Fiber Optics

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Lecture – 27
Recap: Propagation Characteristics

Till now we have studied the propagation characteristics and salient features of optical fiber and optical wave guides. So, I think now it is time to stop for a while, and have a recap of whatever we have done till now.

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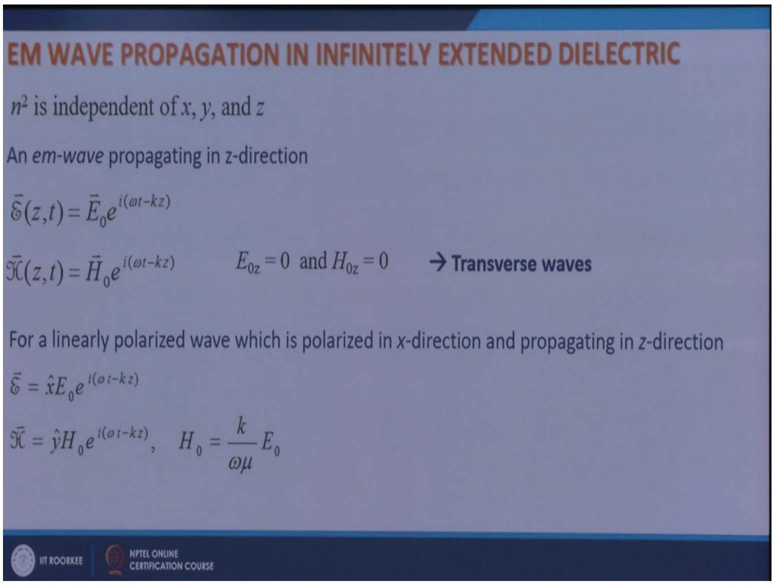

So, in order to understand propagation characteristics of optical fibers, we have first understood how an electromagnetic wave propagates in an infinite extended dielectric medium. So, infinitely extended dielectric medium n square is independent of x y and z. So, it is homogeneous medium then an em wave propagating in z direction, has an electric field and magnetic field given by e z t is equal to E 0 e to the power i omega t minus k z and h is equal to H 0 e to the power i omega t minus k z. So, this has x y and z all the components, and we had shown that for such a case E 0 z and H 0 z are 0 so, this is a transverse wave

For a linearly polarized wave which is polarized in x direction and propagating in z direction, I can write the other components as e is equal to x cap E 0 e to the power i omega t minus k z and h is equal to y cap H 0 e to the power i omega t minus k z, where

H 0 and E 0 are related by this. So, so I see two things one you should notice that this E 0 and H 0 they are constants they do not depend upon a spatial coordinates, and also that this is purely real. So, h and e are in phase. I can also write it down in terms of B, since B is equal to mu times h. So, B would be y cap B 0 e to the power i omega t minus k z. So, the magnitude amplitude of B is 1 over v times E. And so the amplitude of B is really much smaller than the amplitude of e, then we had considered propagation in wave guides.

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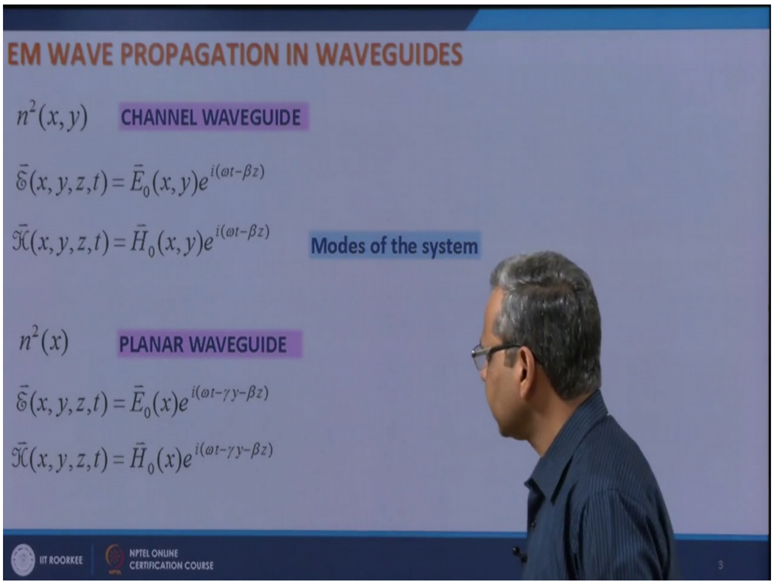

So, now if n square depends upon x and y, and the wave is propagating in z direction, then the electric and magnetic fields associated with the wave, light wave would be given by these and we notice here now these E 0 and H 0 are not constants, but they depend upon x and y. So, there is some profile, there is some electric field profile and magnetic field profile which propagates in z direction with certain propagation constant beta. So, these are the modes of the system; if instead of having n square x y I have variation only in x direction.

So, n is a function of x only then it is a planar wave guide, and in such a case I can write down the electric and magnetic fields as E 0 of x e to the power i omega t minus gamma y minus beta z, and h like this. So, here what I see that this E and H they depend only on x. So, they are the functions of x and they form the mode of the planar wave guide ok.

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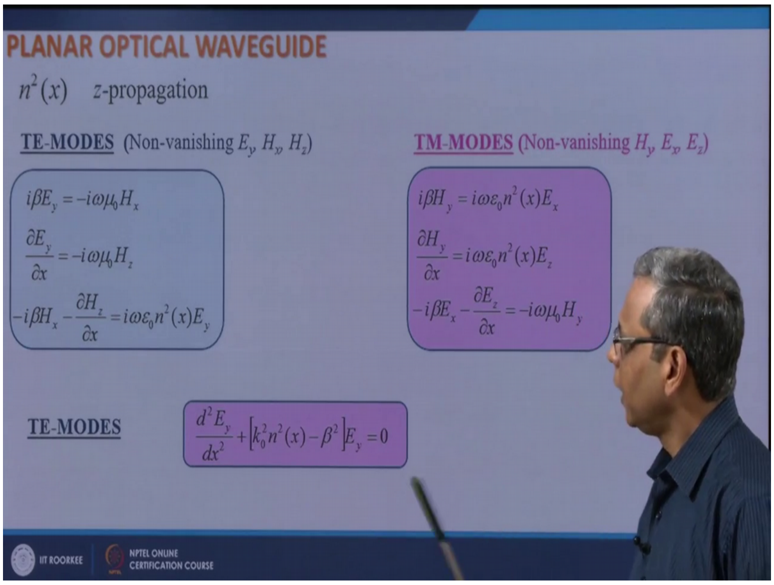

And I can choose either beta here or gamma depending upon in what direction the mode is propagating.

So, if I choose the direction of propagation is z then I can put gamma is equal to 0. Then we considered planar optical wave guide with n square of x variation and propagation in z direction then we had seen that we can categorize the modes of such a wave guide into two categories two polarizations, one is TE modes, where the non vanishing components of electric and magnetic fields are E y H x and H z, and these three components are related to each other by these three equations.

I can have another category that is TM mode TM polarization, where the non vanishing components are H y Ex and E z and these components are related to each other by these three equations. So, for TE modes for example, I can form a wave equation I can eliminate H x and H z and get an equation in E y, which tells me how E y changes with x and this is what I want to find out because I want to find out E y of x which is the mode.

So, that satisfies this equation. So, for any given n square of x can solve this equation find out the values of beta and corresponding E y. So, these are the modes.

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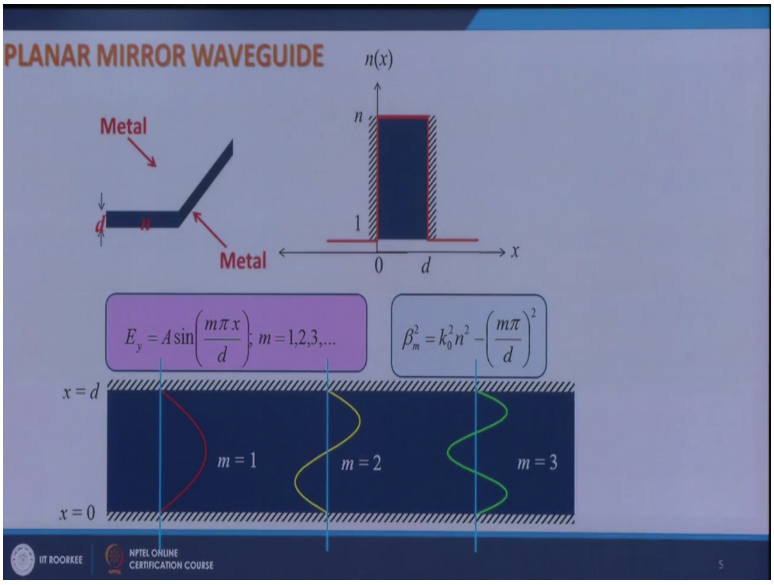

I work out first an example of planar mirror wave guide where I have very thin layer of dielectric medium of refractive index n and width d, which is coated with metal on top and bottom. So, the refractive index profile looks like this and when we worked out the modes of this then the electric field associated with this is like this, and the propagation constants are like this.

So, I have modes discreet modes of such a wave guide corresponding to different values of m. So, they look like this they look like the modes of a string modes of a (Refer Time: 06:32) string; then we consider symmetric step index planar wave guide, dielectric wave guide where the refract index profile is like this.

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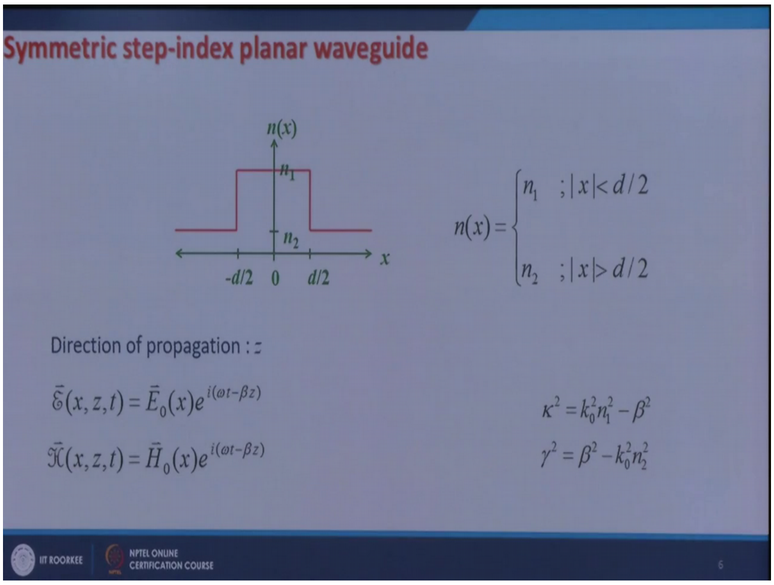

So, I have a high index layer sandwiched between two lower refractive index layers. The width of the high index layer is d which extends from x is equal to 0 to x is equal to d by 2 and 2minus d by 2.

If the direction of propagation again I considered z, then the electric and magnetic fields associated with the light would be given by this and now my task is to find out this E 0 of x and H 0 of x. Refractive index profile of this wave guide is given like this where n x is equal to n 1 for mod x less than d by 2, it is n2 for mod x greater than d by 2, and n 1 is greater than n 2. We define some kappa and gamma as kappa square is equal to k naught square n 1 square minus beta square, and gamma square is equal to beta square minus k naught square n 2 square. So, when I find out the guided modes of this TE modes and TM modes.

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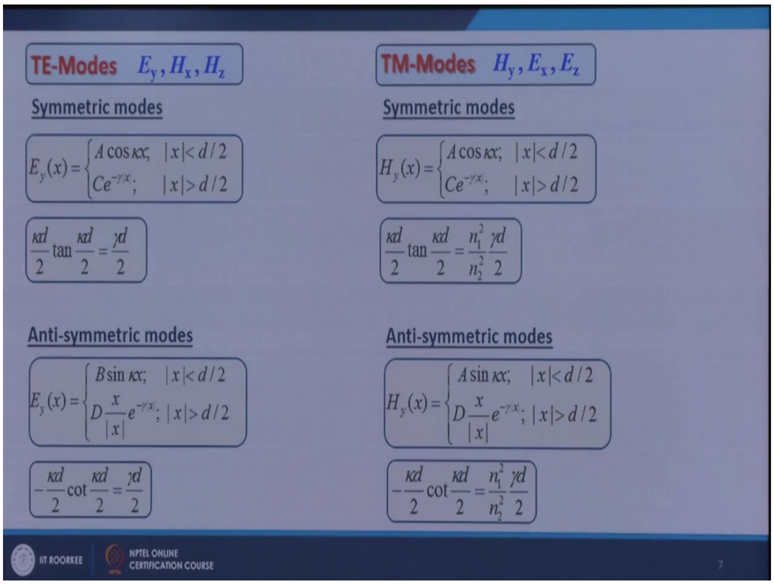

So, for TE modes again I have non vanishing components E y H x and H z, and if I solve the wave equation that the differential equation for E y, then I find that I can further categorize these modes into symmetric and anti symmetric because their structure is symmetric about x is equal to 0. For symmetric modes the field profiles are given by E y of x is equal to a cosine kappa x in the in the guiding film in the high index region, and it is exponentially decaying in the lower index surrounding. The values of beta they satisfy this equation this is the transcendental equations.

So, we solve this transcendental equation get the values of beta which are the solutions of this equation, and for those values of beta I find out E y. So, in this way I get the complete solution the mode field profiles as well as the propagation constants. Similarly for anti symmetric modes this is the field solution and this is the transcendental equation. The same thing I can do for TM modes for which the non vanishing components are H y Ex and E z, I have symmetric modes and anti symmetric modes and these are the transcendental equations. Now only difference that you would see in between this and this is a factor of n 1 square over n 2 square, this comes from the boundary conditions.

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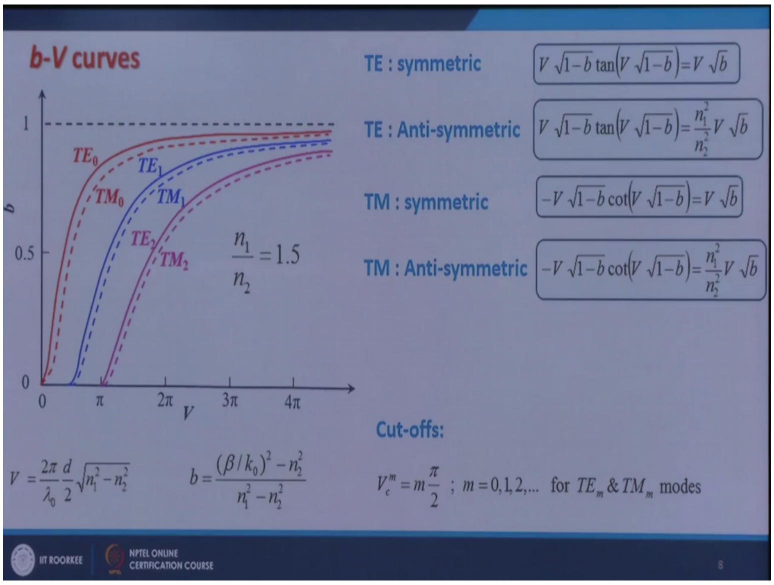

So, in a wave guide a very important diagram is bV diagram, where b is the normalized propagation constant defined like this, and V is the normalized frequency defined like this which contains all the information which you require about the wave guide and the wave length.

So, I can now translate the transcendental equations corresponding to TE and TM symmetric and anti symmetric modes into these normalized parameters like this, and then I can solve these equations to generate these curves. The idea of and advantage of generating these curves is that these curves are universal. So, if I talk about TE modes, then this is the equation for symmetric and this is the equation for anti symmetric. There should not be this is the equation for TE is symmetric mode and this is the equation for TE anti symmetric mode.

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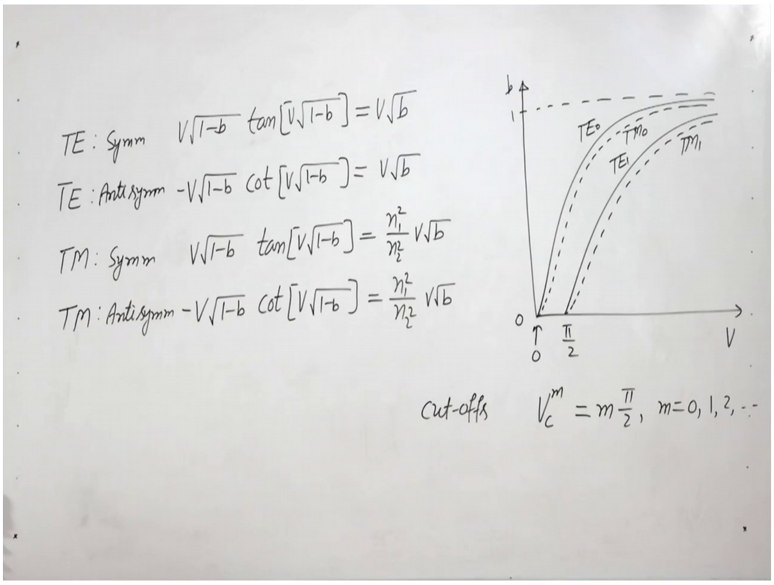

So, let me make this correction here, TE symmetric is V square root of 1 minus b tan V square root of 1 minus b is equal to V square root of b, and TE anti symmetric is minus b square root of 1 minus b cot V square root of 1 minus b, is equal to V q root of b TM symmetric is V square root of 1 minus b tan V square root of 1 minus b is equal to n 1 square over n 2 square TM square root of b and TM anti symmetric is equal to minus V square root off 1 minus b cot V square root of 1 minus b is equal to n 1 square over n 2 square, V square root of b. So, I can have the transcendental equations in normalized parameters, and by solving these equations for different values of V, I can generate v b curves. So, those vb would look like this is V and this is b; for guided modes b would vary from 0 to1. So, this would be TE 0 mode TE 0 this would be TM 0 mode.

This would be TE one mode; this would be TM one mode and so on. So, I can generate these universal curves bv diagrams, and if I now know the wave guide and the wave length then I can just calculate the value of V and go to these curves find out the value of V and find out the value of propagation constants. The cut offs of these modes are here. So, this is 0 this is pi by 2 and the cut offs are given by cut offs Vc m for TE TM both is equal to m pi by 2, where m is equal to 0, 1, 2 and so on. So, how the modal fields look like.

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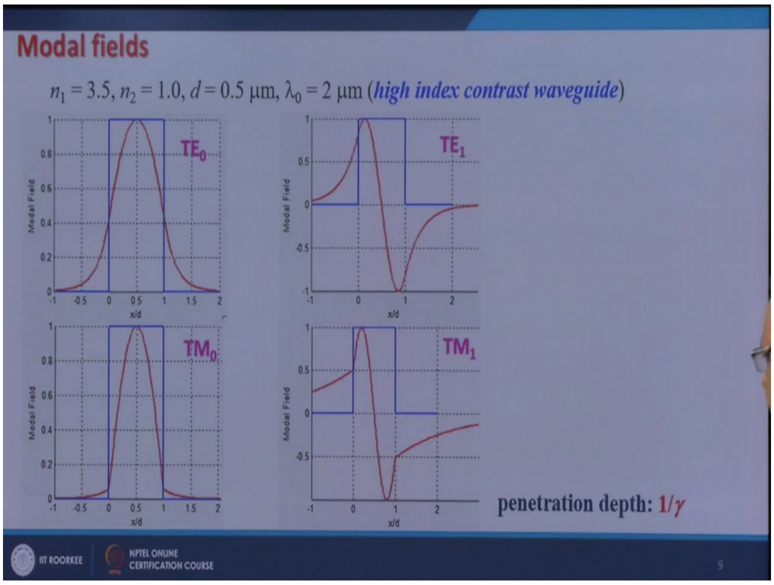

So, if I take high index contrast wave guide, I have taken high index contrast wave guide here in order to show the difference between TE TM modes for n 1 is equal to this n 2 is equal to this, d half a micron lambda naught 2 micro meter. So, this is how the TE 0 mode would look like and this is how the TM 0 mode would look like. You can see the discontinuity in at in the slope at the boundaries. This is how TE one mode look like this is how TM one mode would look like I also define penetration depth, the extent up to which the field would penetrate into n 2 region. So, it is given by 1 over gamma. We have seen that these modes are nothing, but the superposition of two plane waves one going in exact plane like this and one going in exact plane like this.

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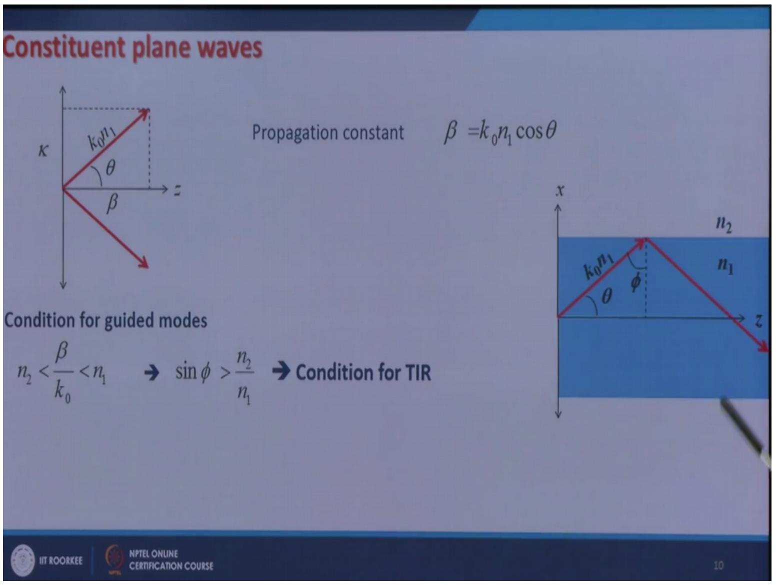

So, upward and downward, making angles plus minus theta from z axis. So, what we see that there is one plane wave going like this, another plane wave going like this. So, if I take the components in x direction and in z direction, then in x direction I get two counter propagating waves which give you standing wave pattern. And these modes these modes are nothing, but the standing wave patterns. So, the field stands in x direction does not propagate in x direction, but this is standing wave pattern propagates in z direction with propagation constant beta, which can be given by k naught n 1 cos theta.

And you had also seen that the condition for guided mode can be translated to the condition for total internal reflection. So, this is also we had seen then what is the power associated with the mode? To calculate the power associated with the mode we found out the pointing vector which gives us the intensity, and since the electric and magnetic field associated with light or fluctuating rapidly, k at a frequency of about 10 to the power 14 or 10 to the power 15 hertz. So, we need to take the average time average of these.

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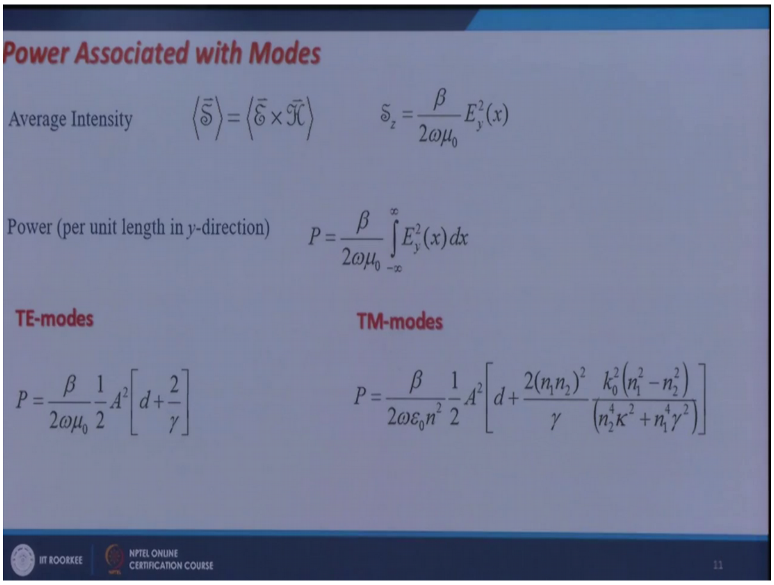

So, when a we do this, then we find that only z component of S the pointing vector survives, and it is equal to beta over 2 omega u naught E y square of x, this is for TE modes. So, from here I can find out the power per unit length in y direction, because y direction is infinitely extended for planar wave guide. So, the power per unit length in y direction would be given by beta over 2 omega mu naught integral minus infinity to plus infinity E y square of x dx. So, when I calculate this for symmetric planar wave guide, then for TE modes the power is given by this and for TM modes the power would be given by this.

These powers are in watts per meter then we had extended the analysis to asymmetric planar wave guide, where I have asymmetry here this is the sub straight this is the cover. So, the sub straight and cover refractive index are different. So, ns here is greater than c, and what we did we wrote down the fields in all the three regions. So, these are the fields electric fields corresponding to modes in all the three regions, and the beta the propagation constant will satisfy this transcendental equation. For TM modes the fields would be given like this and the transcendental equation becomes this.

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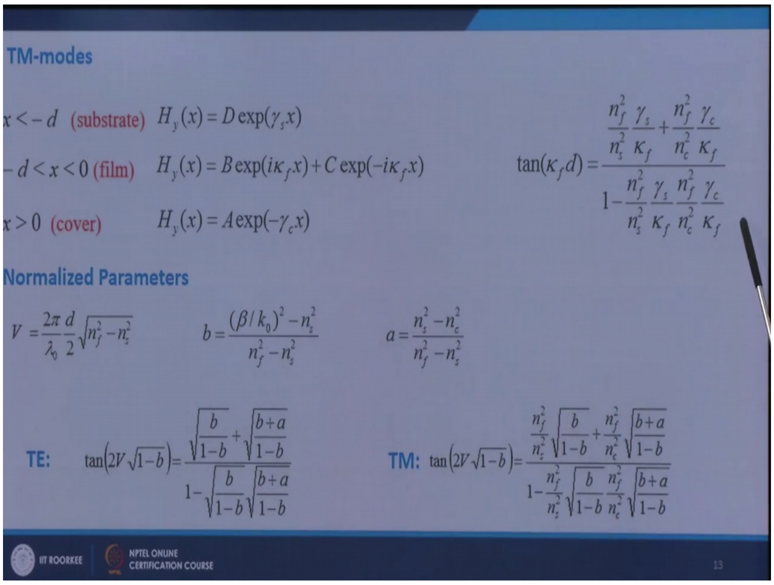

We had also defined normalized parameters for asymmetric planar wave guide. So, V is defined with respect to ns. So, here you see nf square minus ns square not with nc, because it is ns that will govern the guiding condition. B is also defined with respect to ns. So, beta square over k naught square minus ns square divided by nf square minus ns square, here we have another parameter which is asymmetry parameter which tells you how different ns is with nc. In normalized parameters I can have transcendental equation for TE modes and for TM modes like this, if I draw the bv curves here now then I see that the bv curves for asymmetric wave guides they go like this ok.

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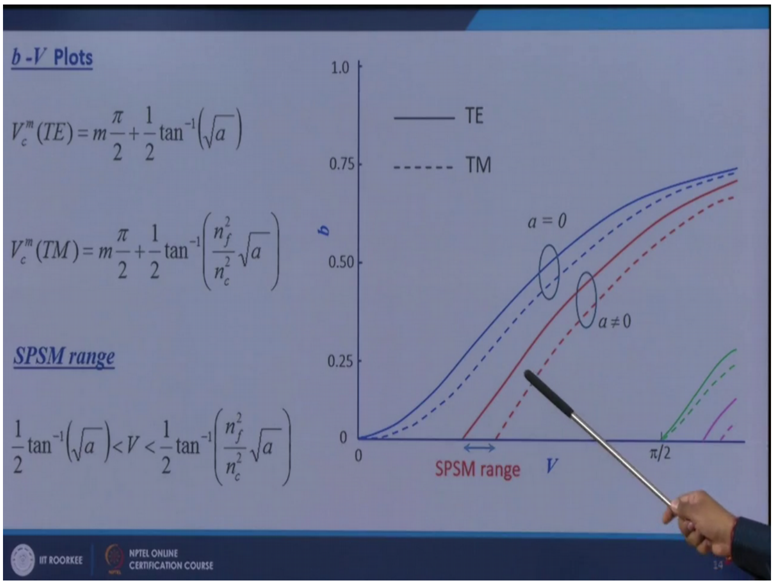

What I notice that they do not have 0 cut off. TE 0 mode also does not have 0 cut off. If you look at the cut offs if you look at the cut offs then you can see that for symmetric planar wave guide the cut offs were m pi by 2, but now these cut offs would be shifted by this amount half ten inverse of square root of a because of this asymmetry parameter. If a is equal to 0 then it becomes the symmetric planar wave guide. So, these I also notice that the cut offs of TE and TM modes are now different. In case of symmetric planar wave guide the cut offs of TE and TM both the modes were the same m pi by 2.

But now the TE and TM modes have different cut offs. So, this is TE 0 mode this is TM 0 mode TE 0 mode have cut off here, TM 0 mode has cut off here. Now if I have v which is somewhere here then I will guide only TE 0 mode and TM 0 mode is cut off. So, in the range of V from which goes from half tan inverse square root a to half tan inverse nf square over nc square is square root of a, I have single polarizations single mode wave guide, and this range is known as sp sm range. This is how the modal fields would look like, this is for TE 0 mode this is TM 0 mode this is TE 1, TM 1, TE 2, TM 2.

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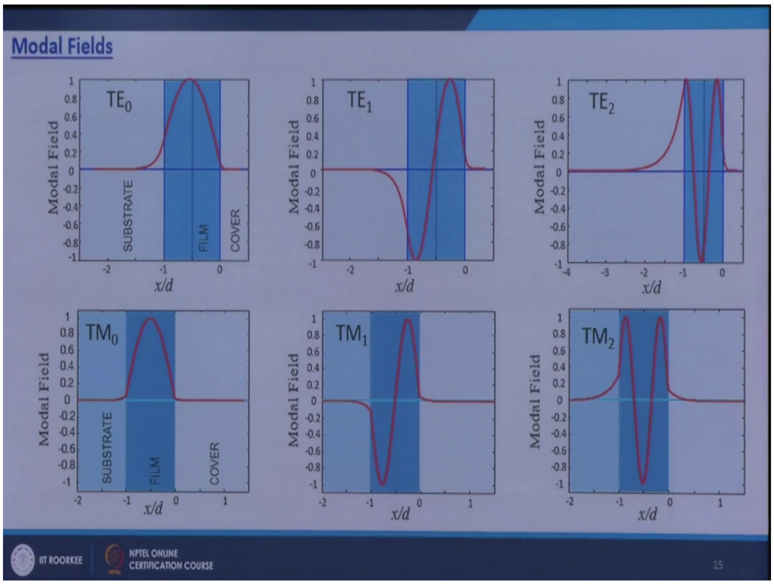

We can notice that in the sub straight the field extends more because index contrast with the film is less while in the cover, the field very quickly decreases goes down to 0, because the index contrast is high. Then we worked out the modes of a step index optical fiber. So, in a step index optical fiber I have a core and cladding I have assumed to be extended to infinity.

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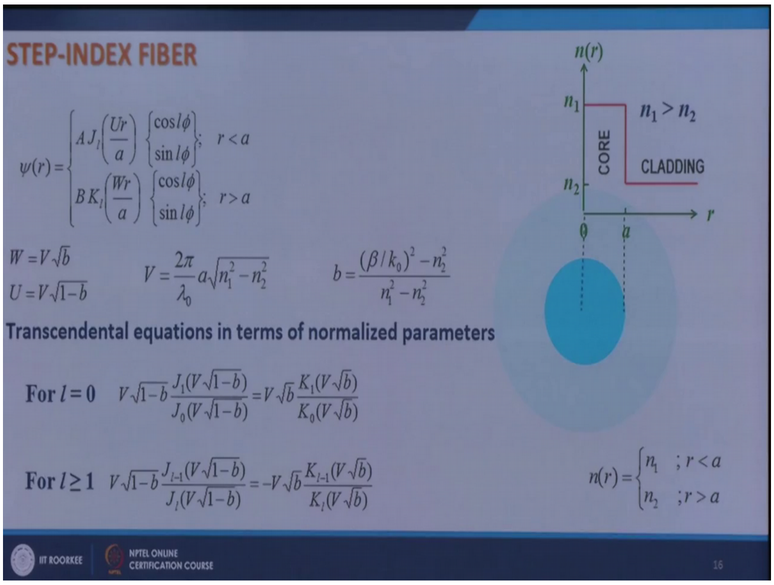

So, here I have the modes which are function of r and phi. So, this is the r solution AJl Ur over a, and phi solution can is cosine l phi and sin l phi this is in the core, in the cladding the r solution is kl w r over a, and phi solution is again cosine l phi and sin l phi. Then we again we had defined TE normalized frequency and normalized propagation constant, and worked out the transcendental equations for different modes corresponding to l is equal to 0 and l greater than or equal to 1, because now I have phi directions. So, there would be discreet modes in phi direction also. So, which are governed by the values of l and the l assumes only integral values 0 1 2 3. So, for those I can then find out what are the r solutions, and the modes propagation constant would then be given by these two transcendental equations.

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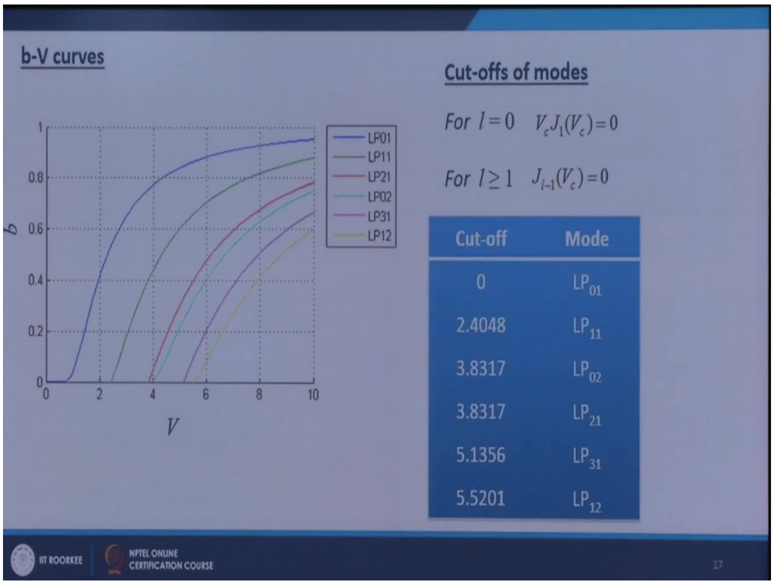

So, if I do that. So, for different values of b different values of v I can find out the propagation constants and I can plot them. So, this is LP 0 1 mode, this is LP 11 mode this is LP 21, 0 2, 3 1, 1 2 and so on. The cut offs of the modes are not straight forward they are the solutions of these equations they are given by this. So, which a where j are the Bessel functions. So, by solving these by finding out the zeros of these Bessel functions I can find out the cut offs.

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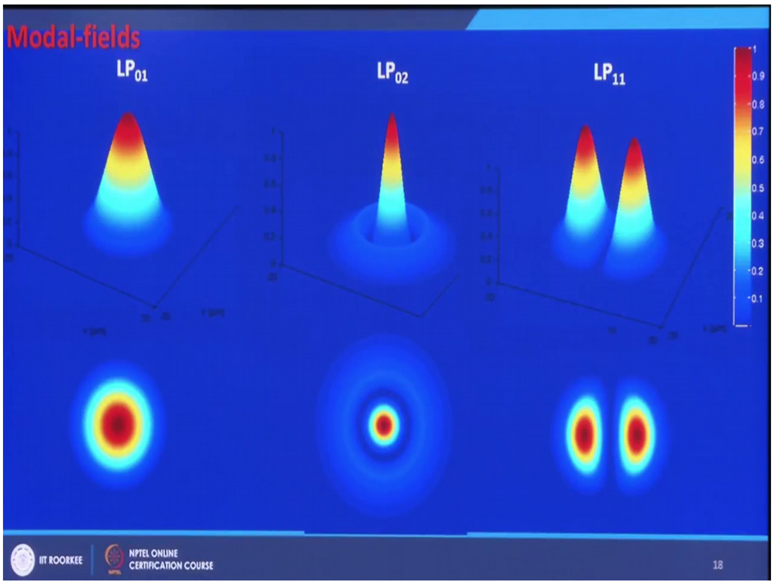

So, these are the cut offs of various modes first few modes. This is how the modal fields would look like this is LP 0 1 mode LP 0 2 mode LP 1 1 mode. So, these are 3 d plots and these are the plots.

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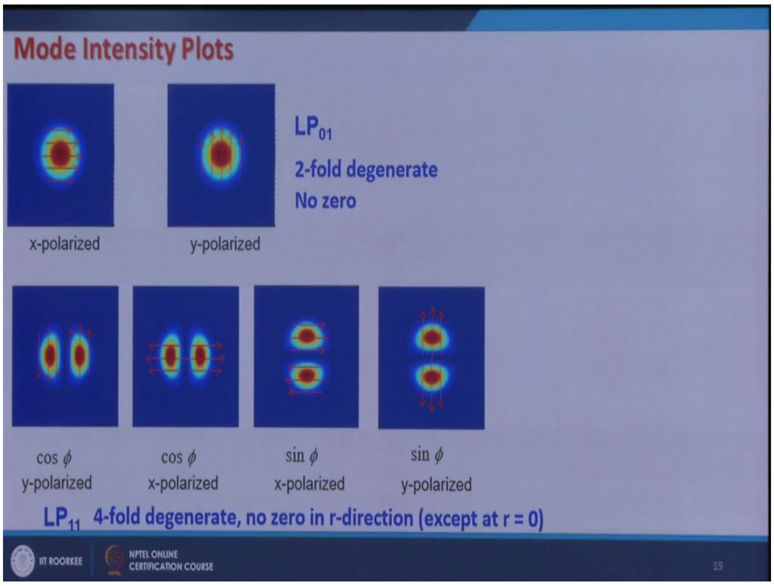

When you see them from the top these are the contouring intensity plots. What I see that l is equal to 0 modes are 2foldy generate, because phi solution is cosine l phi and sin l phi. So, f l is equal to 0 then there is no phi dependence there the degeneracy comes only from polarization, while L naught equal to 0 mode are 4r foldy generate 2 foldy generacy comes from phi and two foldy generacy comes from the polarization. Then we had seen the single mode fiber and a single mode fiber we had defined three very important parameters, one is cut off wavelength which is given by 2 pi over 2.4048 times a square root of n 1 square minus n 2 square.

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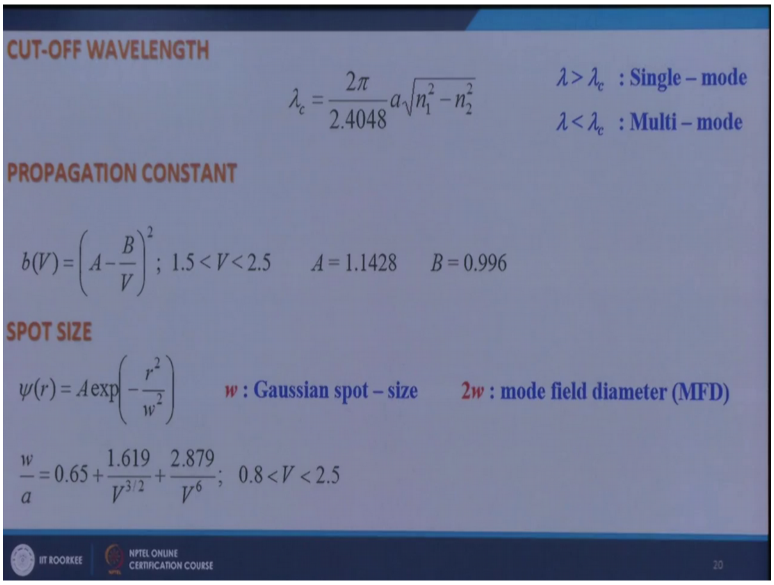

And what is the cut off wavelength of the fiber? It is the wavelength corresponding to the cutoff of LP 11 modes.

So, if lambda is greater than lambda c then the fiber is single modded, and if lambda is smaller than it is multi modded. The mode propagation constants of the fiber can be fitted with an empirical relation, if V lies between 1.5 and 2.5, and this empirical relation is given by this with these values of a and b. The spot size the mo the modal field profile of a single mode fiber if you look back to this, it resembles a Gaussian and it can be very well fitted with Gaussian. So, we can fit a Gaussian to this profile then we define this spot size as Gaussian spot size where the field intensity drops down to 1 over e square of it is value at the center.

So, w is Gaussian spot size and 2 w is mode field diameter, we can also fit an empirical relationship between excuse me w over a and v which is given by this. So, if I have a fiber I just calculate the value of v for a given wavelength, and I can get the value of w over a from this relationship. Then another important parameter for a single mode fiber is what is it is bend loss.

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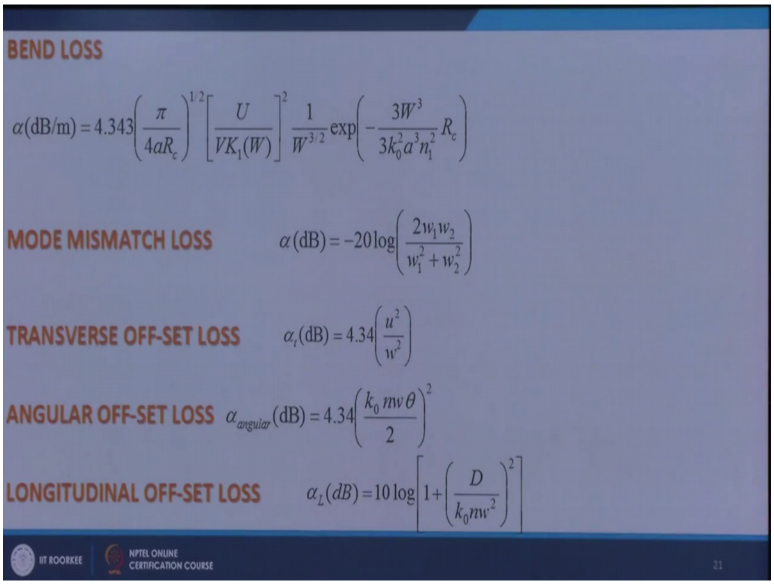

So, bend loss is given by this, then their instances there are instances where we need to join two fibers together, and when we join two fibers together then there would be losses.

If two fibers are not identical, then even if there are no misalignment while joining the fibers there would be a loss which is known as mode mismatch loss, which is purely due to different spot sizes of the modes of two fibers. If you join two identical fibers and then there can be three kind of misalignments transverse, angular and longitudinal, and these are the misalignment losses or splice losses due to different kind of misalignments. So, this we had seen. Then in the end we had seen the wave guide dispersion.

So, ultimately we need to use this fiber in telecom system, and when we send pulses through this optical fiber then dispersion happens we had seen that in a fiber there is inter modal dispersion and material dispersion, when you use single mode fiber inter modal dispersion is diminished.

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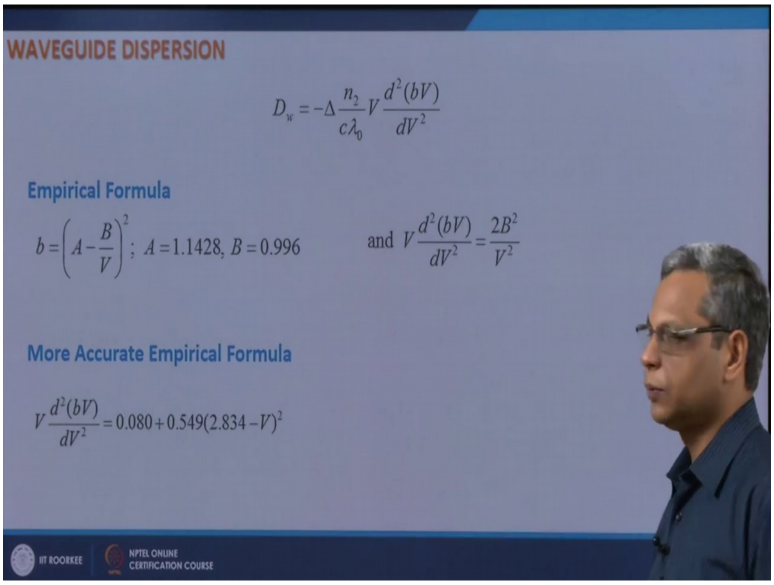

So, we get rid off inter modal dispersion, but then there is another dispersion which is wave guide dispersion which comes out because of the dependence of mode propagation constant of fiber mode on wavelength ok.

So, the wave guide dispersion coefficient is given by this, and we can use empirical formula between b and V to calculate this vector, or we can use a more accurate empirical formula given by marquees for V times d 2 b V over dV square. So, in this way we had understood the salient features of optical fiber, how light is guided into optical fiber, what are the important parameters of a single modded fiber.

So, with all this background about optical fibers and the propagation characteristics of optical fiber, now we are ready to use this fiber in a system or to use this fiber to make components and devices, which we will do in the subsequent lectures.

Thank you.