

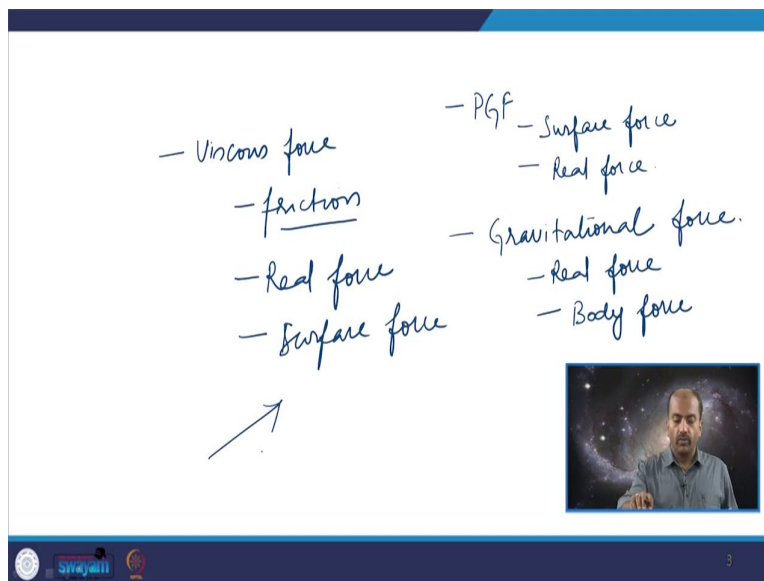
Introduction to Atmosphere and Space Science
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Lecture - 12
Forces-Viscous Force

Hello students. So, in continuation with our discussions, so we have been discussing about various types of forces which are relevant for motions of atmospheric air. So, we have seen that there two broad types of forces which are real forces and apparent forces. Real forces are the ones which you talk about when you are trying to understand motion in inertial frame of reference. And non inertial frame of reference leads to forces which are called as apparent forces.

So, we have seen what is pressure gradient force and what is gravitational force and how these two forces try to balance each other. So, we have seen the pressure gradient force.

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So, pressure gradient force was also called as PGF; this force is a surface force which means this force will not act towards the center of mass and this will not depend on the mass of the object let say and this force is a real force. So, there is no aspect of non inertial frame of reference in this type of force.

In addition, the second type of force that we have seen is called as gravitational force. So, this is also a real force and this is a body force; that means we are going to say that, the force will act towards the center of mass of the object, right.

So, in addition to these two forces we were trying to discuss what is called as viscous force or the analogous is the friction. So, friction is a similar type of force. So, friction; you talk about friction when you consider objects with the mass, objects with a surface with in touch with each other. So, there is a friction between them and this force also comes into picture when you discuss about motion, fine.

Then the other important feature about the viscous force is, viscous force is all is also a real force; that means, inertial motion. And in addition; viscous force is also a surface force. So, this force will act across the boundary of a layer.

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Handwritten notes and diagram illustrating viscous force:

- Equation: $\tau_{zx} = \mu \frac{\partial u}{\partial z}$ (where $\delta z \rightarrow 0$)
- Equation: $\tau_{zx} = \mu \frac{\partial u}{\partial z}$ (labeled as Shearing stress)
- Equation: $\mu = \frac{\tau_{zx}}{\frac{\partial u}{\partial z}}$ (labeled as Viscosity)
- Equation: $F = \mu A \frac{\partial u}{\partial z}$ (labeled as Shearing stress in the x-direction due to velocity shear in the z-direction)
- Diagram: Shows a fluid column in the z-direction with velocity u_0 in the x-direction. A force F is applied to the surface. The diagram also shows the velocity profile u in the x-direction across the z-direction.

So, we have seen a simple construct for the understanding of viscous force and if viscous force is to be discussed; then it is very important for us to construct a simple picture in which we have seen at the end of last discussion we have seen that, what is called as tau z x the shearing stress. If you consider the fluid column to be existing in the z direction and if you want a uniform velocity to be attributed to this fluid column; let us say the fluid column was constructed in this direction. Let say this is the x direction and the column of the fluid is in the z direction.

Let say so, the force was applied on the top plate, attributing a constant velocity of u_{naught} . That is how we have seen that, if you want to achieve a velocity u_{naught} by applying some amount of force on the top layer; this the amount of force is proportional to the velocity that you want to achieve; is proportional to the let us say it is proportional to the size of this let say the length of this particular column. In then additional, in addition it is also proportional to the area of the plate that you have kept on the fluid.

So, the idea is you push this plate, this plate will try to impart or transfer equal amounts of force to the immediate layer of fluid that is just beneath it, and it the this layer will try to impart the same amount of force to the layer beneath it and it goes on. So, what ideally should you expect is that, if there is no shear or if there is no dissipation of force or if there is no viscosity between these layers; I mean what are the idea of viscosity is the fundamental characteristic of the fluid which resists or which tries to hinder movement of the fluid.

So, there is an internal friction between these layers. So, these layers; they have internal friction between these layers; that means, they do not allow or they do not let the fluid parcel to be moving at the same velocity. So that means, that as this force dissipates across the z axis; so, this leads to some shear or so if you have u_{naught} as the initial velocity or the velocity that you want the topmost plate to be given. Then there is a shear in the velocity which you called as Δu and this shear of velocity arises as you travel down with a displacement of Δz .

Now, this we have included all these things by saying that, the F is equals to μ times $A \Delta u$ by Δz . So, μ is the dynamic viscosity coefficient, A is the area of the plate that we have kept on the fluid and which you are trying to push, Δu is the shear in the velocity, and Δz the displacement over which this Δu shear occurs in the fluid, ok. So, we have seen all these things and so, at the end we have to derived the shearing stress the τ_{zx} is called as the shearing stress, which is $\lim_{\Delta z \rightarrow 0}$. I will write which was written as μ times. So, I am converting this Δ 's into du by dz .

So, the idea is τ_{zx} is the shearing stress is in the x direction due to velocity shear in the z direction. So, this is a very important aspect; shearing stress τ_{zx} . So, for example, so we have considered the movement of fluid along x direction and we have taken the fluid, the column of the fluid along the z direction.

So, you have attributed velocity or the force across x axis; and the velocity shear, velocity shear is given by delta u is in the z direction. So, this is the basic idea. So, let us say this is a two dimensional picture where you have motion across x axis and shear across, velocity shear across z axis, fine.

So, you can change this and you can look at this picture in several other directions let say. Once you want to you want to have a complete picture of what is going on; then what you can probably expect is, you can have the velocity shear across the x axis and the movement along z axis. So, it is a very many combination; let us say you can bring x y z together and then you can write the shearing stress components of let us say tau x y, tau y z or tau z x things like that, then you can combine all this things.

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The slide contains the following content:

- Equation:** $\frac{P}{A \cdot t} = \text{Shearing stress}$
- Equation (a):** $\tau_{zx} + \frac{\partial}{\partial z} \left(\tau_{zx} \cdot \frac{\delta z}{2} \right)$
- Text:** "Shearing stress across the upper boundary on the fluid below."
- Diagram:** A 3D cube representing a fluid element with dimensions δx , δy , and δz . A shear stress τ_{zx} is shown acting on the top face.
- Equation (b):** τ across the lower boundary on the fluid above is $\left[\tau_{zx} - \frac{\partial}{\partial z} \left(\tau_{zx} \cdot \frac{\delta z}{2} \right) \right]$
- Diagram:** A 2D velocity profile graph showing velocity $u(z)$ on the y-axis and position z on the x-axis. The velocity is zero at the bottom and increases to u_0 at the top. A force F is applied to the top surface, and a "layer" is indicated below the graph.
- Image:** A small inset photo of a man speaking.

So, basically shearing stress arises; I mean now the basic idea is; when you consider fluid and when you are trying to apply force in some part of the fluid, where you are imparting some amount of force for some at some part in the fluid. So, it is natural that the molecules which are nearer to this plate in this particular experiment let us say; the molecules will have more mean molecular momentum in comparison to the molecules which are at rest.

So, we already taken the velocity which is a function of z. So, at the topmost layer, the velocity was taken to be u naught; and at the bottom most layer the velocity is taken to be zero; at any point the velocity is a function of z, ok. So that means, velocity is a constant change in quantity or velocity has a gradient in this direction, simple right.

Now, the basic idea is; if the velocity is changing, the momentum that particles possess at different points in this fluid column will also be different. And it is also a reality that, the molecules will be travelling in all directions. I mean there are other directions as well; but I do not draw those directions, let us say for a simplicity. So, the molecules are travelling in all the directions.

Now, what it means is that, the amount of the magnitude of momentum that is carried by the molecules which are travelling downwards is larger in compared to the amount of momentum that is carried by the particles which are travelling from the bottom to the top. So, there is a net momentum transfer which happens in this direction, ok. So, there is a net momentum transfer which happens in this direction.

So, the basic idea is let us say. So, the mean x momentum increases as we go from z is equal to zero to z is equal to l. Now the downward transported momentum per unit area, per unit time is called as a shearing stress right, it is called as a shearing stress. So, in a similar situation the mean molecular motion transport down a gradient, this process is called as diffusion. So, what is the result of this diffusion? So, molecules are travelling from one point to another point. So, it results into the process which is called as diffusion; diffusion is nothing, but idea where to keep the mean molecular density at any point in the fluid to be constant. So, if there is no diffusion, there is a possibility that had this been let us say gas; it is a possibility that there are more number of molecules at a point and there are less number of molecules at a different point anyhow.

So, basically, so you define the diffusion or let us say the downward transported momentum per unit area per unit time is called as the shearing stress. Now let us say let us construct a simple picture in which we can extend this argument to other directions as well and calculate the net shearing stress due to the force application in a direction and the resulting shear of velocity in the perpendicular direction let us say.

Let us consider a simple volume of fluid let us say, like this. So, it is again the similar idea that we have seen in the case of pressure gradient force, right. Now let us say, so this is Δx the coordinate system is x, y and z. So, this becomes Δx , this is Δy and this is Δz .

So, now in this picture which is kind of similar to what we have seen already. So, τ_{zx} will be in this direction. Now let us say shearing. So, shearing stress that is acting through the

center of the fluid element is τ_{zx} . So, stress acting on the upper boundary, let us say this boundary. So, you know that, the shearing stress at this centre is τ_{zx} . So, we already seen in the case of pressure gradient force we have taken the pressure at the center of the cubical volume is p naught and with respect to that point we have calculated the pressure on either side, which are at a distance of δz by 2.

So, in this case the shearing stress at the centre of this contour volume is τ_{zx} . And if it is the case, the shearing stress that is acting on the upper boundary on the fluid below is simply τ_{zx} plus $\frac{d\tau_{zx}}{dz}$ of τ_{zx} times δz by 2.

So, what is this, shearing stress across the upper boundary which is acting on the fluid below. So, this is shearing stress across the upper boundary on the fluid below, ok. So, this is this, ; then the shearing stress acting across the lower boundary on the fluid above. So, similarly shearing stress across the lower boundary on the fluid above is simply τ_{zx} minus $\frac{d\tau_{zx}}{dz}$ of τ_{zx} times δz by 2. So, this is let say, this is called as a, and this is called as b.

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net shearing stress is

$$\tau_{zx} + \frac{d\tau_{zx}}{dz} \cdot \frac{\delta z}{2} - \left[\tau_{zx} - \frac{d\tau_{zx}}{dz} \cdot \frac{\delta z}{2} \right]$$

net shearing stress

$$= \frac{d\tau_{zx}}{dz} \delta z \quad \frac{P}{A \cdot t} = \textcircled{P}$$

net viscous force \Rightarrow multiply dA

$$= \frac{\tau_{zx} \cdot A_{\text{area}}}{\delta z}$$

$$= \frac{d\tau_{zx}}{dz} \cdot \delta z \cdot \delta x \delta y$$

The slide also includes a diagram of a fluid element with a downward arrow on the top surface and an upward arrow on the bottom surface, and a small video inset of a man speaking.

So, now what do we do; we have shearing stress acting on the upper surface and shearing stress acting on the lower surface. So, we have both. So, the net shearing stress is τ_{zx} plus $\frac{d\tau_{zx}}{dz}$ of τ_{zx} into δz by 2 minus τ_{zx} minus $\frac{d\tau_{zx}}{dz}$ of τ_{zx} into δz by 2, which will be equal to $\frac{d\tau_{zx}}{dz}$ of τ_{zx} by δz .

So, now what is this? This is the net shearing stress. So, now, the stress is simply the rate of momentum transfer per unit area, right. Now this is in the dimensions of pressure. Now if I want the net viscous force; I will simply have to multiply by with area, right. So, which means that so; that means, that we have to multiply tau z x into area. So, which will be dou tau z x by dou z into delta z into delta x delta y, right.

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$$\text{Viscous force} = \frac{\partial u}{\partial z} \delta x \delta y \delta z$$

$$F. \text{ Per unit mass} = \frac{\partial u}{\partial z} \cdot \frac{\delta x \delta y \delta z}{\rho \cdot \delta x \delta y \delta z}$$

$$\frac{F_x}{m} = \frac{1}{\rho} \frac{\partial u}{\partial z}$$

$$\frac{F_x}{m} = \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right)$$

$$\left(\frac{F_x}{m} \right) = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial z^2}$$

μ : dynamic viscosity
 ρ : density
 u : velocity

So, we can write the viscous force as dou tau z x by dou z delta x delta y delta z, right. So, now, let us say, in per unit mass, then we can write. So, viscous force per unit mass is dou tau z x by dou z into delta x delta y delta z divided by density times volume, so fx by m.

So, shearing stress is. So, the shear is in the z direction and the force. So, shear is in the z direction, velocity shear is in the z direction, the force dissipation is in the z direction; but the original force applied is in the x direction, right. Now because of this I know what is fx; fx per unit mass is simply 1 by rho dou tau z x by dou z, right

Now, if I bring in the relation that we have already derived, which will be for the dou tau z x. So, fx by m, the viscous force per unit mass is 1 by rho times dou by dou z, tau z x was defined to be mu times dou u by dou z. Or let us say we can say that, so we write this as fx by m as mu by rho dou square u by dou z square. So, this is force per unit mass, the viscous force per unit mass in the x direction is dynamic viscosity and the density. So, mu is the dynamic viscosity and rho is the density and u is the velocity.

Now, things I mean I suppose the things are clear, it is very simple in the sense here the force is applied in the x direction, the velocity is, the velocity shear is in the z direction. So, we have calculated; we have obtained an expression for the viscous force per unit mass along the x direction.

So, if you want this much amount of velocity shear, the force per unit mass that should be applied for a fluid with a characteristic viscosity μ and density ρ should be this as simple.

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The slide contains the following handwritten content:

- On the left, a diagram shows a fluid element with a velocity vector u pointing right and a shear rate $\frac{du}{dz}$ indicated by a vertical line with arrows. A force F_x is applied to the top surface. A note says "Shear rate".
- In the center, the equation $\frac{\mu}{\rho} = \nu$ (kinematic viscosity) is written, followed by $F_{vx} = \nu \frac{\partial^2 u}{\partial z^2}$.
- On the right, a list of forces: PGF, GF, and VF with an arrow pointing left.
- Below that, a diagram shows a velocity profile with a shear rate $\frac{du}{dz}$ and a velocity $u_0 = 0$ at the bottom.
- At the bottom right, there is a small video inset of a man speaking.

Now, so the dynamic viscosity μ divided by the density is defined as ν which is kinematic viscosity is defined as kinematic viscosity, ok.

Now, so in simple terms we write F_{vx} as $\nu \frac{\partial^2 u}{\partial z^2}$. So, basically what we have learnt about viscous force. So, now, we have the pressure gradient force, the gravitational force and the viscous force. So, the viscous force comes into picture, which tries to hinder movement of the fluid in one direction. So, you want the fluid to be moving in one direction, you apply the force along the same direction; but the force that is applied is at a point and this force dissipates in this let us say in the perpendicular direction. And as a result you see some velocity shear, as a result of this velocity shear the force that you apply will not be effective enough for the bottommost layer to be moving at the same velocity that you intend; that means, this velocity will probably be zero.

So, assuming this kind of a picture we have derived several components of the viscous force. So, these viscous forces; so, this is something about the various types of inertial forces which exist in the atmosphere. So, inertial force; the basic idea inertial force is let us say if you consider a frame of reference which is at rest with respect to; let us say if you consider a few of reference which is at rest with respect to another frame of reference.

So, this frame of reference is going to be called as let us say even with the uniform velocity; this frame of reference is going to be called as inertial frame of reference. So, in inertial frame of reference any objects movement with respect to this inertial frame of reference with a velocity or with a constant velocity can be attributed to sum of all the forces. So, this forces which you take and add are generally called as the inertial forces.

So, inertial forces are important as long as the frame of reference is at rest or moving with a uniform velocity. As long as there is no acceleration involved, the frame of reference can be assumed to be inertial frame of reference; when there is a relative motion with acceleration, then the forces will be different. We will talk about those different types of forces in the subsequent lectures; because where they we will discuss something about the centrifugal force or the Coriolis force stuff like that, ok.

Thank you.