Fiber Optics
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Lecture – 25
Waveguide Dispersion- I

In this lecture we would look into another important characteristic of a single mode optical fiber and that is Waveguide Dispersion. We have seen that in a multi mode fiber if it is a step index multi mode fiber, then the data rate is limited by what is known as inter modal dispersion, which is due to different modes carrying light into the fiber and these different modes to have a with different velocities, they have different propagation constants that gives raise to what we call as inter modal dispersion. We can minimize inter modal dispersion in a multi mode fiber by having a graded index profile, and then in such a fiber we have inter modal dispersion which is very small.

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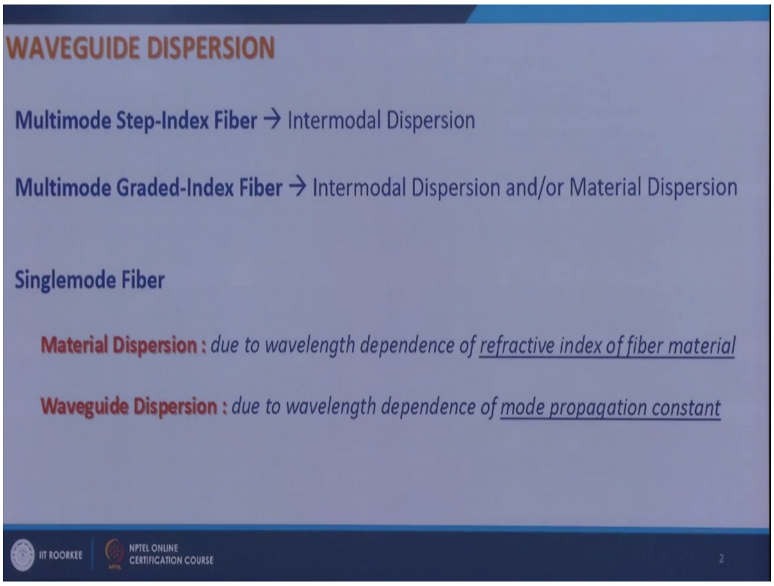

And then another kind of dispersion comes into picture which is known as material dispersion, which is nothing, but the wavelength dependence of the refractive index of the material of the fiber. So, in a graded index multi mode fiber we have seen that the material dispersion and inter modal dispersion can become comparable. So, we will have to take into account material dispersion as well as inter modal dispersion, while in a multi mode step index fiber, inter modal dispersion is so, large that we ignore material dispersion.

Although by using a graded index fiber, we can minimize inter modal dispersion, but we cannot completely get rid of it. So, the obvious way to get rid of inter modal dispersion is to use a single mode fiber. Then in a single mode fiber material dispersion would always be there because the refractive index of the material would always depend upon the wavelength of light and we are using light source which has which always has finite spectral width. So, all the wavelength and components will contribute; then apart from this material dispersion a single mode fiber also exhibits another kind of dispersion which is known as wave guide dispersion what is this wave guide dispersion? Because we have finite line width of the source, so, different wavelength components of the source have different propagation constants of modes.

So, in a way each wavelength component has its own mode, and that mode propagates with different velocity, it has different propagation constant, and that give raise to what is known as wave guide dispersion.

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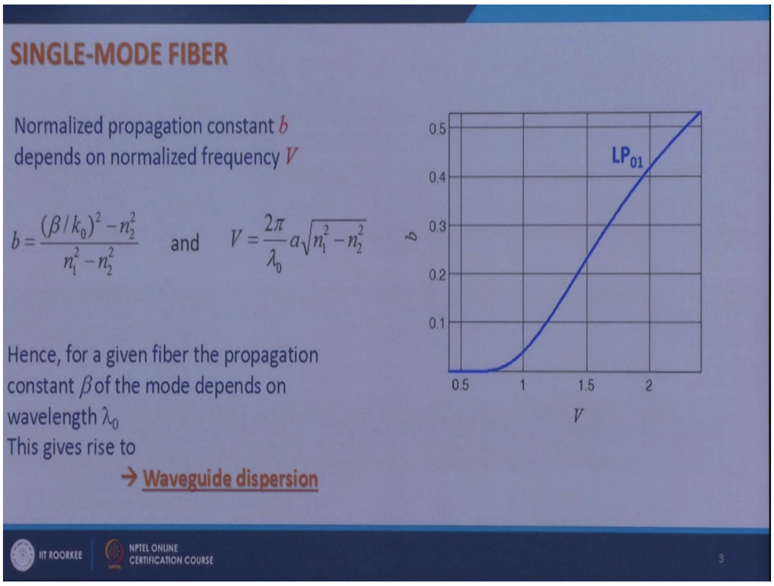

We call it wave guide dispersion because it is purely due to purely due to the wave guidance in a fiber, because wave guidance in a fiber is different for different wavelengths.

So, if I look at single mode fiber then it supports LP 01 mode and if I plot its b V curve b V diagram, then it goes like this and I know b is related to propagation constant like this, and V is related to the wavelength like this. So, propagation constant depends upon lambda and this relationship between the propagation constant and lambda can be obtained from the relation between b and V. So, for a given fiber the propagation constant beta of the mode depends upon lambda naught, and this gives raise to what is known as wave guide dispersion.

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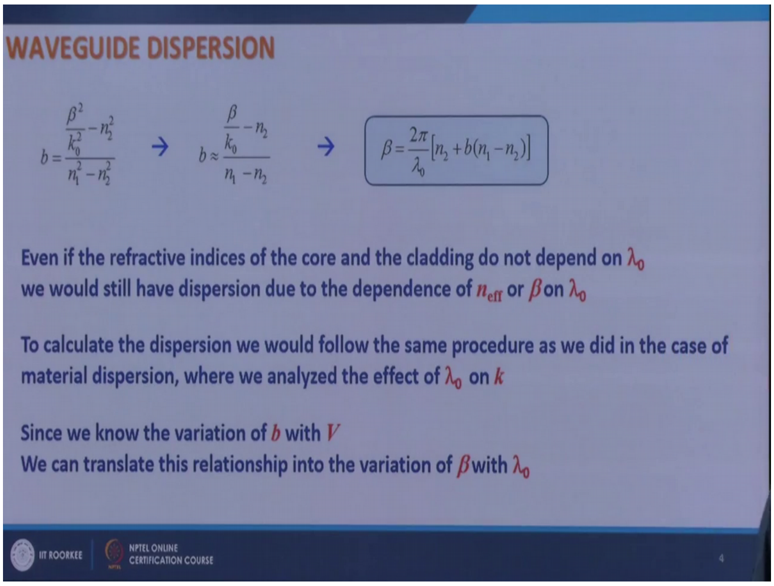

So, what I can do now? I can obtain the propagation constant of the mode in terms of fiber parameter and the normalized prorogation constant B. How to find out the broadening of pulse due to this? So, the idea is exactly the same the procedure is exactly the same as we had done in the case of material dispersion, in case of material dispersion we consider optical waves propagating in infinitely extended medium that is in bulk medium.

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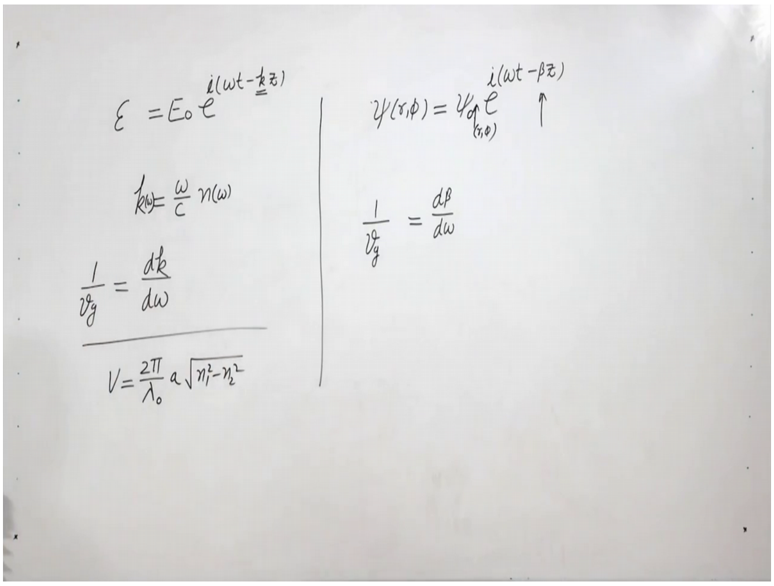

So, we considered plane waves, and what we had done if you remember that in a bulk medium the plane waves go like this E is equal to E 0 which is a constant, e to the power i omega t minus k z. Where k is the propagation constant E 0 is constant this is a plane wave and k can be written as omega by c times n, and since n is since n depends on frequency or wavelength k also depend upon frequency. So, this is a function of omega and so, this is a function of omega and I know that if there is a group of waves if there is a pulse, then the group velocity can be given by one over vg is equal to dk over d omega, and that is how we obtain the group velocity and then transient time and subsequently the material dispersion.

in case of optical fiber what I have I do not have plane waves, but I have modes, and these modes propagate as psi r phi is equal to let me put psi 0 e to the power i omega t minus beta z, and this is a function of r and phi both this is this is not a constant this is a function of r and phi. Then instead of having k have beta which is a propagation constant and if I want to now find out the pulse broadening then again I will have to find out the group velocity and in this case the group velocity will be given by d beta over d omega, and this beta is a function of lambda naught or the frequency omega. So, the procedure would exactly be the same. So, that is why I have written beta in terms of this and since b is b depends upon v which subsequently is a function of lambda or omega.

So, that is how can now find out d beta over d omega. So, if I look at this, this I have obtained for a weekly guiding fiber because if you look into it b is equal to beta square over k naught square minus n 2 square divided by n 1 square minus n 2 square. So, this I can approximate by beta over k naught minus n 2 over n 1 minus n 2 for a weekly guiding fiber, where n 1 is very close to n 2. So, if I look at this expression then eve if the refractive indices of the materials of the core and cladding do not depend upon lambda, we would still have dispersion due to dependence of beta over lambda or dependence of any effective over lambda. So, to calculate the dispersion we will follow exactly the same procedure as we had done for material dispersion and then in this way I will find out the group velocity by d beta over d omega and subsequently the pulse broadening and wave guide dispersion coefficient.

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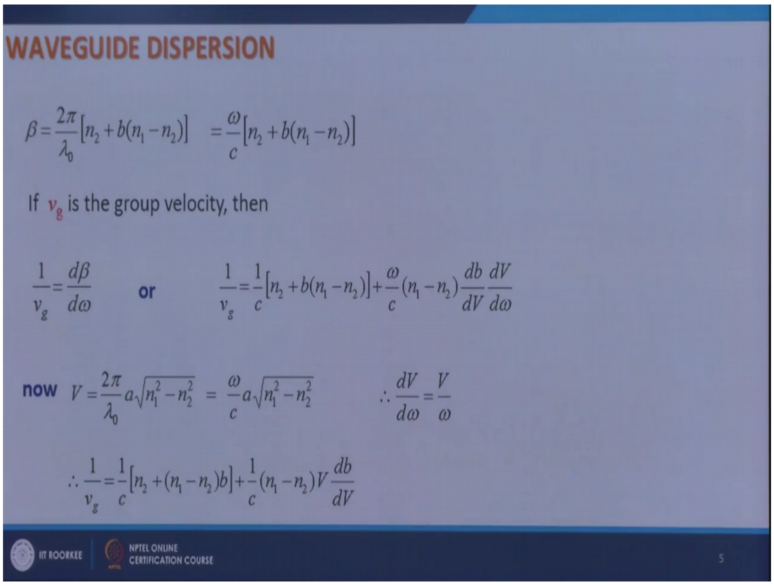

So, I again write down the expression for beta which is 2 pi over lambda naught which is nothing, but k naught, n 2 plus b times n 1 minus n 2 and this I can write as omega by c times n 2 plus b times n 1 minus n 2, and if vg is the group velocity then 1 over Vg is equal to d beta over d omega. So, I do d beta over d omega from here. So, I will get 1 over c n 2 plus b n 1 minus n 2 plus omega by c then the differential of this would be 0, and if I take the differential of this then it would be omega by c n 1 minus n 2 db over bV times bV over d omega.

So, please pay attention here that I have ignored here the material dispersion, I have assumed that n 1 and n 2 do not depend upon omega in order to isolate the effect of wave guide dispersion. So, in order to purely find out the effect of wave guide dispersion or to purely find out the wave guide dispersion, I have assumed n 1 and n 2 to be constant with respect to omega. So, what is bV over d omega? I have V is equal to 2 pi over lambda naught times a times n 1 square minus n 2 square which is nothing, but omega by c times a times square root of n 1 square minus n 2 square. So, bV over d omega would simply be V over omega. So, I put it there then will get 1 over Vg is equal to 1 over C n 2 plus n 1 minus n 2 times b plus 1 over c n 1 minus n 2 times v times db over dV.

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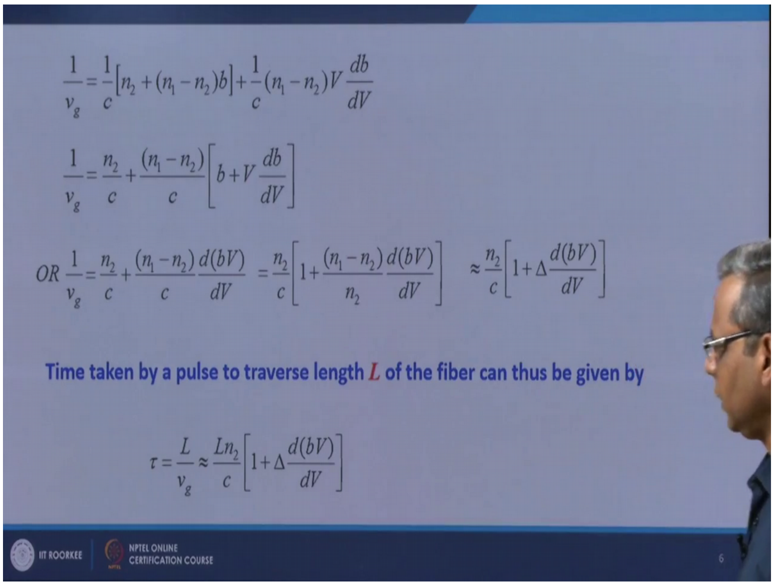

So, now I simply rearrange the terms here. So, I take this n 2 by c here and n 1 minus n 2 by c here which I take common from this term also. So, here I have in the square brackets b which is coming from here as V times db over dV. So, this I can simply write as n 2 by c plus n 1 minus n 2 by c, and this I can write as d of bV over bV right. Now I take n 2 by c common and then I can write it as n 2 by c times one plus n 1 minus n 2 over n 2, d bV over bV and this I can approximate by delta for a weekly guiding fiber delta is n 1 minus n 2 over n 1 or n 1 minus n 2 over n 2 because it is weekly guiding fiber. So, I can approximate it by this.

So, 1 over Vg becomes this much once I have vg then I can find out what would be the time taken by pulse to traverse length L of the fiber, and that would be given by L by Vg which is equal to L n 2 by c, 1 plus delta d bV over bV now if I have a source of spectral width delta lambda naught.

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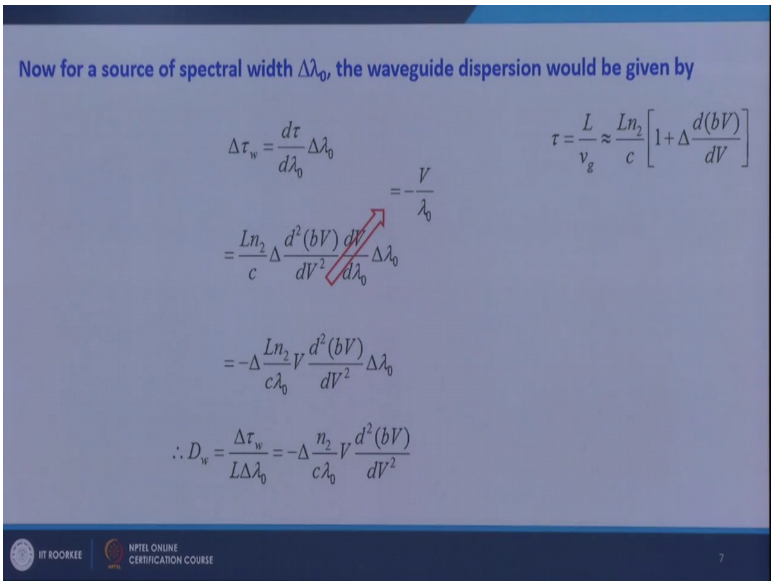

Then the broadening would be given by due to wave guide dispersion delta tau w is equal to d tau over d lambda naught, times delta lambda naught and tau is this much from the previous slide. So, if I just take d tau over d lambda naught from here. So, what I will get L n 2 by c, times delta times d 2 bV over bV square, times dV over d lambda naught times delta lambda naught which comes from here. What is bV over d lambda naught? We know v is equal to if you write back the expression for V, then V is equal to 2 pi over lambda naught times a times square root of n 1 square minus n 2 square. So, if you do bV over d lambda naught from here you will get it as minus V over lambda naught.

So, if I substitute it there then I get the pulse broadening delta tau w as minus delta L n 2 over c lambda naught times V d 2 bV over b V square times delta lambda naught. As usual I define the dispersion coefficient as broadening per kilometer length of the fiber per nano meter spectral width of the source. So, d w is delta tau w over l delta lambda naught. So, it would be minus delta n 2 by c lambda naught times V d 2 to bV over bV square.

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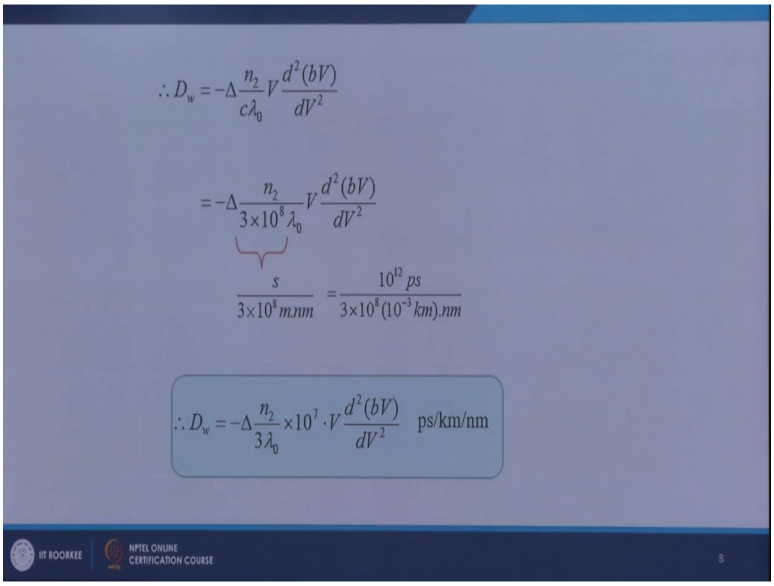

So, I have got this wave guide dispersion coefficient now. I can convert it into or express it into pico seconds per kilometer nano meters. So, what I do I put c the value of c here, which is 3 into 10 to the power 8 meters per second and. So, if now I put if I put lambda naught in nano meters then the dimensions of this would be seconds per meter nano meter, and this would be 10 to the power 12 pico seconds this meter I can convert into kilo meters and I retain this nano meters. So, it will become minus delta times n 2 by 3 lambda naught, times 10 to the power 7 times V d 2 bV over bV square in pico seconds per kilometer nano meter, where lambda naught should be put in nano meters.

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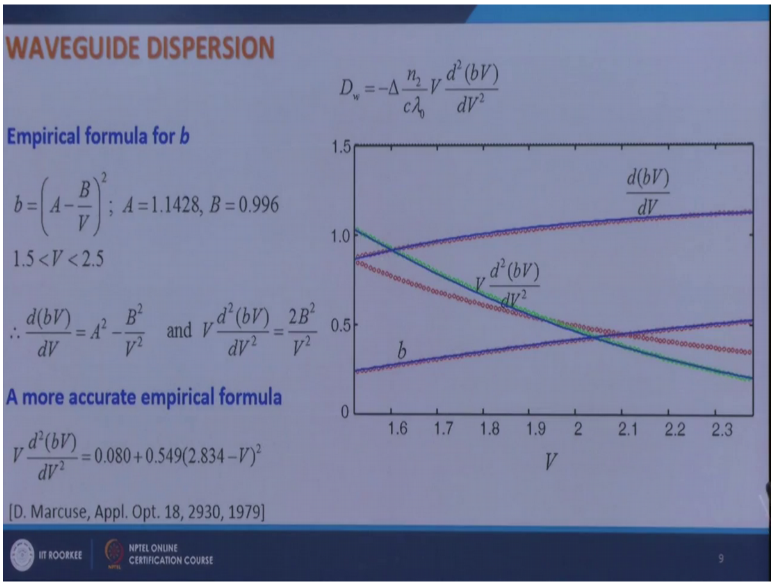

So, what I see clearly that is I want to if I want to find out the wave guide dispersion of a given fiber then I need to calculate this term v times d 2 bV over bV square and then multiply it by fiber parameters n 2 and delta and of course, divided by the wavelength central wavelength of the pulse. So, I know how b varies with v for a single mode fiber. So, if I draw that then the variation of b with v goes something like this, then what I can do can find out first because I need to find out this second derivative of bV.

So, first I find out the first derivative of bV which is d bV over bV which goes like this, and then I find out d 2 bV over bV square times v which comes out like this. So, what I have done for different values of v, I have solved the transcendental equation corresponding to LP 0 1 mode and obtained the values of b. After having obtained the value of b for different values of v, then I have plotted this and then numerically differentiate it this bV with V and then by numerical differentiation I have obtained this. So, this I refer to as exact variations that are by exactly solving the transcendental equations. Now I also know that that there is an empirical relationship between b and v and that empirical relationship is given by this formula b is equal to a minus b over v square where a is this b is this and this relationship is valid in the range of v going from 1.5 to 2.5.

From here it is not difficult to obtain d bV over bV, which will come out to be A square minus B square over V square and then V times d 2 bV over bV square, which comes out to be 2 B square over V square. So, from here I can if I want to calculate the wave guide dispersion then I can simply put 2 B square over V square in place of this and I can immediately get the wave guide dispersion, but how accurate it is how accurate this empirical relationship is for obtaining wave guide dispersion. So, for that what we have done here I have plotted the variation of B as a function of V from empirical relationship, and then this and this obtained from empirical relationship also. So, I have plotted it here as red circles. So, these red circles show the variations of various terms with respect to V as obtained from empirical relationship. So, what I see that variation of V fits well, variations of db over bV also fits quite well, but this term deviates a quite a lot except at a particular values of V which is close to 1.9.

So, at around V is equal to 1.9 the agreement between the empirical relationship and the exact values is good, but when you deviate from this value then they are the agreement is not good. So, what should I do well then I cannot use this for the entire range of V, then mark use in 1979 gave another empirical formula which is much accurate more accurate, it is again empirical relationship between V times d 2 vv over dV square and V and a relationship is given by this. When I plot this then I which has (Refer Time: 21:21) shown here in green circles, then I find that this relationship fits very well in the entire range of V from 1.5 to 2.5. So, I can safely use this relationship for calculating wave guide dispersion in a single mode fiber. Now let us look at an example take a typical conventional single mode fiber whose parameters are given by this and this I have adopted from the text book introduction to fiber optics. So, for these values of fiber parameters.

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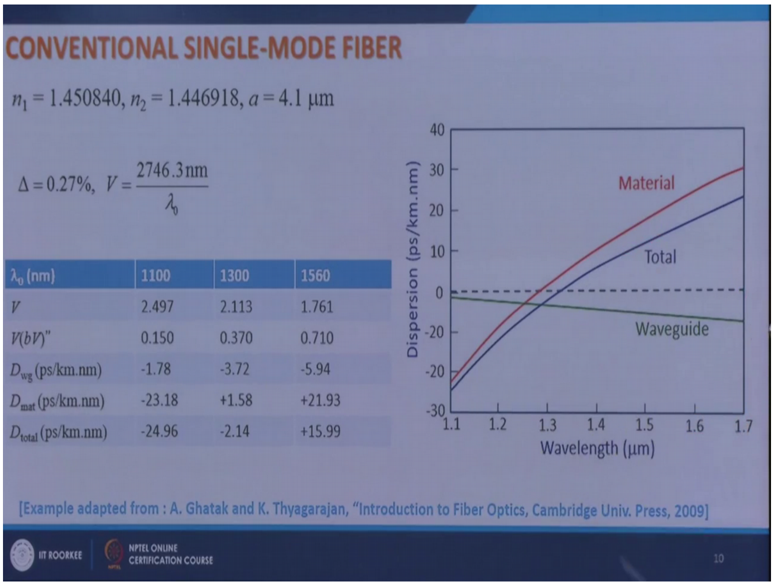

If I calculate the value of V then the value of v comes out to be 2746.3 nano meter divided by lambda naught ok.

So, when I put lambda naught in nano meters then for that value of lambda naught I can find out the value of V, and these 2 values of n 1 and n 2 correspond to delta which is about 0.27 percent. Now let me analyze this and calculate all the dispersions and analyze this fiber a. So, I analyze and this fiber at three different wavelengths by calculating various parameters. So, when I put lambda naught is equal to 1100 nano meter then V is 2.497.

So, it is close to single mode, but it is not exactly single mode and the wave guide dispersion if I calculate comes out to be minus 1.78 pico seconds per kilometer nano meter, material dispersion is minus 23 pico seconds per kilometer nano meter, and total dispersion is about minus 25 pico seconds per kilometer nano meter. So, these are the values at 1100 nano meter wavelength, at 1300 nano meter wavelength wave guide dispersion is minus 3.7, material dispersion is very low plus 1.5 weight, a as we can expect because this is few silica glass fiber and for fused silica glass fiber the 0 dispersion wavelength is around 1270 nano meters.

So, the total dispersion is minus 2.14 pico seconds per kilo meter nano meters. Near to communication window at 1516 nano meter, wave guide dispersion is about minus 6 pico seconds per kilometer nano meter, while material dispersion is plus 22 pico seconds per kilometer nano meters. So, the total dispersion comes out to be about 16 pico seconds per kilometer nano meter. So, what I see in this fiber that this fiber has very low dispersion of about minus 2 pico second per kilometer nano meters at 1300 nano meter wavelength, while it has plus16 pico seconds per kilometer nano meter dispersion near the lowest loss wavelength. So, to have a much better picture and wider picture, I have plotted here dispersion as a function of wavelength in the range 1.1 micron to 1.7 micron.

So, this green curve shows that this is wave guide dispersion which is always negative, this red curve shows the material dispersion, and this blue curve shows the total dispersion. Here we have calculated the total dispersion by adding the material dispersion and wave guide dispersion which is not rigorously correct, but it is it is quite accurate when the dispersions are not very high. If the dispersions a the values of dispersions are very high, then I should not just add the material dispersion and wave guide dispersion and. In fact, what I should do while calculating wave guide dispersion I should also take into account the variation of n 1 and n 2 with respect to lambda. So, I should automatically include material dispersion while calculating wave guide dispersion, that would be much more rigorous analysis, but for small values of dispersion like this, if I just add them up then it is sufficient.

So, this is what I have. What is the meaning of negative dispersion and positive dispersion? We have seen that in material dispersion also that for the wavelengths which are less than 1.27 micro meter, the dispersion is negative material dispersion is negative for the wavelengths longer than 1.27 micrometer the material dispersion is positive, and wave guide dispersion is always negative. So, the total dispersion what I see is now shifted from 1.27 which is 0 material dispersion wavelength from 1.27 micro meter to about 1.3 micro meter. So, I have 1.3 micrometer near 0 total dispersion wavelength.

So, in the next lecture we would see what is the meaning of negative and positive dispersion, how various fiber parameters effect wave guide dispersion, how we can tailor the dispersion in a fiber by having different profiles.

Thank you.