Introduction to Atmosphere and Space Science
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Lecture - 14
Coriolis Force and Curvature Effect

Hello students, so in continuation with our discussion about the Coriolis Force and deriving the Coriolis force components. We have considered this geometry as shown in this figure.

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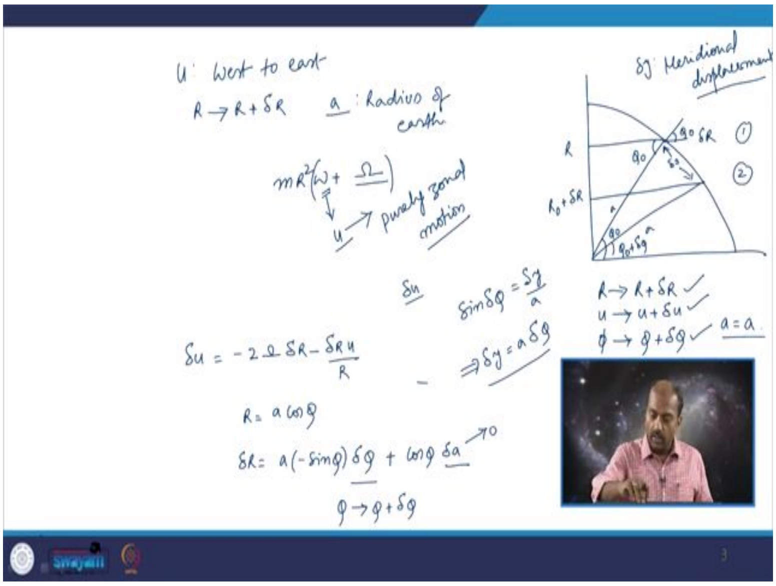

So, the object or the air parcel that we have considered is having it is velocity in the zonal direction, that is, it is moving from let us say from west to east. The object is slightly displaced in the meridional direction causing it to change from R to R plus delta R. So, as you see R capital R is the distance of the object to the axis of rotation, but not to the center of the earth. The radius of the earth is given as small a; small a is the radius of earth right.

So now, the object is displaced such that it is value changes from R to R plus delta R. So, here in the picture it may not appear that R is changing from R plus delta R, rather it appears to be R minus delta R, but we can keep a provision that delta R can be positive or negative. So, in any way so at this point the as the object is moved from R to R plus delta R something that we have seen so far is that its velocity; Of course, the zonal velocity has to change because, it is the only component of velocity that we have involved in the in the discussion so far.

So, here so it is very important that we keep a track; so this is the angular momentum of the object of the object yes and when it is combined with the angular velocity you have omega combined with the angle velocity of the earth fine. Now, as the radius as the distance to the axis of rotation changes by some magnitude, since we have involved only with omega; you have involved only u. So, it is purely zonal motion, so the object is executing purely zonal motion; it is not moving in any other direction.

So, this purely zonal motion has to change by a small magnitude delta u and in addition from this from this geometry what you will also be able to realize is that the latitude changes from phi to phi plus delta phi this is a very important concept. So, for the ease of calculations we can say that the delta phi; the change in the incremental change in angle can be a very small angle.

So, in continuation so what we can do is; so what we have done so far is derive an expression for delta u. So, which is minus 2 omega delta R minus delta R u by R fine. So, from this geometry we can write that R is equals to a cos phi, simply R is equal to a cos phi yes or phi, phi naught. So, it is the same thing. So, delta R can be a times minus sin phi delta phi plus cos phi delta a. Now, the very important thing is that. So as the object is displaced; delta phi change in the angle is it 0 or nonzero.

So, what you can say is the angle is changing from phi to delta phi; so there is a non zero change. So, there is this derivative, this differential cannot be 0 and rather delta a as the object is still on the surface on the surface of the earth. So, this the distance to the center is the radius of the earth. So, at both the times let us say at time 1 and at time 2 the object is still on the surface, so delta a can be treated as 0.

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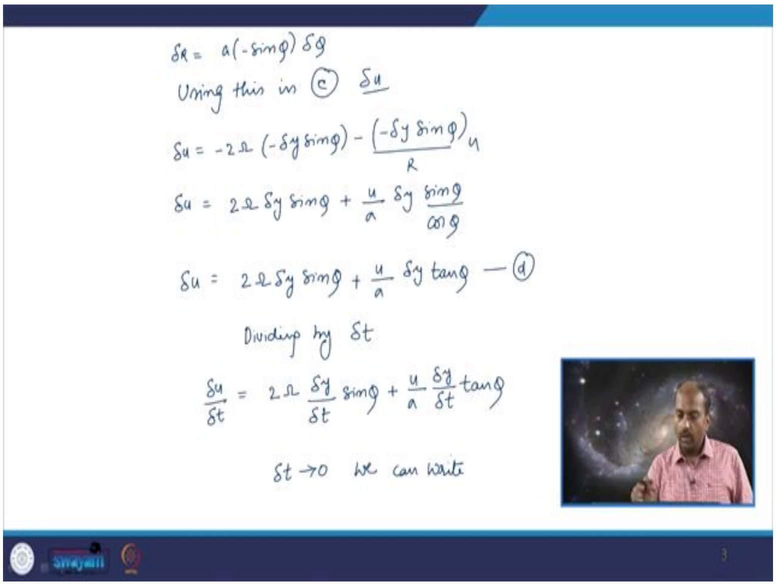

So, if you do that so what we will realize is that delta R becomes now a times minus sin phi times delta phi right. Now, this we can use this in equation c. What is this equation c? Equation c is the expression for delta u. So, you can use this so, delta u can now be written as minus two omega times minus delta y sin phi minus delta y sin phi divided by R u ok. I forgot to tell you how come we how did we get delta y in the picture. So, using this small angle.

So, this is the; this is the displacement, so delta y; what is delta y I have written in this figure delta y is the meridional displacement. So, if the force has done anything it has displaced the air parcel by a magnitude of delta y, using the simple trigonometry we can write that sin of delta phi is equals to delta y by a. That means, delta y is equals to a delta phi provided the angle delta phi is very very small ok. So, you so I have used this into this expression, so which we can write as delta u is equals to; now delta u the change in the velocity is simply 2 omega delta y sin phi plus u by a delta y sin phi by cos phi R is equal a cos phi. So, delta u becomes 2 omega delta y sin phi plus u by a delta y tan phi. Now, dividing this expression dividing this expression by delta t, let us say this expression is equation number d. So, delta u by delta t is equal to 2 omega delta y by delta t into sin phi plus u by a delta y by delta t into tan phi. So, this change delta R is now expressed in terms of delta y actually that is what I see here right. So, if I say no equation is in the deltas now.

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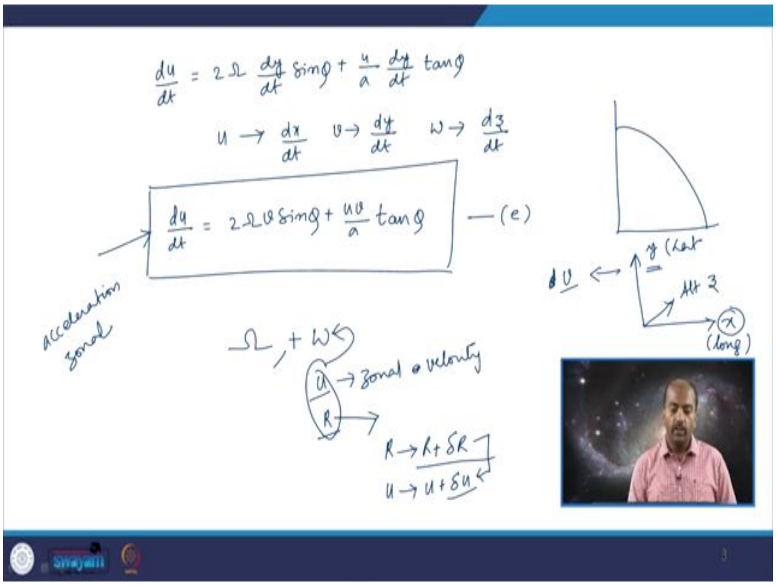

So, which taking the time limit let us say when delta t tending to 0. We can write we can write that d u by dt is equals to 2 omega dy by dt into sin phi plus u by a dy by dt into tan phi. So, u is the velocity which is equivalent to dx by dt, v is the velocity which is equivalent to dy by dt, w is the velocity which is equivalent to dz by dt. So, in the coordinate system that we have taken if you see the earth laterally, so displacement so this coordinate system is like this.

So, this is the x direction which is analogous to longitudes, the y direction is analogous to latitudes, z direction which is vertically upwards either belongs to altitude z. So, the object is having zonal velocity, so the object is traveling across x axis, x direction. Now, the displacement was in the y direction or in the latitudes, so it has now resulted in a component of velocity which is can be written as v right. So, d u by dt is equals to 2 omega v sin phi plus u v by a tan phi right.

Let us call this equation as so, if we have used d this equation can be called as equation e. Now, what is that we have obtained. So, where did we start; we started from having the angular velocity of earth given by capital omega and the angular velocity of the object by the virtue of its linear velocity u in the zonal direction zonal is and the radius; and the distance to the axis of rotation due to these two things we have written the angular velocity of the air parcel as omega. Since the object has a velocity with respect to the earth these two velocities gets added up.

So, this is the basic construct that we have done, I mean the basic, the beginning the is like this now you displace this object meridionally. So, what happens is if you displace the object meridionally towards the equator, then the distance of this object to the axis of rotation will increase let us say right will increase. So, that means, R will go from R to R plus delta R, so meridional displacement by the application of a force has resulted in the velocity to change to u plus delta u fine. Now, what you realize is that meridional displacement has resulted in zonal acceleration.

So, you see the displacement was in the middle direction by changing the value of R right. But now what you see is acceleration, so this is acceleration; which direction is this acceleration in is now zonal acceleration.

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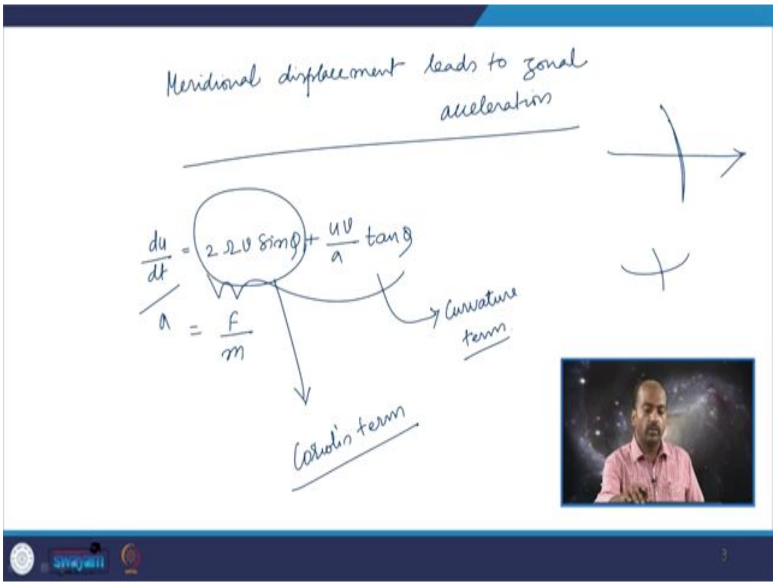

So, in conclusion what you can say is that Meridional displacement leads to zonal acceleration. So, in conclusion of this picture you see meridional displacement leads to zonal acceleration right. So, what is particular about it I mean if you displace an object like this, the object will accelerate in this direction, as simple as that. Now, anything else to it I mean let us say we have now we have two very important terms du by dt; let us say 2 omega v sin phi plus u v by a tan phi.

So, this is whatever what are the physical dimensions of this equation? So, this is the rate of change of velocity which is the acceleration. So, per unit mass the quantities on the right-hand side, let us say per unit mass the quantities on the right-hand side should be equal to F by m right. So that means that, these quantities will have the dimensions of force per unit mass. For example, this 2 omega v sin phi is generally referred to as the Coriolis term this term is referred to as coriolis term.

Now, there are many different possibilities of this I mean moving the object, that means you can displace it meridionally then you will see you can see what are the effects of changes in the meridional displacement. Let us say if you displace it meridionally or you can displace it zonally, let us say if the object is having a velocity component in the y direction and if you displace it in the x direction you can you may want to see what will be the effect of this particular displacement. Then you will write an equation of motion like this right. So, many things can be worked out.

That means that, so in different possibilities you will always realize that the acceleration will let us say we will sometimes will contain two terms. This is this the first term is called as the Coriolis term and the second term is called as the curvature term curvature term. Now, let us say in two in order to understand this more rigorously let us consider the displacement of an object in the vertical direction.

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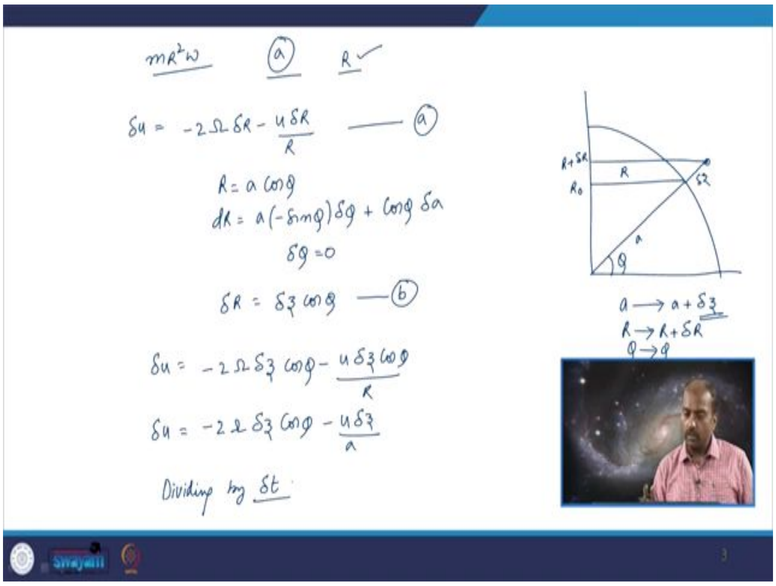

So, let us say the object to begin with is in the same frame of reference, that is; it is moving with respect to the earth at a velocity u. Now, let us say if in this picture what we will do is we will take the object let us say so the object is at this distance. So, at this with respect to the axis of rotation it is at a distance R and with respect to the centre of the earth it is at a distance a.

Now, the object is displaced vertically up. So, this is now when the object is displaced vertically upwards let us say. So, this will go like this; so this is R naught and let us say this is R plus delta R. So, this R is also changing here right and the displacement is by a magnitude delta z. So that means, now the parcel; the parcel or the object that we have taken after the displacement it is now the center the distance to the center of the earth is now changing by a plus delta z ok. Now, if so if you look in the if you look it look at it in a two dimensional picture it may appear.

So, but what you will realize if you if you so the R is also changing from R plus delta R. Now, what you will realize; so this is at the same angle. So, the angle or the latticed the angle at this point subtends at the center of the earth with respect to the equator will not change. So, phi will remain phi. So, here in the earlier case what we have seen is that phi changes; in the earlier case what we have seen is that; in this case R of course is changing u is changing and phi the angle is also changing.

But here a is not changing a remains the same a is the distance to the center of the earth; a is not changing. But in this picture what you will realize is that the value of value to the centre of the earth is changing by a small distance delta z and the angle phi will remain a constant ok. Now, in this picture we can start directly from the even if you. So, here or the angular momentum equation; let us say the angular momentum does not depend on a of course, it does not depend on a.

But it does depend on R right. So that means, that if you work out the same angular momentum conservation you will realize that you will get the same value of delta u, delta u will now be written as minus 2 omega delta R minus u delta R by R right. Now, so in this way so R is equals to; so here R is again a cos phi, so dR or delta R is a times minus sin phi delta phi plus cos phi times delta a right.

So, here so delta phi is 0 in this picture. So, delta R can simply be written as delta can simply be written as delta z times cos phi. So, delta so the change in the value of a is delta z right. So, you can substitute this into; let us call this equation as a now and let us say this equation is b. We can use b in equation a and we can rewrite that delta u is equals to minus 2 omega delta z cos phi minus u delta z cos phi divided by R. Now, R is equal to a cos phi using that delta u is equals to or yeah minus 2 omega delta z cos phi minus u delta z by a right.

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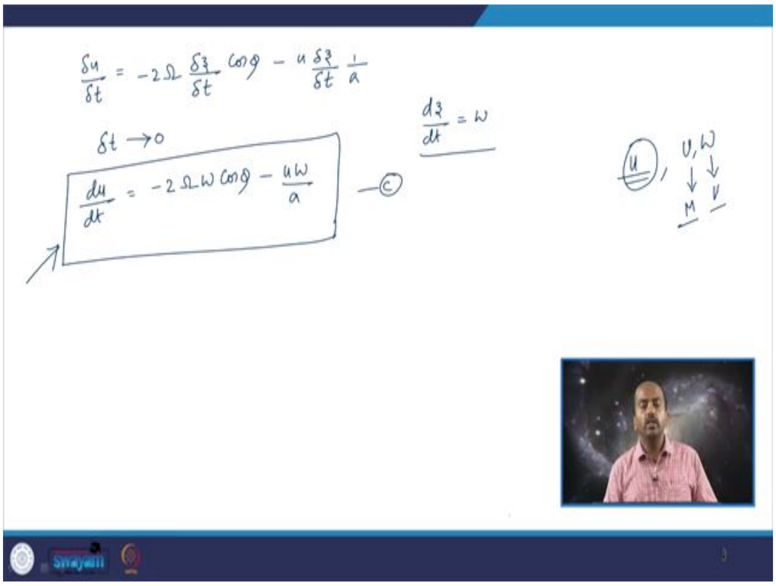

Now, dividing by delta t, if you divide with delta t you will get delta u by delta t is equals to minus 2 omega delta z by delta t into cos phi minus u delta z by delta t into 1 by a. When delta t tends to 0 you can write du by dt is equals to minus 2 omega w cos phi minus u w by a right. So, when so here d z by dt is w there is nothing new about it. So, finally we have got another equation similar to the earlier equation, let us say we call this equation as we call this equation as c. So, what does it mean it means that we have now we have got.

So for an object having it is velocity in the zonal direction. So, zonal direction is the object of object's velocity. So, spare that particular direction you have two other direction v and w right. So, v is the meridional direction and w is the vertical direction right. So, if the object having zonal velocity; having only zonal velocity is displaced in the meridional direction or in the vertical direction, we have seen both the cases. We will realize that the both the cases will lead to acceleration in the zonal direction.

So, just like we have equation e in the last picture we have this equation again representing both the cases where it is leading towards the zonal acceleration right. So that means, let us rewrite these two equations for understanding more details about them.

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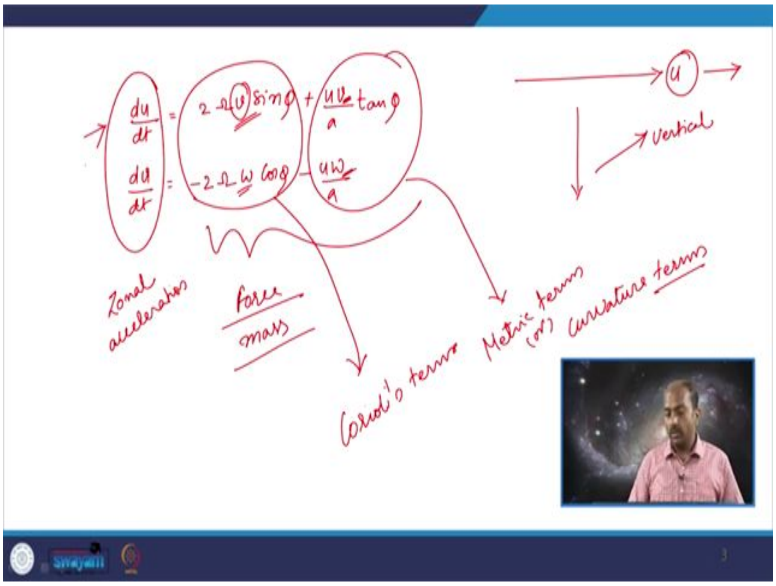

Let us say let us say du by dt is equals to 2 omega v sin phi plus u v by a tan phi and d u by dt is equals to minus 2 omega w 2 omega cos phi minus u w by a right. So, what are these two equations both the equations are zonal acceleration and this is both the equations have zonal acceleration on the right-hand side you have force per unit mass ok, now no matter so.

So, now let us say the other possibilities could be let us consider above object which is moving in the vertical direction and let us displace it in the horizontal direction or in the zonal direction or let us displace it in the vertical direction. Or the other possibility could be let us consider an object which is moving in the vertical direction, then displace it in the zone and meridional direction, so this could be many possibilities.

So, what you realize is that if the objects original velocity is in this direction, if you displace it in this direction or let us say this vertical direction both of them will result in accelerating in the same direction; this is a very important aspect of it. So, in conclusion what you can say is that the first part this these terms these terms are referred to as Coriolis terms and these terms are referred to as the metric terms; metric terms or curvature terms, they arise because of the curvature of the earth curvature terms.

Now, we can extrapolate this argument saying that the object is having it is original velocity in a particular direction and then you can apply external force and displace it in a in a certain direction that you want. And then you can also get these expressions get these expressions similar to this and then we can make a generalized description of the Coriolis force ok.

So, this is the basic so what is the; what is the origin of all this acceleration, the origin of this acceleration is basically the conservation of angular momentum. And what is the need for conserving the angular momentum because you are changing the distance with respect to the axis of the rotation right. So, what is the need if you talk for you to talk about angular momentum conservation is that, because the object which you are thinking to be moving or to be at rest with respect to a stationary frame of reference.

Because in atmospheric physics we always use what is called as the geocentric reference frame, that means a frame of reference that is fixed with respect to the earth. But then things get complicated because the earth itself is not at rest rather it is moving.

So, the in order to if you want to describe the set an object's motion, then you take into account you describe it as it is and then you take into the effect of the frame of reference which is moving. So, then you will realize that you will need additional force components which you call as apparent forces right. So, just to comprehend all these things what we have done is we have taken the object to be moving in a particular direction we conserved this angular momentum and after conserving the angular momentum, we realize that the meridional displacement or the vertical displacement whatever it is, will result in zonal acceleration not in the meridional way or in the.

But one more very important thing is that you see you see that the displacement that you are you have you have done is resulting of course in a velocity across the same direction and let us say in this. But the effectively; the acceleration is in the zonal direction ok.

So, this is where we stop, so in the next class we will try to supplement these two equations with few more components of the Coriolis force and then we try to make a generalized description of the Coriolis force and the effect of Coriolis force on the wind moment along in the different hemispheres and in different directions.

Thank you.