be less than the mean molecular weight of the dry air. That means, dry air is heavier and moist air is lighter ok. Let us see how the consequences of that. So, this is the gas constant of water vapor you always remember this, right.

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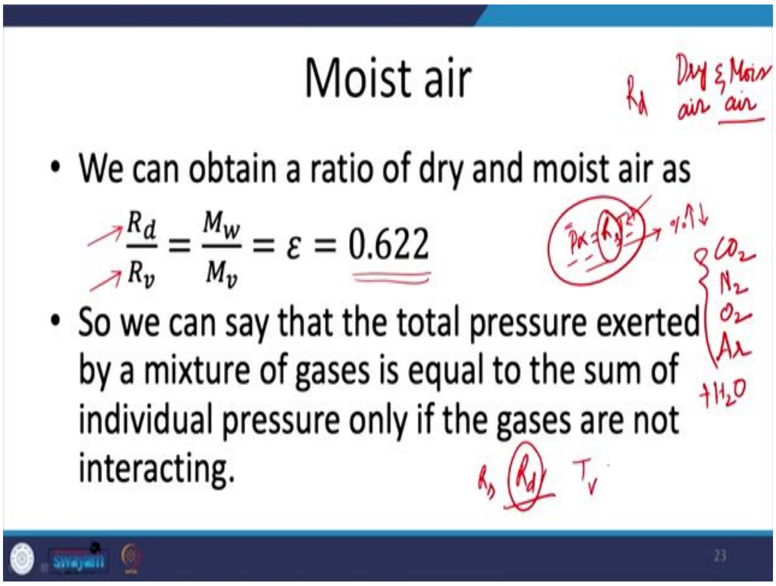

So, moist air we can obtain a ratio of dry air to moist air. So, the amount of water vapor that is present to the amount of water dry air is generally the ratio of the gas constant. The specific gas constant of dry air and the specific gas constant of pure moisture is 0.622. So, if you have completely moisture or if you have a completely dry air, so the ratio of these two is 0.622. So, this is a very important number that you should remember. So, we can simply say that the total pressure exerted by a mixture of gases is equal to the sum of individual pressures exerted by the constituent gases as if the gases are not interacting and each constituent is occupying the entire volume at a particular temperature.

So, I hope you must have listened to it carefully. So, to convey this in a simple term, it is the mixture of it is the sum of two partial pressures, but as if the partial pressures have been calculated as if it is occupying the entire volume by itself right. So, this is some this has some important facts about moisture right.

Now, one thing before we start our discussion about virtual temperature is that, so here any gas has to be has to be defined with the help of its specific gas constant. So, that means if you take 1 kg of gas and then if you measure its pressure at a particular volume and its temperature at a particular temperature and its volume if you measure the pressure, then this p is equals to R T p alpha is equals to RT should be with a specific gas constant.

So, you are taking 1 kg of gas, then you are measuring its pressure, then it should be accompanied with the specific gas constant of 1 kg of that particular gas simple right. So, if it is the case then ultimately so we have seen that the how the mean molecular weight is calculated, how do we calculate it. We take we take the number of moles, we take the masses of each gases divided by the number of moles, you sum it, you will get the mean molecular weight fine.

Now, the simple question is if you change the concentration, if you change the percentage by mass of any constituent gas, it is natural the gas constant will change because it has to accommodate the variation in the pressure that could account due to the change in the constituents right because you are not talking about a mole, you are talking about 1 kg right.

So, where if you add up more number of heavier atoms, you will exceed 1 kg. If you add up if you want to add up lighter molecules to 1 kg, you will need more molecules as simple right. So, if it is the case one thing is if we change keep changing the concentration and if you still want to use let us say if you still want to use p alpha is equals to R let us say R s T if you want to use this ok.

Now, R s is clearly dependent on a particular concentration of the mixture of gases say specific right. Now, for each time when you change this concentration up and down, you need to evaluate the specific gas constant every single time, only then the pressure that can be calculated from this equation knowing the values of alpha and T will match this expression otherwise their a gas constant will not be useful, right. So, the point is every time when you change the concentration, the specific gas constant needs to be evaluated every single time right.

So, what the convenient way of doing it is rather than we do not, we do not let say for the case of dry air and let us say and for the case of moist air. So, the point is you take dry air to be a combination of Co2; CO2, N2, O2 and Argon right. Now, what you do is; if you want to be realistic, air will also have some molecules of H_2O at the expense of any of these as such right.

So that means that what you do is you do not calculate the gas constant every single time rather you transfer the dependence of variation in the concentration over to temperature such that temperature each and every time all implicitly carries the information of how much amount of water vapor is present in your dry air.

So, that is so the transition is instead of using R s you use R d, the specific gas constant of dry air and you use this in combination with the temperature which is called as the virtual temperature. Now, let us see what is the virtual temperature and how we can derive an expression for the virtual temperature right.

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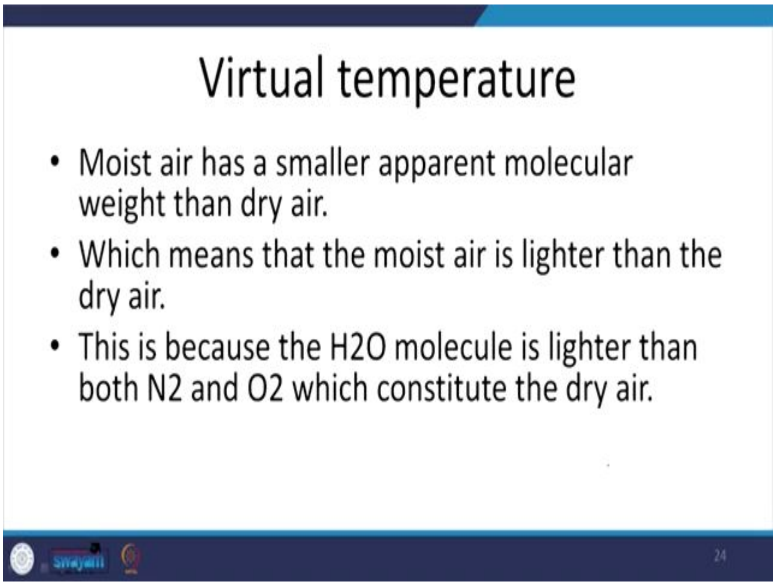

So, moist air has small apparent molecular weight we know that right because of water vapor than the dry air. So, always remember that the dry air is denser and moist air is lighter, ok. So, for a given volume the density of moist air will be smaller when compared to the density of dry air for the same volume right.

So, which means that the moist air is lighter than the dry air which is usually conceived the other way. Generally, the perception is that moist air is heavier and the dry air is lighter, but it is not like that. Dry air is heavier and moist air is lighter if you take the same amount of gas right. So, this is because the H_2O molecule is lighter than both N_2 and O_2 which constitute the dry air. So, even if you replace any of these two and substitute it with H2O molecule, you are decreasing the mean molecular weight as simple right. So, we expect the same amount of pressure.

So, at the same amount of pressure let us say the water molecule replaces some H2 or O2 molecules, right. So, we have already seen that the dry airs specific gas constant can simply be evaluated as this. So, as the molecular weight of dry air will be larger, right we know that the molecular weight of that is what we have seen right.

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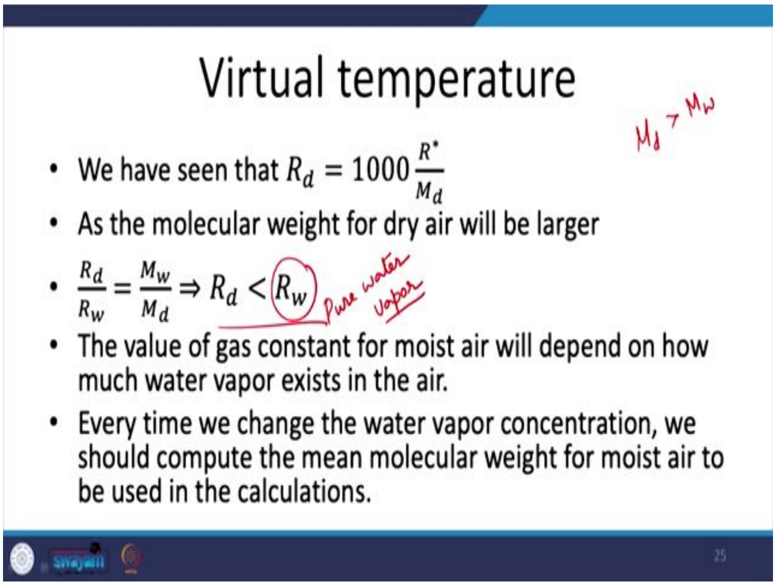

So, molecular weight; so simply molecular weight of dry air will be larger than the molecular weight of moist air as if water acts, simple. So, as the molecular weight for dry air will be larger, you will see that R d by R w from this formula itself is equals to M w by M d which means the specific gas constant of dry air is less than the specific gas constant of the water.

So, always remember this is pure water, this is pure water vapor right. So, this so this is already established. I mean we have already kind of established this right. This is already we have this is what we have already established right and they also fall to a ratio which is 0.622 right.

So, the value of gas constant for moist air will depend on how much water vapor exists in the air, its moist air that I am talking about. I am not talking about pure moisture; I am talking about moist air which means dry air with some water vapor right. So, every time we change the water vapor concentration, we should compute the mean molecular weight for that given concentration and with then only we can use this specific gas constant in the calculation. So, that is the basic way things are following right now, right. So, what is the need for virtual temperature. Right.

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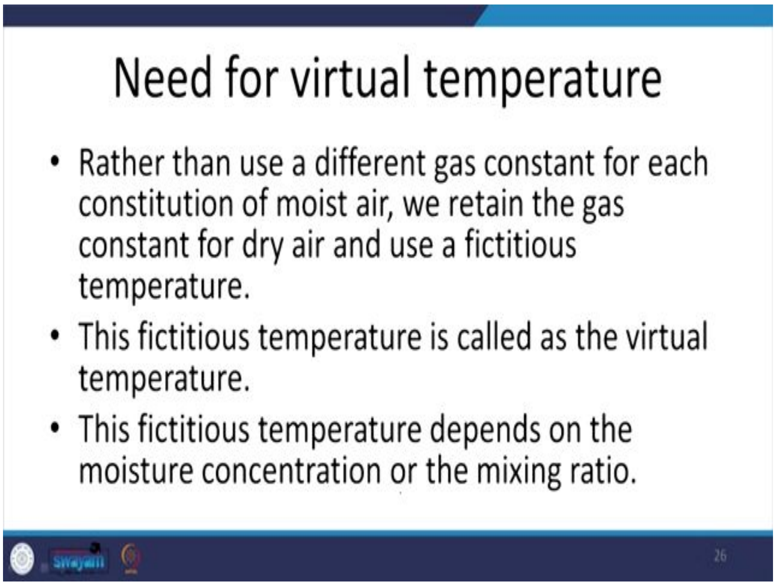

So, rather than use a different gas constant for each constitution of moist air, we retain the gas constant of dry air and we use a fictitious temperature right. So, what do the point is rather than changing the gas constant for each chemical composition of moist air. We retain the gas constant for dry air as it is and which we shift this dependence of moist air at different concentrations on the temperature and call it as a fictitious temperature which is also called as the virtual temperature right. So, this fictitious temperature is called as the virtual temperature simple right. This fictitious temperature depends on the moisture concentration or the mixing ratio.

So, this fictitious temperature has the has the variability of the amount of moisture in the dry air which is also called as the mixing ratio to what point what percentage right. Now, let us say; let us see how we derive an expression for the virtual temperature right. Let us consider a volume v of moist air.

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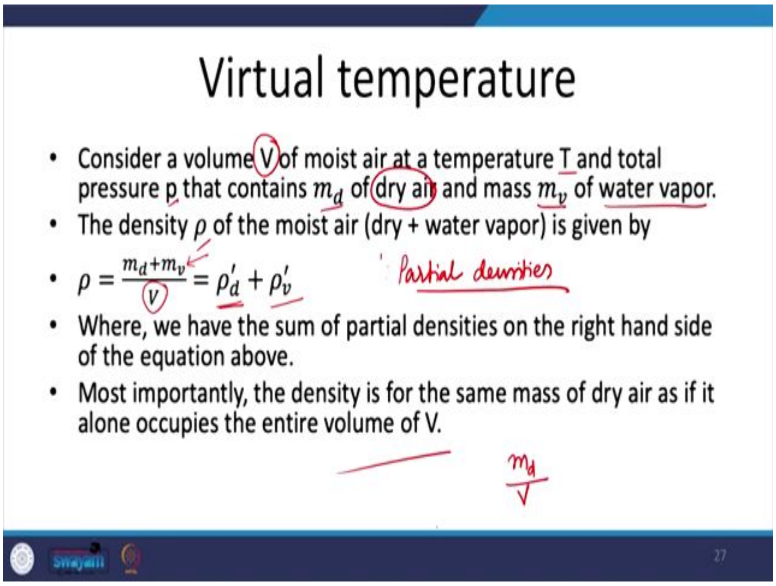

Let us consider volume V of moist air at a temperature T and the total pressure P and which contains m d of dry air. So, here by dry air I mean the regular composition of the air and m v amount of water vapor. So, within the gas that has been; that has been confined with a volume V, there are m d accounting dry air molecules and m v accounting water vapor molecules. So, the density of moist air is assumed to be rho which is a combination of dry plus moisture.

So, the density is given by the total mass divided by volume; volume is the same right. So, total mass can now be divided into mass of dry air plus mass of water vapor right. So, we can since the density is the same, we can write it as the density of dry air plus density of water vapor, but with a prime. So, these primed variables are the partial densities. So, these are not the total densities partial densities. So, the primed variables are. Now, let us say so where we have the sum of partial densities on the right-hand side of the equation of all.

So, what we simply reduce we take we took a volume of V containing a moisture air and we calculate its density to be the sum of partial densities, right. So, most importantly the density for the same mass of dry air. So, the rho d is the density that is coming out of mass of dry air divided by the total volume.

See the total volume is occupied by the total gas I mean the total moist air. It is not that the only dry air is occupying the same volume, but the idea of partial density is like that the mass of dry air which is small than the total mass is still occupying the same volume or the entire volume to itself. So, that is why we call it as partial densities right,

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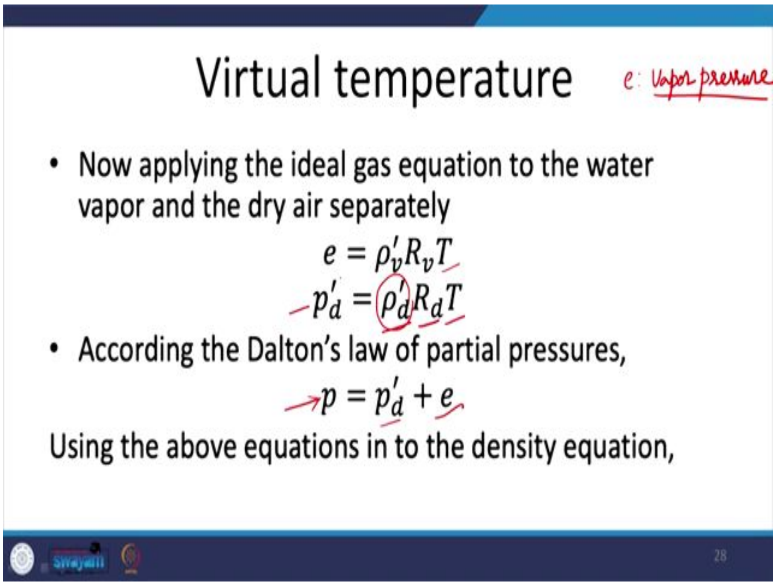

So, if it is taken that way; so then now applying the ideal gas law. So, now we have now we have a mixture of gases two species gases. One species is the dry air species and another species is pure moisture. We take it, we apply the ideal gas law for both of them. So, here always remember we always refer small e as the partial pressure or vapor pressure. Small e is the vapor, it is it is a convention that we use small e for writing vapor pressure. We do not use a small pw or small pv right. So, partial pressure e is rho v prime times R v times T, all right. So, rho v prime is the partial density of the water vapor, R v is the gas constant of the water vapor, T is this is the temperature.

So, the enclosure now we cannot define something called as a partial temperature. The temperature of the entire volume is the same for any gas molecule, right. Now, here p d prime is the partial pressure of the dry air is equals to rho d prime, the partial density of the dry air times Rd gas constant of the dry air and the total temperature.

So, according to the Dalton's Law of Partial Pressure the total pressure that is exerted by the gas is equal to the sum of the partial pressures as if the partial gases are occupying the entire volume is the same story, right. So, the part total pressure p is equals to p d prime plus the vapor pressure. So, using the above equations into the density equation. So, what is the

Density equation? Density equation was this; right using density into this equation we can write that.

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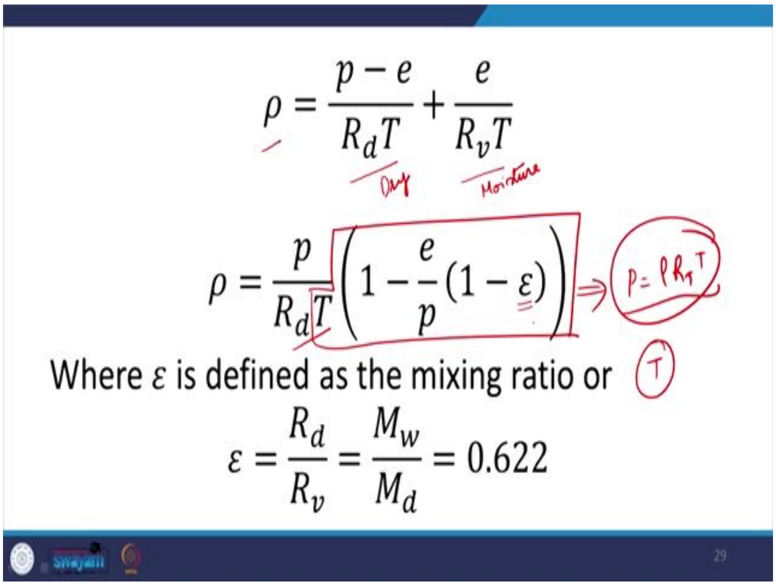

So, total density is the partial density of water vapor plus partial density of the dry air. There is this is the dry air and this is the water vapor right. So, we can write p is equals to p minus e by R d T. So, this is simple as this. So, p d prime is this. So, this is written as this and when you write p, you write p d prime is equals to total pressure minus water vapor; right p minus e by R d T.

So, this is the dry air component of the density and this is the moisture component of the density, right. So, we take p by R d T outside and we then we can write 1 minus e by p into 1 minus epsilon. Epsilon is the mixing ratio or epsilon is the ratio of mean molecular weights of water vapor to dry air, right.

So, we can say that rho is equals to 1 minus e by p into 1 by epsilon naught, right. Now, if you want this to if you want this equation to be in the form of p is equals to rho RT, now if you want this to be written as p is equals to rho R d T right, if you want this to be in this form, so you need all this. Let us say this everything that is inside this box should be of the dimensions of temperature, right.

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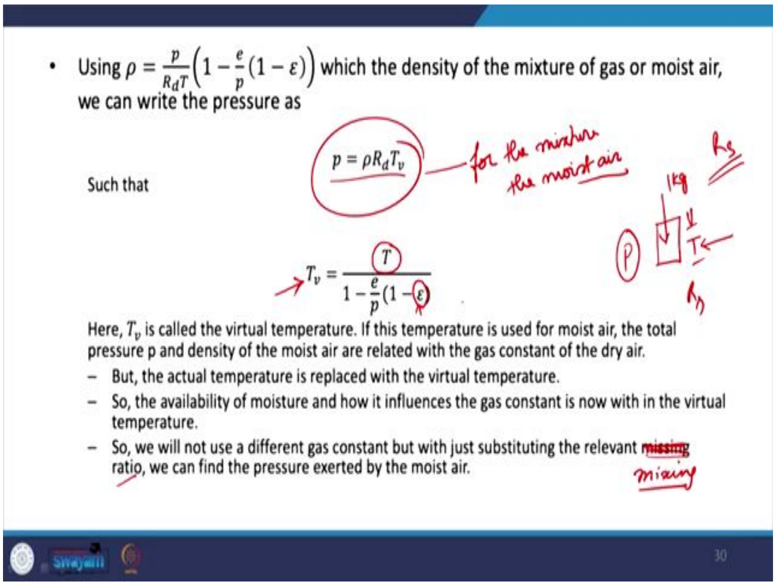

So, then we can say that which is the density of mixture. So, if you we can write the pressure as p is equals to R d T V, then whatever this 1 minus; 1 minus e by p times 1 minus epsilon divided by T should be of the dimensions of 1 by T v. So, here T v is simple. I mean the point is; we have this equation by adding the partial pressures, we got this equation.

So, if the total density has to be this equation, if that means that p is equals to rho times R d T, so this entire box; thing should be in the dimensions of temperature. So, this temperature which is divided by this denominator is generally referred to as the virtual temperature. So, here T v is called as the virtual temperature. If this temperature is used for moist air, the total pressure p and density of the moist air are related with the gas constant of the dry air right. So, here, this is the gas equation for this time this is not for the dry air rather this is for the mixture or for the moist air. This happens only if you use temperature T v.

So, this T v is the virtual temperature. If this temperature is used, if T v is used for moist air, we have the gas constant R d, but not R v so, but the actual temperature is replaced with the virtual temperature. The gas equation is the same, but the actual temperature is now replaced with the virtual temperature.

So, the availability of moisture and how it influences the mean molecular weight this aspect is now transferred to the virtual temperature, but this is not in the gas constant right. So, we will not use the different gas constant, but with just substituting the relevant missing ratio this is wrong, but substituting the relevant mixing ratio we can find the pressure exerted by the moist air every single time.

So we will so what we will do is; we will see how much is the amount of water vapor that is present in the dry air. We know that ratio. Let us say we just put it into epsilon, we are still using the value of temperature. We are not we have not lost the; we have not lost the temperature information as such that means so ideally what is the what is the point is if you take a volume of gas; you know the volume let us say. And if you put it at a particular temperature capital T, the ideal gas law allows you to calculate the pressure that is it, right. Now, if you take 1 kg of this gas and if you know the volume and temperature, the ideal gas law requires the input of R s right.

Now, the point is as long as it is you know what is inside, you can calculate R d; you can calculate R s, you know what is inside if you know, but if you do not know what is inside or rather if you keep changing what is inside, then you cannot use the same value of R s. You will keep you will have to keep changing, right. Now, what the point is you will still need the temperature information as it is, but you will need a ratio which tells you how much amount of water vapor is present in dry air, right.

So, you will substitute that dry air ratio into this and then you will simply calculate the pressure right. I mean not pressure if you know the pressure, you can calculate the volume as well right. So, this is the point. So, here so instead of using a temperature you are using a fictitious temperature which is called as the virtual temperature. So, this is the basic idea of the virtual temperature. The virtual temperature is the temperature that the dry air would need to attain; would need to attain the same density as the moist air.

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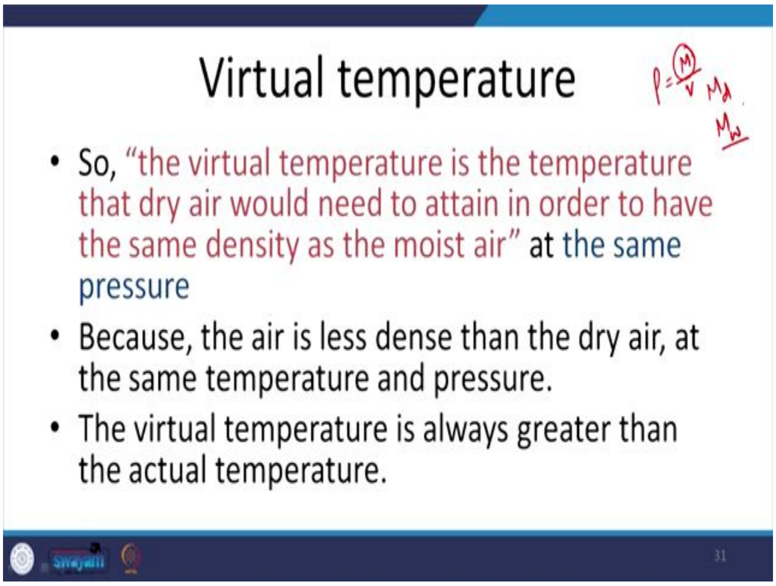

So, I know I mean I have already said the dry air has a density and the moist air also has a density. Now, we have to fix the volume here. If you fix the volume, the density of the dry air will be different. So, density is mass by volume.

So, mass M d for dry air and M w for moist air will be different. So, the mass of the moist M w is larger in comparison to M d right. So, the virtual temperature is the temperature that the dry air would need to attain in order to have the same density as the moist air. That means it has to decrease in its density.

So, the virtual temperature is that temperature that when you take dry air and you decrease its density to match the density of moist air, if you do that you will have to increase the temperature right. So, this expression also says that because the air is less dense than the dry air, the same temperature at the same temperature and pressure the virtual temperature is always greater than the actual temperature. So, what happens generally? When you increase the temperature, the density of a particular gas will decrease right.

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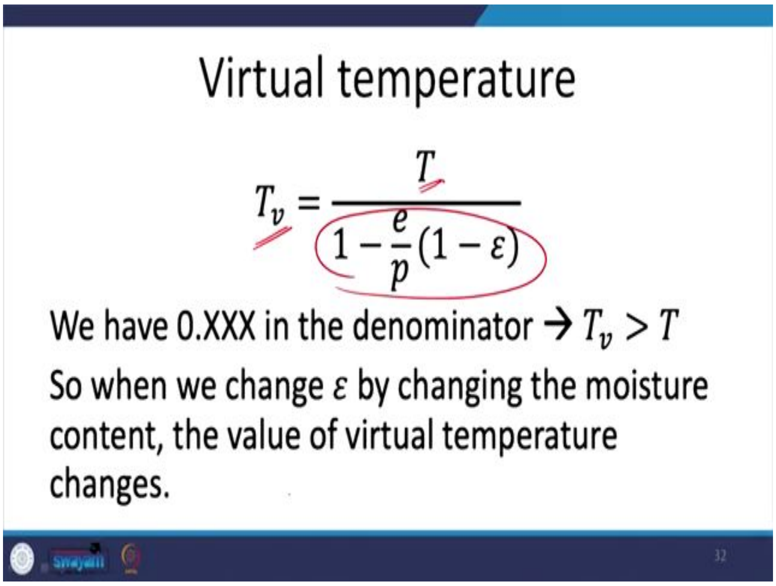

So, that is why so here if you see carefully on in the denominator, you have a term which is less than 1. So, that is why T v is always greater than the temperature. So, T v you divide a number with zero point something on the left-hand side, you will get a larger number right. So, we have the zero point xxx in the denominator. That means T v, the virtual temperature is always greater than the real temperature. That is why that is why the virtual temperature is defined as that particular temperature to which you have to heat the gas, so that its density becomes equal to moist air density right.

So, when we change epsilon by changing the moisture content, the value of virtual temperature also changes right. So, this is the basic idea of virtual temperature. So, what we will do is we will continue our discussion about other aspect of other very important aspect of atmospheric thermodynamics right. So, I hope you must have understood what is virtual temperature and when do we use virtual temperature, right. So, we will stop here. We will continue our discussions on Geopotential in the next lecture.