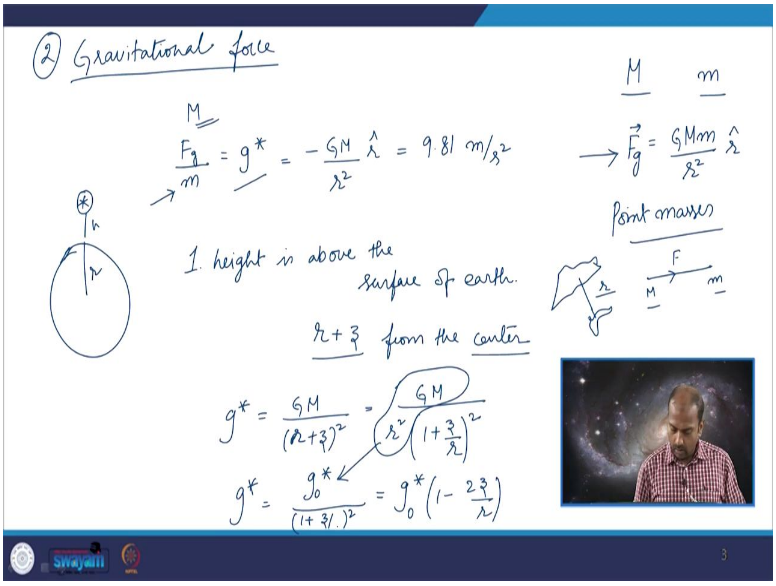
Introduction to Atmosphere and Space Science
Prof. M. V. Sunil Krishna
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 11
Forces - Gravitational Force

Hello students, so in continuation with our earlier discussion we were trying to derive mathematical expressions for various types of forces which are relevant for motion in atmosphere. So, here so we have already defined we already derived a relation for the pressure gradient force which in which we have realized that the pressure gradient force depends on the gradient of pressure, but not the pressure itself right. The second most important force which is also called as let us say this force is the Gravitational Force; gravitational force.

(Refer Slide Time: 00:57)



So, we all know that like wherever there are two masses, let us say of objects which have two which have masses of let us say capital M and small m. Let us say the amount of force that acts between them is called as the gravitational force is nothing but G M m by r square along r cap right.

So, in principle this applies two point masses I mean the basic law is applicable to Point masses. But what do is point masses, but then we say that the center of mass this. So, this mass acts between these center of masses of the two objects. So, this is the small m and this is small m this is the capital M. So, this the force acts between the center of masses of the objects ok.

So generally, so when we take the idea of r also becomes more complicated if you have a mass distribution in a irregular way. Let say then the two objects of course, they are bound by the gravitational pull of course. But the idea of let us say r becomes also different. So, you in principle what you should do is you should calculate the force between each point mass at each of the r, then you sum it then you will get a more appropriate magnitude of force.

But then what you do is you just calculate the force magnitude between the center of masses of the objects. But let us say in saying that r refers, so we can say that the r refers to the distance between the center of mass of the two objects. So, if you consider the earth to be a mass of capital M the force that is exerted per unit mass is called as a gravity, Fg is the gravity force, which is nothing but minus GM by r square across r cap.

Since you know the mass of earth, so any object on the surface will be attracted so this is G M by r square. So, if you do the mass of earth the magnitude, the number that mass of the earth object on the surface of the earth will experience a force Fg due to the earth's mass. Let us say and the force that is experienced by this object let us say a smaller mass, small m per unit mass is called as g.

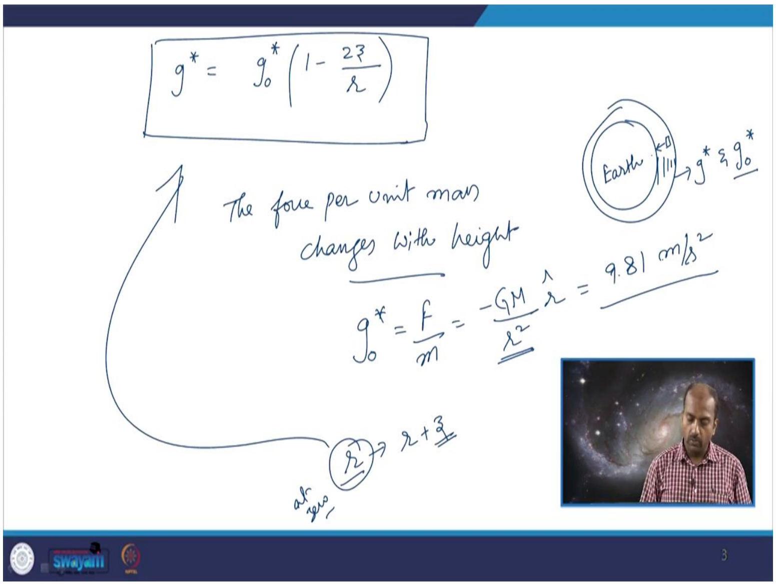
So, we have we are referring to it as g star we will realize; we will try to understand why it is called as g star. We will talk about something called as an effective gravity, apparent gravity things like that ok. So, this is this the value of this is going to be 9.81 meter per second square. So, this is the 9.81 meters per second square is the force per unit mass the acceleration due to gravity that we call as right.

So, in atmospheric science it is customary to use the height above the mean sea level, I mean so the idea of height is let us say if the air parcel is sitting here let say is here the high the idea of height is r is the radius of the earth and h is the height above the surface right.

Now, generally the what is the in atmospheric sense we use that the height above. So, use the idea of height is above the surface of the earth above the surface of earth, so this is the this is customery. So, for a typical mass that is existing above, let say so we say that if r is the radius of Earth and z is the height above the surface. So, the object is at this height from the centre. So, from the center the object is at this height or at this distance let us say.

Now, we say that in this case the acceleration due to gravity can simply be written as GM by r plus z whole square or GM pi r square into 1 plus z by a whole square. Or g star is equals to g0 star at the ground by 1 plus z by a whole square. There is nothing but g star into one minus 2 z by a right. So, g0 star is this right, so this part is called as g0 star right.

(Refer Slide Time: 05:46)



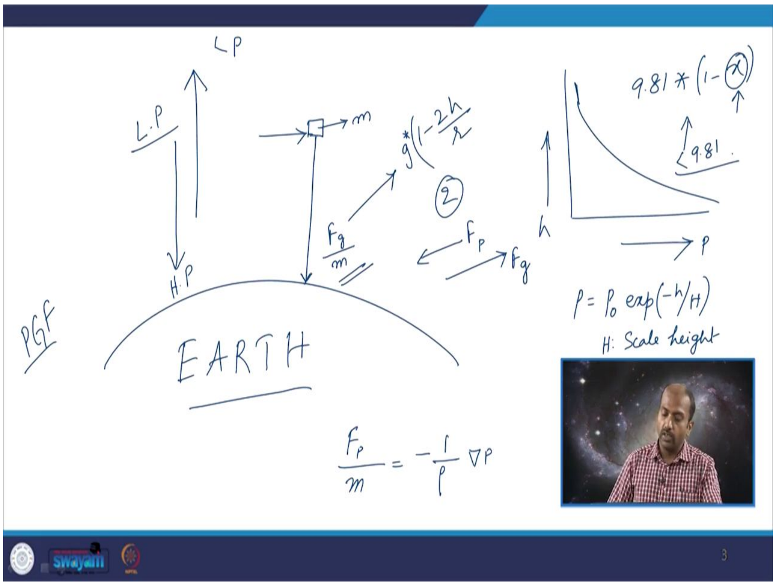
So, what we can simply write is the gravitation or the acceleration due to gravity at any other height is given as 1 minus; ok, so, here there is there is small confusion this should have been r ok. So, in that way so g zero star 2 z by r right.

So, what does it realize? we realize that the force per unit mass changes with height right. So, g0 star is the force per unit mass which we call as minus GM by r square along r cap, which was realized to be 9.81 meter per second square this is fine. But when you displace the object when you write r as let us say r plus z instead of r that means r is there r is at the 0 at 0. So, when you put an object at a height set above r, we will we will realize that it will change by this right.

Now, what does it mean? I mean if you have the earth's atmosphere as an envelope surrounding the earth. Let say in that case what most importantly you will see that the acceleration due to gravity at the height 0 and at any other height will be different. So, g star and g0 star. So, what is the highest g0 star is the highest the maximum magnitude of acceleration due to gravity will be at the surface right.

Now, that means that, so any object at any height will experience a force towards the surface. What is the driving mechanism for this force to be pulled, it is the mass of the earth that means the earth's mass creates a force by the gravitational effects that this mass will be pulled towards the earth right.

(Refer Slide Time: 08:10)



Now, very important concept that you will have to understand here is that, let us say you take the surface of the earth you consider an air parcel to be here right. That means, that this air parcel certainly has some mass right has some mass and this air parcel is drawn towards the earth due to Fg per unit mass right.

Now, in the you know earlier discussions what we have realized is that the atmospheric pressure will vary like this, that means this is what; this is the pressure and this is the height. That means, the pressure at any height is going to be p naught exponential minus h by H. Where h is small H is the height and capital H is the scale height. So, capital H is the scale height right.

Now, what is the idea; what I am trying to do is so we are we are talking about forces here I mean what type of forces are existing or like governing the movement of air in the earth's atmosphere, because earth atmosphere is a gaseous envelope we do not talk about any solid objects here right.

Now, here what you see is any air parcel if it is experiencing Fg by m, so obviously this air parcel should be drawn towards the towards the earth. I mean it should it should just follow this arrow and it should get stuck to the earth right.

Now, more importantly more importantly we know that the pressure at the surface is the highest and the pressure at the at the as you go up will decrease exponentially this is a very fast decay right. So that means, that we can take it for granted that near the surface there is a high pressure and above as you go above there is a gradient in the pressure. Such that at a certain at this height let us say at this height that we have taken there is a low pressure right.

Now, when you invoke the first force that we have discussed which is the PGF, we have discussed that pressure gradient force. Let us say we call this as we call this as Fp the pressure gradient force is called as Fp. The pressure gradient force per unit mass is found to be minus one by rho times del p.

So, now you see there is a pressure gradient in this direction right and what is what should be the movement of air. So, the moment of air should be let say in this direction. So, the air parcel should invariably go from high pressure to low pressure, I mean low pressure is not just here anyway the more you go up is just low pressure right.

So, now there is a balance of forces, so gravitational force is trying to pull this air parcel towards the high pressure and the pressure gradient force is trying to push this air parcel to the low pressure; high pressure to low pressure right. So, this balance, kind of this balance leads to the concept of buoyancy anyway. But the most important thing is you have discussed two forces and you have realized that in some situations, these two forces will act exactly opposite to each other; pressure gradient force and gravitational force right.

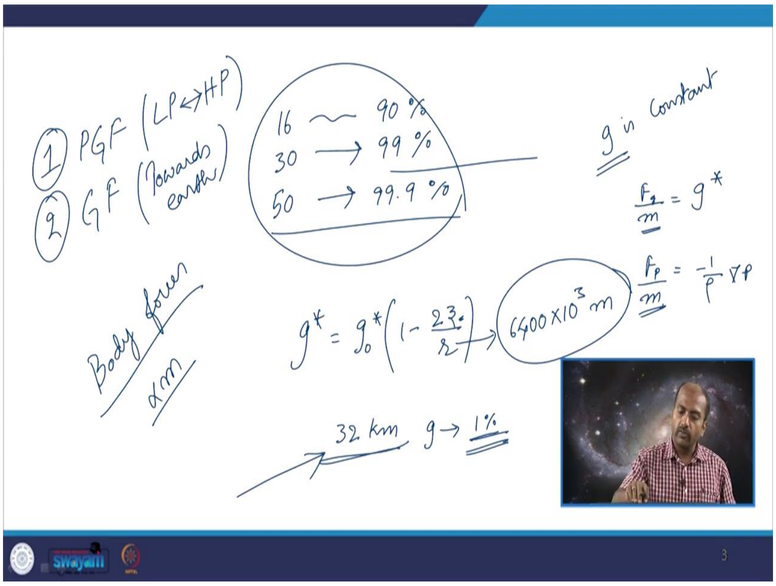
So, these two forces will try to bring the balance in which the air parcel will probably be not displaced towards the high low pressures. I mean so this is the basic idea that so the gravity is trying to keep the atmosphere intact. Otherwise if the if there is no gravity, then obviously things will just escape; the all the air that is stuck to the stuck by the gravity will just escape into the low pressure that is it and the atmosphere will be lost. So, this is the basic idea why the atmosphere is held to the planet, is because of the restoring force in terms of the gravitational pull of the earth.

So, let us say how important is it for us, I mean now we have realized that there is a gravitational pull by the earth which builds up a force and the effect of this force is that the air is drawn or air is attached to the planet right.

The second effect that is most important is that, so how important I mean gravitational force and this gravitational force changes with this formula I mean you have a small value you have. So, you have 9.81 times 1 minus something let us say let us call this as x.

So, as you go up this x will change, I mean x will change in such a way that it will increase, so x will increase. So, as a result any value that you are going to put as x will always be less than 9.81. That means, the gravitational force; the gravitational pull will be less when you go up in the atmosphere right. Now, how are the thing how are things going to change. So, we know for a very good idea that.

(Refer Slide Time: 13:01)



So, up to 16 kilometers let us say 90 percent of the atmosphere mass exists or by weight of volume and 30 kilometers nearly 99 percent of the atmosphere exists and up to 50 kilometers it is 99.9 percent of the atmosphere exists below 50 kilometers.

Now, using this formula let us say g star is equals to g0 star times 1 minus 2 z by r, where r is the radius and z is the height above the surface. We will realize that; so you substitute r as

6400 into 10 to the power of 3 meters and you will realize that up to. So, it is only let us say if you travel 32 kilometres or 32 into 10 to the power of 3 meters the g will be reduced by 1 percent. The gravitational force will reduce only by 1 percent in the first 32 kilometers.

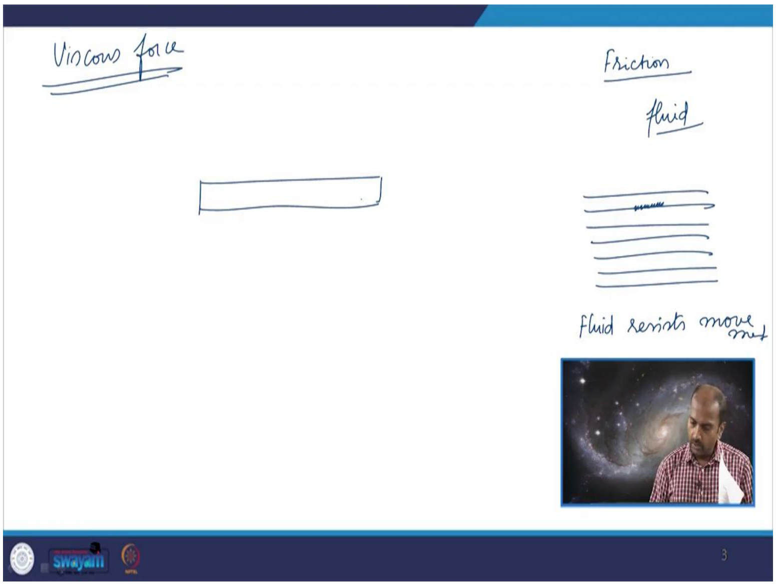
Why is so small percentage is; because of this large number in the denominator as simple as that, the height to which you have to travel should be very large in comparison to this number only then the gravitational change will be much reasonable. So, for our understanding what is important is so up to 13 32 kilometers nothing changes as such right. So, why 32 kilometre you keep your limit here, that means 99 percent of the atmosphere is existing up to a height in which gravity does not change much.

So, for our calculations or for our understanding, we can always say that g is constant. So, if you are performing calculations let us say a several of kilometers then probably you will have to account take into account way the way in which g also changes. So, you cannot take g to be a constant in those pictures.

But for our calculations in the entire atmosphere 50 kilometers up to 50 kilometers nothing changes and g can be taken as a constant for all the practical purposes ok. So, this is the basic idea. So now, we have now we have realised what is the summary so, far is that; now we have realized we have a pressure gradient force which acts between low pressure and high pressure and the other force is the gravitational force which acts towards earth right ok.

Now, this what are these two forces are basically the body forces; they are body forces right. So, they do not body forces and they depend on the mass. So, simply I mean; we know that F by m is g star. So, there is a mass dependence on the force and F by m F p by m is minus 1 by rho del p, so there is a mass dependence right. So, apart from this the most important force the other most important force is the viscous force, viscous force or viscous force.

(Refer Slide Time: 16:13)

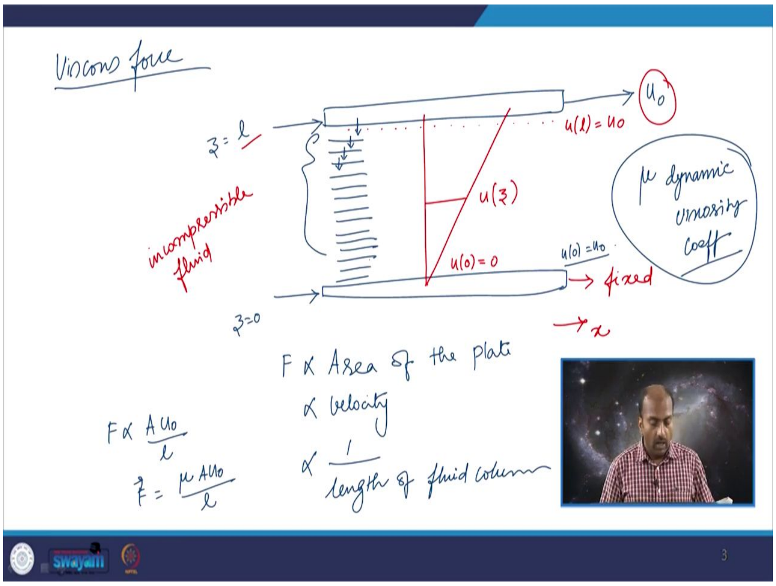


So, what is the idea of viscous force. So, viscous force is similar to friction there is no absolutely there is no difference, I mean you talk about viscous force when you have fluid.

What is the fluid? Let us say gas or liquid. So, when you take fluid you talk about viscous force. So, what is the basic idea of viscous force. So, in fluid you take the fluid to be consisting of several layers and there is an amount of frictional force between these layers of fluid, such that it will resist the moment of fluid. I mean the fluid resists by the virtue of viscous force fluid resists movement.

So, any real fluid which will demonstrate viscous force will cause it to resist movement. There are other types of fluids where the viscous force tends to be 0 at any particular condition that is not the subject of matter ok. So, viscous force causes the fluid to resist the tendency to move ok, a layer of incompressible fluid is confined between two horizontal plates separated by distance l let us say.

(Refer Slide Time: 17:37)



Let us consider for our understanding let us consider two plates force. Now, let us consider two plates what are these two are two plates which are separating a fluid ok. Let us say that this plate is at z is equals to 0 and this plate is at z is equal to l and there is a fluid in between ok.

Now, let us say this the top fluid is pushed into motion such that it moves with a velocity with a constant velocity u naught. Now, what is the basic idea? Basic idea is that there is an incompressible fluid in between. So, in between this so in between this there is an incompressible fluid between these two plates. Where are the positions of these two plates are given as z is equal to 0 and z is equals to l.

So, this fluid cannot be compressed and on the top of this fluid there is a plate which is which can be moved or which can which is moved along the let us say along the x axis along the x axis at a constant velocity u naught.

Now, how does the things change things change, let us say the layer of u is confined between two horizontal plates separated by a distance l. So, this is the distance of separation is l which is obviously indicated here right. So, the lower plate is fixed, so this is fixed this plate is fixed and the top plate is set into motion such that it achieves a velocity it achieves a constant velocity u naught.

So, viscosity forces the fluid particles that are in contact with this plate also to move with the same velocity. So, there are particles the fluid particles which are in contact with this top plate will also be set into motion and they will also be trying to reach the velocity u naught.

We will realize that the entire fluid volume that is there inside this, we will not be able to move at the same velocity. The reason is there is an internal friction in the fluid which will try to dissipate the amount of force that is trying to move the fluid in one direction at a constant velocity.

Now, you what you have done is you have moved the moved the top plate and by moving the top plate you will realize the fluid that is immediately next to the plate will also try to move at the same velocity. As you travel down into the fluid what will what you will realize is the velocity with which the fluid will move will decrease as you travel down because of the internal friction.

So, we will say that the fluid layer that is immediately below the top plate will move the same velocity and the fluid at the bottom will remain at rest. So, u as a function of the distance or the z coordinate is 0 and u at l is equals to a constant value u naught and at any point the velocity will be a function of z. So that means, that so what I have drawn is so at the top the velocity will be the same as the plate and due to the dissipation of the force that is trying to move the fluid. Because of the internal friction of the fluid the velocity at the bottom of the fluid or the bottom layer of the fluid which is in contact with the fixed plate will be 0. So, there is a differential movement of fluid as a function of z coordinate right.

So, now let us say let us say if you how much amount of force you want to apply? So, let us say if you consider the amount of force, the force that is that needs to be applied in order to move this fluid; in order to move this plate with a constant velocity u naught will depend will depend on the area of the plate. It will depend on the area of the plate; we also depend on the velocity with which you want to move the plate how much amount of velocity. I mean if you want the plate to be moving very fast you will have to apply more force that is it as simple as that and it will also proportional inversely let us say two over one over length. So, length of this fluid column.

What it means is that the amount of force that you have to apply, if there is more fluid there is more amount of frictional force which is trying to oppose the movement. So obviously, you will need more force and in this fluid the amount of force will always be will also be less. So, if you include these dependences; you will write F is proportional to A u naught by l or in order to get rid of this proportionality we will say that f is equals to mu times A u naught by l.

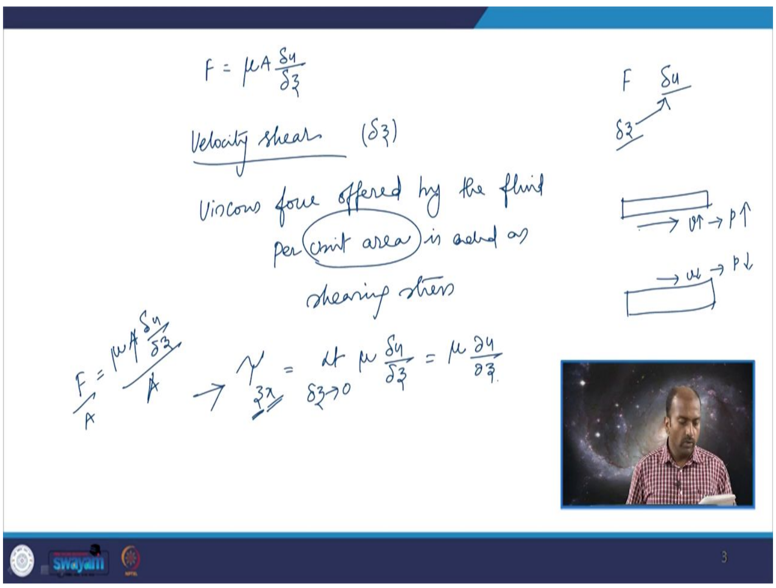
So, mu is called as the dynamic viscosity coefficient, so mu is called as a dynamic viscosity coefficient. So, this f is the amount of force that is required to be applied on the top plate for it to move at a constant velocity u naught.

So, for uniform motion of for a state of uniform motion every horizontal layer. Now let us say if you want this if you want a steady state, for uniform motion every layer let us say you separate this into let us say so many layers. So, many that I can write on this whole, let us say so for understanding. Let us say if you want the entire fluid volume to be flowing with the same velocity u naught as given and you expect that there is no dissipation of the force.

What do you expect what should be done? In that case what every fluid layer let us say this force is transferred from here to this here. So, without any offering any resistance every fluid layer must push the fluid layer that is below with the same amount of force that the plate pushes it.

So, if there is no dissipation in the in this process, the entire fluid layer with. So, you will say that u at 0 will also be u naught. So, for an equilibrium situation you will require that every fluid layer will exert equal amount of force on the layer that is sitting below it. But there is an ideal case there if there is no dissipation as such.

(Refer Slide Time: 24:29)



So, in that case if there is dissipation then the dissipation. So, due to the dissipation of the force every fluid layer that is let say which is in the size of delta z, you will realize that due to the dissipation or due to the viscous force you realize that every fluid layer the velocity changes by a magnitude delta u. If this dissipation has to be 0, you will rewrite that what is the amount of force that that goes down or that that dissipates as mu times A delta u by delta z right. So, this decreases in the velocity is called as a velocity shear.

So, due to the dissipation of force the decrease in the velocity of the fluid layers in the subsequent steps is called as the velocity shear, this is a very important term. So, the decrease in the velocity is called as a velocity shear across the layer delta z.

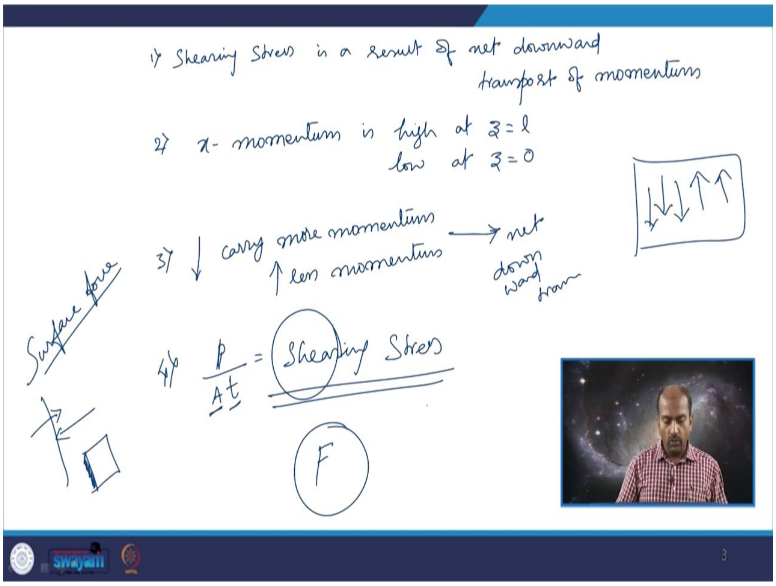
So, the well viscous force offered by the fluid per unit area is also called as the shearing stress. So, the viscous force that is offered; so what is offering the viscous force, viscous force offered what is offering the viscous force, offered by the fluid per unit area is called as shearing shearing stress which can be defined as which tau zx.

So, here the fluid column is in is in the z direction and the displacement is trying to happen along the x direction. So, this is the convention. So, which can be defined as limit delta z tends to 0 mu times delta u by delta z is equals to mu times dou u by dou z. So, this is what is this per unit area, so we got rid of the unit area.

So, the viscous force so force is m A delta u by delta z per unit area is called as a shearing stress ok. So now, that we have defined the shearing stress. So, we will try to see how the force can be written in this picture. So, in order to let say in order to understand what is going on.

So, now so shearing stress is the result of net downward movement of transport of momentum. Now, so here the particles that are close to the plate above are travelling with a higher velocity. Obviously, right they are travelling with a higher velocity. So, they will have more momentum obviously; momentum the p will be high and the particles will close to the lower plate are travelling with a small velocity. So, there is a lower momentum. So, there is a momentum transfer of course, so shearing stress is the result of net downward transport of momentum by the random molecular motions.

(Refer Slide Time: 27:46)



So, molecules are moving across this fluid. So, shearing stress; what is shearing stress force per unit area, shearing stress is a result of net downward transport of momentum. So, what we can say is that the momentum in the x direction, x momentum is high at z is equals to l and low at z is equals to 0 obviously right. So, molecules travelling with molecules which are travelling downwards at any instant will carry more momentum. So, why do why did I talk about the molecules which are travelling downwards. Since you have a fluid, molecules are travelling in all directions obviously right.

So, molecules which are travelling downwards carry more momentum in comparison to the molecules which are travelling upwards, where they have less momentum. So, so as a result of this imbalance there is a net downward transport of momentum right. So, we can safely say that there a net downward transfer of momentum.

What is the direction of this momentum transfer is the directions of this x momentum, because it is imparted by setting the plate into motion of x along the x direction. Let us say the most important point is this net downward transported momentum per unit area per unit time is the shearing stress right. So, the idea of shearing stress comes from the net transport of momentum per unit area per unit time,

Let us say for example, this is again a so this shearing stress or viscous force is again a surface force right, unlike the other forces the gravity or something. So, this is the surface force in pressure gradient force what we have discussed is that due to the random molecular motions on the wall.

So, there is a there is a net transport of momentum this momentum transport, there is the transport that is happening across the surface area per unit area per unit time was called as the was called as the force. Now, here is the is the pressure here what we have is the net transfer transported momentum per unit area per unit time is the shearing stress.

Now, we will try to derive I mean from here we will try to derive what is the relation for viscous force, mathematical relation for viscous force ok. So, the fundamentally the concept is the net transport of downward transport of momentum leads to the idea of shearing stress. Why do you call it a shearing stress? The shear is coming from the velocity; shear is dissipation of force causes the velocity to decrease as you travel downright ok. So, this is. So, in the next class we will try to understand how do we derive an expression for shearing stress and how do we derive an expression for the viscous force that acts in the fluid ok.

Thank you.