Introduction to Atmosphere and Space Science
Prof. M. V. Sunil Krishna
Department of Physics
Indian Institute of Technology, Roorkee

Lecture – 13
Forces – Coriolis Force

Hello students. So, in today's class we will try to understand the apparent forces. We have seen the origin of apparent forces, why do these apparent forces come into picture or what is the role of this apparent forces? So, apparent forces are only the reaction terms of the inertial forces; that means, so, if you want to account for the motion of the accelerate of the reference frame or non-inertial frame, if you want to account the this particular non inertial frame of reference then you have to add few terms in the forces. So, these forces are generally called as the apparent forces.

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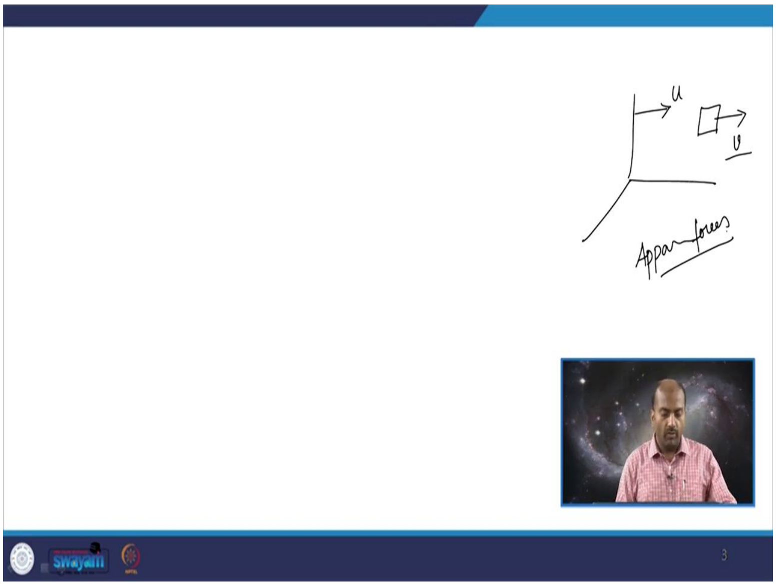

Now, one every important thing is if you consider a frame of reference which is let us see which is rotating or which is moving; any object which is considered to be at rest with respect to this frame is indeed these neither at rest nor in uniform motion. So, the point is you have a frame of reference which is moving with a velocity which is changing in with magnitude or in direction and this object if it is also changing or if it is moving with a velocity this object this object's velocity is to be described or this object motion is to be described with respect to the frame of reference and with respect to the frame of reference which is moving in nature.

So, here we use what are called as the apparent forces fine. Now so, in continuation with our discussion we have already learnt, what is this centrifugal force, how does it come into picture and how does it modify the gravity, how does it modify the direction of the gravity and why the apparent gravity points away from the center of the earth and how is it responsible for creating the bulge that you see on the planet earth near the equator right.

So, in today's class what we will try to do is, we will try to extend the understanding of apparent forces. So that we can formulate or we can derive expression for the Coriolis forces.

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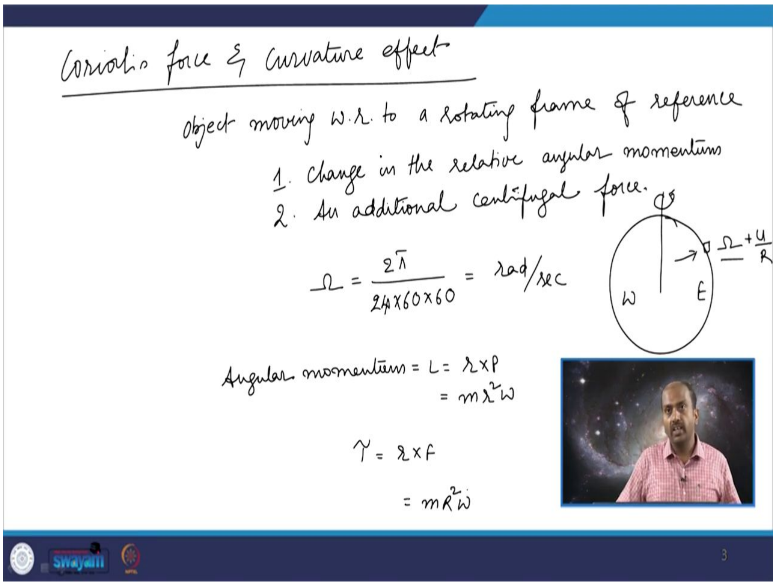

So, today's class is going to be about Coriolis force and curvature effect; curvature effect ok. So, Coriolis force is a is a very important part in our discussions is going to be is going to play a very important role in many of the atmospheric motions. So, because the Coriolis force effect is the one which tilts the direction of motion of an object which is which just traveling or which is moving with respect to moving or rotating frame of reference ok.

So, for an object which is in motion relative to a rotating frame of reference we need to consider two things. So, for an object moving with respect to a rotating frame of reference rotating frame of reference we need to consider two very important things, one change in the relative angular momentum of the object. So, we have to consider change in the relative angular momentum of the object relative angular momentum.

Secondly, we need to add an additional centrifugal force, an additional component which is called as the centrifugal force fine. Now, let us consider; let us say that let us consider an object of unit mass on the surface of the earth. Now, let us say you have the earth let us consider an object which is moving or which is let us say which is at rest with respect to the earth. Now, the earth is rotating from west to east. How fast does the earth rotate if you want to calculate the angular velocity of the earth covers an angular displacement of 2 pi radians over one day so, which is 24 hours; 24 hours into.

So, this will be the angular velocity of the earth in radians per second ok. So, what it means is that, so and there is an angular velocity component with respect to earth. So, earth travels at the speed it covers this mean radian per second. So, if you consider an object which is at rest with rest on the surface of the earth on just on the surface of the earth let us say. So, this object can be considered to be moving at the angular velocity of the earth as long as it is at rest.

If the object is moving by itself if it has it is own velocity so, then that velocity has to be added to this angular velocity of the earth. So, in this case what I have done is, omega capital omega is the angular velocity of the earth and u by R is the angular velocity of the object. So, this so, you add this when the object is moving with respect to the earth. Now, what we have a situation in which the object is moving with respect to the rotating frame of reference ok. So, object has it is own velocity and the velocity component due to the earth itself.

Now, the angular momentum in this case; angular momentum is equals to, it can be written as m r square omega. So, here let us say r is the distance from the center and omega is the angular velocity ok. So, and the resulting torque can be written as r cross F ok. So, the angular momentum on the surface of the earth is given as m r square omega, if you take the radius of the earth to be capital R then you write as m r square omega ok.

Now, this angular momentum will remain a constant as long as there is no torque. So, if there is no torque that is acting on the body the angular momentum will remain conserved. So, it will be it will stay the same as long as ok.

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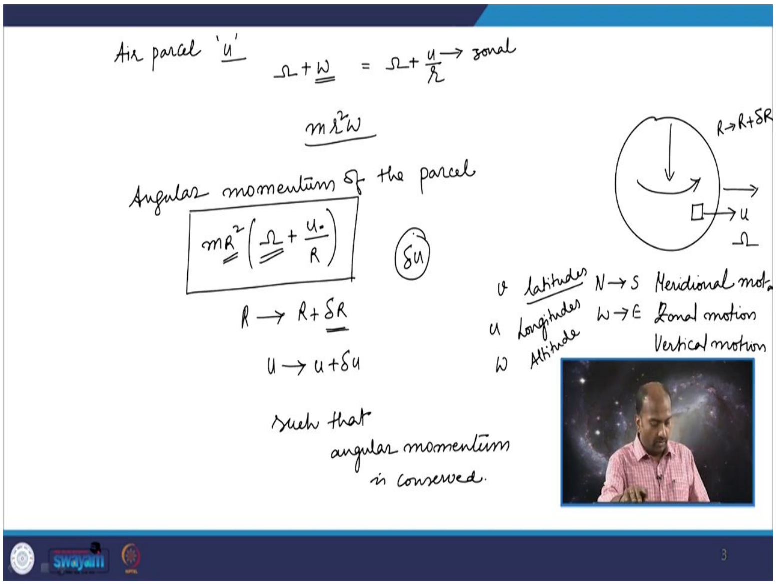

So, for let us say for an air parcel; let us consider an air parcel the angular velocity, if the air parcel has; let us consider an air parcel consistent with our description of the air parcel with which has a velocity u ok. Now, if this air parcel is at a distance of let us say small r from the center then the angular velocity of the air parcel would be omega plus omega. So, small omega is the angular velocity of the parcel with respect to the rotating frame of reference and omega capital omega is the well angular velocity of the earth itself so, which can be written as omega plus u by r omega plus u by r right.

Now, let us say that the air parcel is moving. So, now, it is moving this velocity is called as zonal velocity. Now, let us say in order to be able to identify the different directions of motion or different components of velocity what we do is, we consider the earth let us say. So, any movement in this direction let us say from so, this movement is characterized from north to south from the pole towards the equator or towards the pole is called as the meridional motion.

And any moment in this direction let us say from west to east is called as from west to east is called as zonal motion. And any movement starting from the ground is called as the vertical motion. So, this so, when the object is displaced is set to be displaced in meridional way, in meridional direction then the object is changing is changing latitudes. So, with time the object is traveling across the latitudes when the object is set to be displaced in zonal direction then the object with time is changing longitudes.

When the object is displaced in the vertical direction so, considering generally what happens, let us say if it is at a very brief instance of time, if the object is displaced in the vertical direction then the object is changing in altitude ok. So, latitudinal displacement leads to meridional motion, longitudinal displacement leads to zonal motion and altitudinal displacement leads to vertical motion as simple.

Now, you also identify this direction this is generally; let us say this is generally the u the velocity component in the meridional motion is generally u no, velocity component in the zonal motion is generally u, the velocity component in the meridional motion is generally taken as v and the velocity component in the vertical motion is taken as w.

Now, this particular air parcel that we are talking about. So, the situation is very simple here, the situation is such that situation is such that we have an air parcel which is on this frame of reference or let us say earth this is air parcel. Now, the air parcel has its own velocity in the zonal direction which is small u, this velocity gets added to the angular velocity of the earth which is omega and if it is the case.

So, we have identified the directions of motion we have named them and we also named the directions; the velocity components along these particular directions fine. Now, it is clear that now the air parcel it is moving ahead zonally with respect to earth, I mean since it has its own velocity component it is very easy to understand why it is moving ahead with respect to the earth. So, it is moving ahead with respect to the earth. So, before moving ahead we need to understand the fundamental conservation what is need to what needs to be conserved.

So, one thing as long as there is no external torque applied the angular momentum will remain constant. So, m r square omega will remain a constant ok. Now, let us say you know now this object is having it is own zonal velocity component. Now, the angular momentum of the parcel; now since it is having it is own velocity component,. The angular momentum of the parcel angular momentum of the parcel can be written as m R square omega plus u by R ok.

Now so, this is the angular momentum of the parcel before; when it is moving with a zonal velocity component u right so, zonal velocity is given as u. Now, let us say that the air parcel is slightly displaced such that its radius from the center changes from R to R plus delta R. So, you are going to displace this air parcel slightly in this direction let us say in this direction such that it changes from R to R plus delta R.

Now, what is going to be the consequence? Simple, what is going to be the consequence is that now the object is moved by applying an external force and so that it is the value of R changes from R to R plus delta R then the objects object will try to conserve the angular momentum. How does it conserve the angular momentum, the only other quantity in this angular momentum, let say in this angular momentum that can be changed of course, mass cannot be changed, you are trying to change R by this magnitude by delta R omega the capital omega is the angular velocity of the earth which of course, cannot be changed.

The zonal velocity of the object is the only other physical quantity which can change so, as to maintain the equilibrium in terms of angular momentum or maintain the conservation for the angular momentum. So, as a result of R going to R plus delta R; the object will try to change it is velocity from u to u plus delta u such that angular momentum is conserved; angular momentum is conserved ok.

Now, what we can do is, let us work out the conservation as such let us say what was the angular momentum before and if the air parcel is displaced by a small magnitude what will be the angular momentum afterwards and in this picture let us try to get an expression for delta u by solving the conservation of angular momentum.

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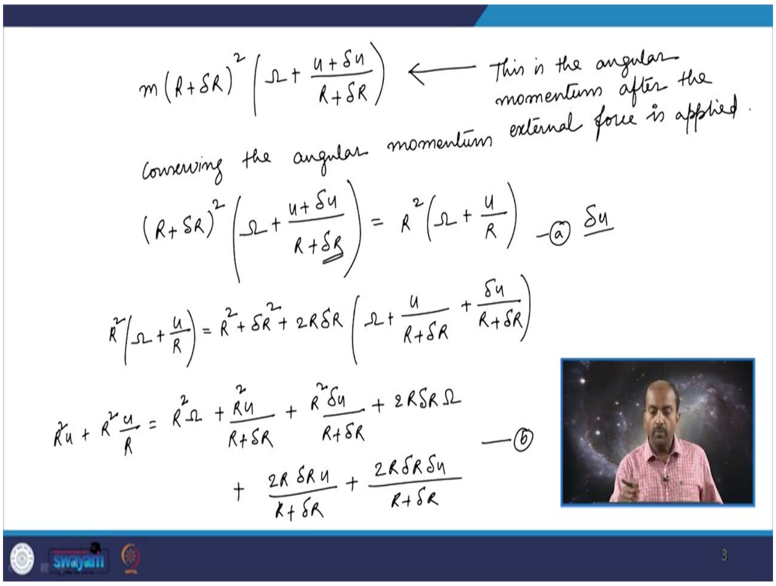

Now let us do to the so, it is natural to expect. So, the new angular momentum would simply be R plus delta R whole square into omega plus u plus delta u by R plus delta R. What is this, this is the angular momentum; angular momentum after the external force is applied external;

force is applied fine. So, what we can do is let us say. So, this is the angular momentum after the external force is applied.

Now, let us equate; let us try to conserve the angular momentum; conserving the angular momentum you can write R plus delta R whole square into omega plus u plus delta u by R plus delta R should be equal to R square times omega plus u by R. Now, the basic idea is if you apply an external force and change R by solving this conservation you should be able to find out what will be delta u.

So, as to equate this conservation always so, we will simply we will solve this. So, we will write R square mass gets cancelled anyway R square times omega plus u by R is equals to R square plus delta R square plus 2 R delta R times omega plus u by R plus delta R plus delta u by R plus delta R, which you can write as R square u plus R square u by R is equals to R square omega plus R square u by R plus delta R plus R square delta u by R plus delta R plus 2 R delta R omega plus 2 R delta R u pi R plus delta R plus 2 R delta R delta u divided by R plus delta R ok.

Now, in this let us say in this equation let us say this equation is a or let us say this equation is this equation is a and this equation is b.

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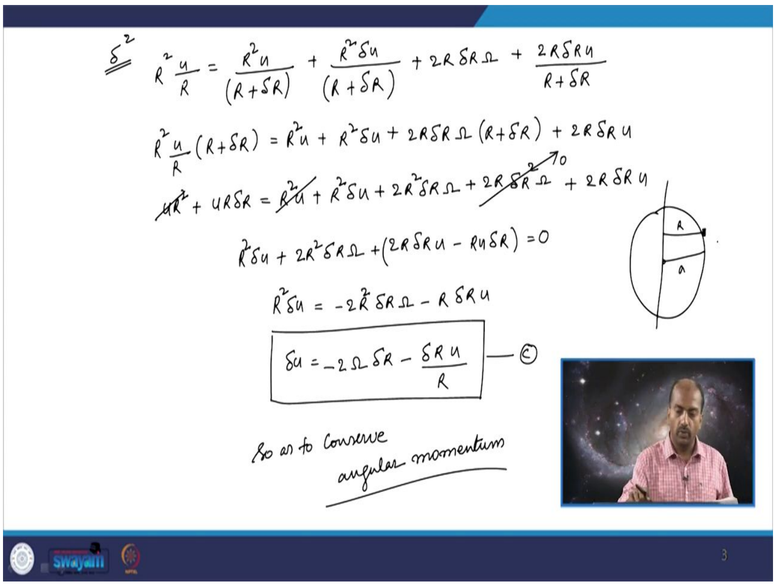

Now, neglecting all terms in the second order terms of delta; neglecting this second order del square terms we can write; we can write, R square u by R is equals to R square u by R plus delta R plus R square delta u divided by R plus delta R plus 2 R delta R omega plus 2 R delta R u divided by R plus delta R, which can be subsequently written as R square times u by R into R plus delta R is equals to R square u plus R square delta u plus 2 R delta R omega into R plus delta R plus 2 R delta R u.

So, which can be simplified by multiplying this into the bracket which will be u R square plus u R delta R is equals to R square u plus R square delta u plus 2 R square delta R omega plus 2 R delta R square omega plus 2 R delta R u. So, this is the second order term in delta R, so that can be neglected and these two terms get cancelled.

So, as a result what you have is we have R square delta u plus 2 R square delta R omega plus 2 R delta R u minus R u delta R is equals to 0 which can be written as R square delta u is equals to minus 2 R square delta R omega minus R delta R u delta u, can be written as minus 2 omega delta R minus delta R times u divided by R. So, the velocity should change by this magnitude so, as to conserve angular momentum now let us look at the geometry of this conservation let us say.

So, this is the change in the velocity which the object has to do by itself so, as to keep the angular momentum constant. Why should the angular momentum be constant, because we have changed the value of capital R by some magnitude. So, the angular momentum should conserve in this picture right.

So, let us look at the geometry of this capital R with respect to the radius of the earth. So, we can simply say that you have let say the radius is given as a and if the object is here the object's distance to the axis of rotation is capital R ok. So, if this is the center. So, if this is the center then the radius is small a. So, let us look at the geometry.

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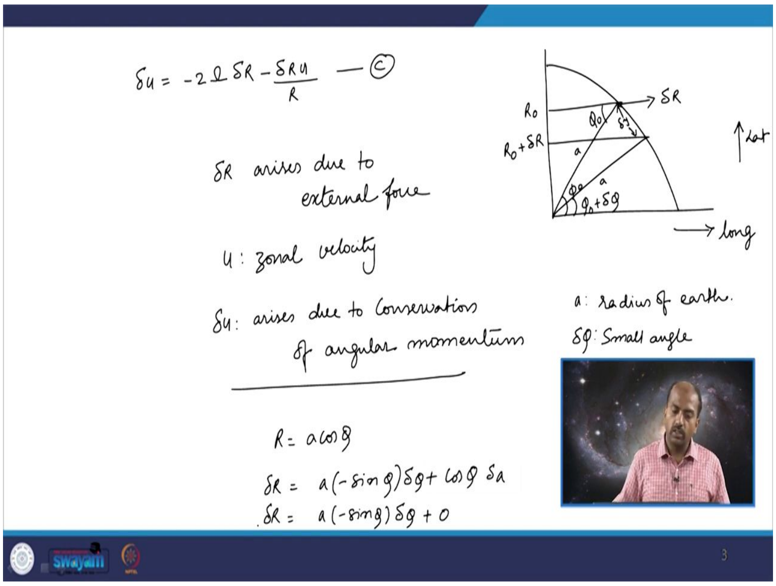

So, now what we can do is, we can consider the surface of the earth like this, let us say no this is this distance is small a. So, for reference small a is the radius of earth. Now so, in this case; let us say this is R naught, capital R itself and this becomes R naught plus delta R. So, this angle is by the way it can also be called as the latitude and this angle becomes phi naught plus delta phi and delta phi is a small angle ok.

Now, you have caused the displacement in this direction and so, this distance can be called as delta y and a is also given. So, by default this angle becomes phi naught. So, you have to understand this picture. So, this is the point where the air parcel was located and we have changed this distance by applying an external force and it has moved by distance of delta R ok.

Now, looking at this geometry what we can do is, we can go back to our expression of delta u and see how these quantities can be implemented into that. So, delta u is given as minus 2 omega delta R minus delta R times u divided by capital R. So, for reference let us say delta R is the delta R arises due to external force; external force u is the air parcels original direction of motion, zonal velocity u is the zonal velocity, delta u arises due to the conservation of angular momentum right.

Then we will see how this particular. So, this is along this direction it is the lat and along this direction is the lon so, if you displace it. Now, let us say, let us see how we can implement this geometry into this particular equation and see how we can get the velocity components also from the coordinate system from this coordinate system we can simply write R is equals to a cos phi right simply. So, delta R can be written as a times minus sin phi delta phi plus cos phi to delta a or delta R is equals to a times minus sin phi delta phi plus 0. So, this delta R can be used in this expression.

So, let us see so, this is the basic idea of the conserve how the conservation of angular momentum will lead to the development of change in the velocity along the zonal direction. So, delta R is changed in a particular direction and we are expecting we are already expecting velocity change in the delta u direction ok. So, the object has been displaced meridionally and this displacement has led to the change in the velocity in the zonal direction and this the main reason for this is the conservation of angular momentum.

So, we will continue this mathematics and we will see how we can use the this mathematical formalism to understand the Coriolis force ok.

Thank you.