

Fiber Optics
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Lecture – 23
Optical Fiber Waveguide- V

In the last lecture, we had approximated the modal field of a single mode fiber by a Gaussian. We had studied the Gaussian mode and defined the Gaussian spot size. We had also studied how the power is distributed between the core and the cladding of a fiber. In this lecture we will define a spot sizes in few more ways and see how the bending of the fiber affects the performance.

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PETERMANN-2 SPOT SIZE

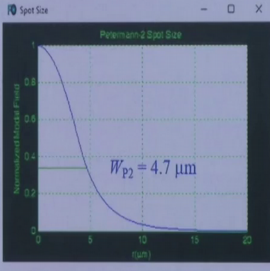
$$W_{P2} = \sqrt{\frac{2 \int_0^{\infty} \psi^2(r) r dr}{\int_0^{\infty} \left(\frac{d\psi}{dr}\right)^2 r dr}}$$

$\psi(r) \rightarrow$ transverse field pattern of the fundamental mode

Related to the transverse off-set loss at a joint between the two fibers

For a step-index fiber


$$W_{P2} = \sqrt{2} \frac{J_1(U)}{W J_0(U)} a$$




Gaussian : 4.75 μm

$W_{P2} = 4.7 \mu\text{m}$

$n_1 = 1.45, n_2 = 1.444, a = 4 \mu\text{m}, \lambda_0 = 1.55 \mu\text{m}$

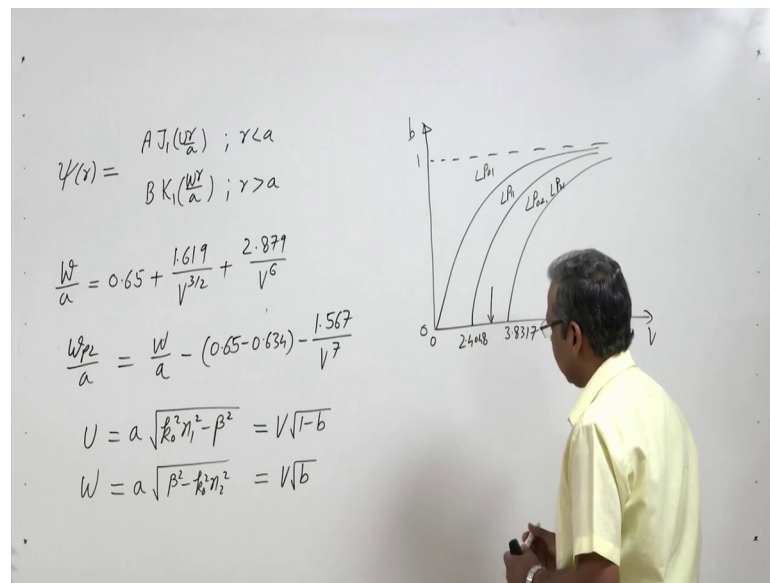
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So, there is another way of defining the spot size which is Peterman 2 spot size, which is related to transverse offset loss at a joint between the 2 fibers, we will study about these losses at the joints in the next lecture. So, Peterman 2 spot size is defined in terms of transverse field pattern of the fundamental mode $\psi(r)$, and is given as W_{P2} is equal to square root of 2 integration 0 to infinity $\psi^2(r) r dr$ divided by integration 0 to infinity $\psi'(r)^2 r dr$.

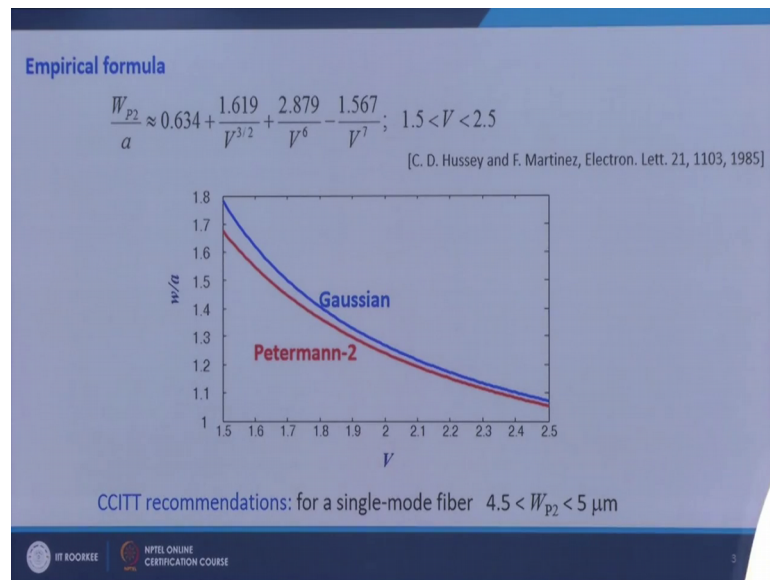
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Now, for a step index fiber I can write $\psi(r)$ in the core as $A J_1(u r / a)$, and in the cladding $B K_1(w r / a)$, where a and b can be related by the boundary conditions that $\psi(r)$ is continuous at $r = a$, and if I use that then I can get the expression for W_{p2} as square root of $2 J_1(u) / W_{p1}$ times a . Here I have plotted the modal normalized modal field of the fiber defined by $n_1 = 1.45$, $n_2 = 1.444$, core radius $a = 4$ micrometer at wave length $\lambda_0 = 1.55$ micrometer. So, this blue line shows the exact modal field of the fundamental mode of the fiber, and this marks this green line marks the pedestal to spot size of the mode calculated by this modal field $\psi(r)$. Since W_{p2} for these fiber parameters and the wave length is 4.7 micrometer, and if I calculate the Gaussian spot size for these parameters it comes out to be 4.75 micrometer.

As we had done in the case of Gaussian, we can also define an empirical relation between the Peterman 2 spot size W_{p2} and the normalized frequency V .

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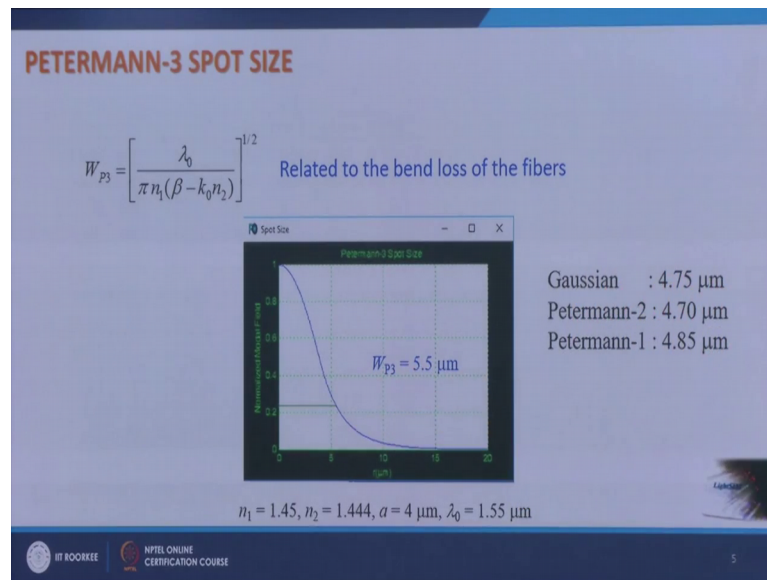


And it is given as W_{p2}/a is approximately equal to 0.634 plus 1.619 over V to the power 3 by 2, plus 2.879 over V to the power 6, minus 1.567 over V to the power 7, and this is quite good in quite accurate in the range of V from 1.5 to 2.5. If I compare it with the Gaussian spot size then in the case of Gaussian this term was 0.65 and this term was not there. So, I am reducing this term from 0.65 to 0.634 and further I am I am reducing the spot size by this much amount. So, if I compare it with the Gaussian then then the Peterman 2 spot size would always be smaller than the Gaussian. For large values of V this this term will tend to 0, the contribution of this term would be very small and the difference between the 2 would shrink; however, this much difference would always be there.

So, now I compare the Gaussian spot size and Peterman 2 spot size, and I can immediately see that the difference is large at smaller values of V , and it shrinks when I go towards higher values of V . This Peterman 2 spot size is accepted as the standard by a committee which sets the standards for telecom fiber, and this committee is known as CCITT according to its recommendations for a single mode fiber to be used in long haul telecommunication system, the Peterman 2 spot size should lie between 4.5 and 5 micrometer

Yet another way of defining a spot size is Peterman one spot size and this is related to angular offset loss at the joint between the 2 fibers.

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And is defined as W_{P1} is equal to a square root of 2 times integration 0 to infinity size square r cube $d r$ divided by integration 0 to infinity, ψ square r $d r$. So, here in this figure I have again plotted the normalized modal field for the same fiber and at the same wave length and the Peterman one spot size. The Peterman one spot size comes out to be four point eight five micrometer and if I compare it with the Gaussian was 4.75 micrometer and Peterman 2 was 4.7 micrometer, for the same fiber and the wavelength.

I also have Peterman 3 spot size which is related to bend loss of the fiber and it is defined as W_{P3} is equal to square root of λ_0 naught divided by $\pi n_1 \beta - k_0 n_2$, and in this figure I again see the normalized modal field, and the Peterman 3 spot size which is shown to be 5.5 micrometer calculated by this formula. If I compare it with the other 3 spot sizes, then I find that it is much larger. Petermann 1 is 4.85 micrometer, Petermann 2 is 4.7 and Gaussian is 4.75, but this is 5.5 micrometer which is quite large as compared to the other three. Here in this table I have listed some important parameters of a single mode fiber in the range of V from 1.5 to 2.4.



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Some important parameters of a single-mode fiber

V	b	U	W	w/a	W_{p2}/a
1.500	0.2292	1.3169	0.7182	1.7840	1.6763
1.525	0.2396	1.3299	0.7464	1.7386	1.6409
1.550	0.2498	1.3425	0.7747	1.6966	1.6077
1.575	0.2600	1.3549	0.8031	1.6577	1.5765
1.600	0.2701	1.3670	0.8315	1.6216	1.5472
1.625	0.2801	1.3788	0.8600	1.5879	1.5196
1.650	0.2900	1.3903	0.8885	1.5565	1.4935
1.675	0.2998	1.4016	0.9171	1.5272	1.4688
1.700	0.3095	1.4127	0.9457	1.4997	1.4455
1.725	0.3190	1.4235	0.9744	1.4739	1.4234
1.750	0.3285	1.4340	1.0030	1.4496	1.4024
1.775	0.3379	1.4444	1.0317	1.4267	1.3824
1.800	0.3471	1.4545	1.0604	1.4051	1.3635
1.825	0.3562	1.4644	1.0891	1.3846	1.3454
1.850	0.3651	1.4741	1.1179	1.3652	1.3281
1.875	0.3740	1.4836	1.1466	1.3468	1.3116
1.900	0.3827	1.4928	1.1753	1.3294	1.2958
1.925	0.3912	1.5020	1.2041	1.3128	1.2808
1.950	0.3997	1.5109	1.2328	1.2969	1.2663

V	b	U	W	w/a	W_{p2}/a
1.975	0.4080	1.5196	1.2615	1.2818	1.2524
2.000	0.4162	1.5282	1.2902	1.2674	1.2391
2.025	0.4242	1.5366	1.3189	1.2536	1.2264
2.050	0.4321	1.5448	1.3476	1.2404	1.2141
2.075	0.4399	1.5529	1.3763	1.2277	1.2023
2.100	0.4476	1.5608	1.4049	1.2156	1.1909
2.125	0.4551	1.5686	1.4336	1.2039	1.1799
2.150	0.4625	1.5762	1.4622	1.1927	1.1693
2.175	0.4698	1.5837	1.4908	1.1819	1.1591
2.200	0.4770	1.5911	1.5194	1.1715	1.1493
2.225	0.4840	1.5983	1.5479	1.1615	1.1397
2.250	0.4909	1.6054	1.5765	1.1519	1.1305
2.275	0.4977	1.6123	1.6050	1.1426	1.1216
2.300	0.5044	1.6191	1.6335	1.1336	1.1130
2.325	0.5110	1.6259	1.6620	1.1249	1.1046
2.350	0.5174	1.6325	1.6904	1.1165	1.0965
2.375	0.5238	1.6389	1.7189	1.1084	1.0887
2.400	0.5300	1.6453	1.7473	1.1005	1.0811

** spot sizes have been calculate from empirical formulae*



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These parameters are normalized propagation constant b , U you remember that u is defined as u is equal to a times square root of $k_{naught}^2 n_1^2 - \beta^2$ or you can also write it as V times square root of $1 - b$, and w is defined as a times square root of $\beta^2 - k_{naught}^2 n_2^2$ and it is defined as V times square root of b .

So, here I have listed the values of U , W then Gaussian spot size and the Peterman 2 spot size for different values of V . Please note that these spot sizes I have calculated by using the empirical relations. These values of different parameters become very handy when we analyze a single mode fiber or we design a fiber for a given application, let us work out a few examples.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.45$, $n_2 = 1.444$, and $a = 4.2 \mu\text{m}$. Calculate:

- (i) The cut-off wavelength of the fiber
- (ii) The wavelength range in which the fiber supports 2 modes
- (iii) Mode effective index at $\lambda_0 = 1.55 \mu\text{m}$
- (iv) The Gaussian spot-size at $\lambda_0 = 1.55 \mu\text{m}$
- (v) The Petermann-2 spot-size at $\lambda_0 = 1.55 \mu\text{m}$

Solution

(i) The cut-off wavelength

$$\lambda_c = \frac{2\pi}{2.4048} a \sqrt{n_1^2 - n_2^2}$$
$$= 1.446 \mu\text{m}$$

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Here let us consider a step index optical fiber with core refractive index and 1 is equal to 1.45, cladding refractive index n_2 is equal to 1 point triple 4 and the core radius a is equal to 4.2 micrometer. Let us calculate the cut off wave length of the fiber. So, I know that the cut off wave length of the fiber is given by λ_c is equal to 2π over 2.4048 times a times square root of n_1^2 minus n_2^2 . So, if I plug in all these numbers in to this formula, then I immediately get the cut off wave length as 1.446 micrometer.

The second thing is the wave length range in which the fiber supports 2 modes. So, what is the range of wave lengths in which the fiber supports 2 modes? So, first I need to find out what is the range of V in which the fiber supports 2 modes. So, if I look at it b V curves for a fiber. So, this is b is equal to 0 this is b is equal to 1, this is LP 01 mode this is LP 01, this is LP 11 and this is LP 02 and LP 21 and, but I see this is 0, this is 2.404 and this is 3.8317. So, if the fiber supports 2 modes then the value of V should lie somewhere here. So, that these 2 modes are supported and the other boards are cut off. So, the value of V should lie between 2.4048 and 3.8317.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.45$, $n_2 = 1.444$, and $a = 4.2 \mu\text{m}$. Calculate:

- (i) The cut-off wavelength of the fiber
- (ii) The wavelength range in which the fiber supports 2 modes
- (iii) Mode effective index at $\lambda_0 = 1.55 \mu\text{m}$
- (iv) The Gaussian spot-size at $\lambda_0 = 1.55 \mu\text{m}$
- (v) The Petermann-2 spot-size at $\lambda_0 = 1.55 \mu\text{m}$

Solution

(ii) The fiber supports two modes if $2.4048 < V < 3.8317$

$$V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2}$$

$V = 2.4048 \rightarrow \lambda_0 = 1.446 \mu\text{m}$ $V = 3.8317 \rightarrow \lambda_0 = 0.907 \mu\text{m}$

$0.907 < \lambda_0 < 1.446 \mu\text{m}$

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So, what I need to just now find out the values of lambda corresponding to these values of V, and V is given as 2π over lambda naught times a times square root of n_1^2 minus n_2^2 . So, V is equal to 2.4048 will correspond to lambda naught is equal to 1.446 micrometer, which we had done in the in the previous part itself. This is nothing, but the cut off wave length and corresponding to 3.8317 lambda naught comes out to be 0.907 micrometer. So, lambda naught would should lie between 0.907 and 1.446 micrometer. So, that there are only 2 modes guided L P 0 1 and L P 11. Third part is what is the mode effective index at lambda naught is equal to 1.55 micrometer. At lambda naught is equal to 1.55 micrometer, V is equal to 2.2435.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.45$, $n_2 = 1.444$, and $a = 4.2 \mu\text{m}$. Calculate:



- The cut-off wavelength of the fiber
- The wavelength range in which the fiber supports 2 modes
- Mode effective index at $\lambda_0 = 1.55 \mu\text{m}$
- The Gaussian spot-size at $\lambda_0 = 1.55 \mu\text{m}$
- The Petermann-2 spot-size at $\lambda_0 = 1.55 \mu\text{m}$

Solution

(iii) at $\lambda_0 = 1.55 \mu\text{m}$ $V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} = 2.2435$

$$b(V) = \left(A - \frac{B}{V} \right)^2 \quad A = 1.1428, \quad B = 0.996 \quad \rightarrow b = 0.4884$$

$$n_{\text{eff}} = \sqrt{n_2^2 + b(n_1^2 - n_2^2)} = 1.4469$$

So, there is only one mode because it is less than 2.4048. So, there is only one mode this is LP 01 mode. And I know from that table that from that table also I can do or from the empirical formula also I can find it out, if I do it from using empirical formula then b is given by a minus b over V whole square where a is equal to 1.1428, b is equal to 0.996 and if I calculate the value of b from here it comes out to be 0.4884. From here I can calculate the n effective as square root of n 2 square plus b times n 1 square minus n 2 square, and it comes out to be 1.4469.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.45$, $n_2 = 1.444$, and $a = 4.2 \mu\text{m}$. Calculate:



- The cut-off wavelength of the fiber
- The wavelength range in which the fiber supports 2 modes
- Mode effective index at $\lambda_0 = 1.55 \mu\text{m}$
- The Gaussian spot-size at $\lambda_0 = 1.55 \mu\text{m}$
- The Petermann-2 spot-size at $\lambda_0 = 1.55 \mu\text{m}$

Solution

(iv) at $\lambda_0 = 1.55 \mu\text{m}$ $V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} = 2.2435$

$$\frac{w}{a} = 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} = 1.15$$

$$w = 4.85 \mu\text{m}$$

Fourth part is what is the Gaussian spot size, at λ_0 is equal to 1.55 micrometer. So, I had already calculated the value of V at 1.55 micrometer which is 2.2435, now I know the empirical relation for Gaussian spot size which is given by this. So, if I just put the value of V here I can immediately get the value of w over a as 1.15 and since a is equal to 4.2. So, I can get the value of Gaussian spot size as 4.85 micrometer. Fifth part is what is the Peterman 2 spot size at λ_0 is equal to 1.55 micrometer.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.45$, $n_2 = 1.444$, and $a = 4.2 \mu\text{m}$. Calculate:

- The cut-off wavelength of the fiber
- The wavelength range in which the fiber supports 2 modes
- Mode effective index at $\lambda_0 = 1.55 \mu\text{m}$
- The Gaussian spot-size at $\lambda_0 = 1.55 \mu\text{m}$
- The Petermann-2 spot-size at $\lambda_0 = 1.55 \mu\text{m}$

Solution

(v) at $\lambda_0 = 1.55 \mu\text{m}$ $V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} = 2.2435$

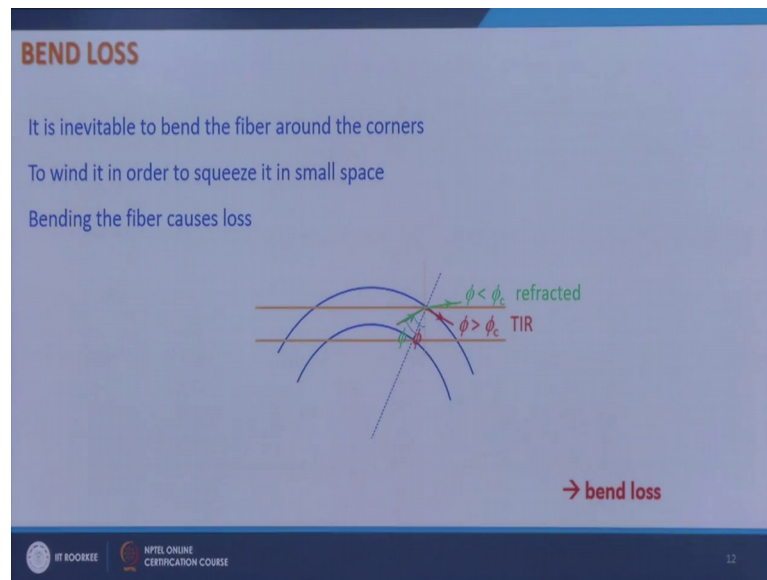
$$\frac{W_{P2}}{a} = 0.634 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} - \frac{1.567}{V^7} = 1.13$$

$$W_{P2} = 4.75 \mu\text{m}$$

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So, I already have the value of V at this wave length which is 2.2435. Now I just put it in the empirical relation of Peterman 2 spot size, then the Peterman 2 spot size comes out to be 4.75 micrometer. Now let us find out what happens if I bend a fiber. If I deploy the fiber in the field it is inevitable to bend the fiber because I will have to take fiber across the corners there are instances when I will have to coil the fiber and keep it somewhere if there is some extra length of the fiber.

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So, it is important to find out what happens when you bend the fiber, and we will see that bending of the fiber causes loss. How much loss does it cause, on what parameters of the fiber it depends on let us study that. So, if I have a straight fiber this is the core and this is the cladding, then I know that if I launch a wave like this, then if this angle is greater than critical angle then there would be total internal reflection and most of the power will remain in the core large amount of power will remain in the core, and various little power will extend in to the cladding. Now if I bend the fiber which is shown by these blue lines, then just look at this wave itself, now this angle of this wave changes with normal because normal does not remain this now, the new normal is this.

So, what I find that this ϕ is now smaller than the critical angle. If ϕ is smaller than ϕ_c the critical angle then this wave would be refracted in to the cladding and there is no total internal reflection. So, the power would be lost. So, when you bend the fiber it causes loss in the fiber how sensitive the fiber would be for this bending it will depend up on how close this ϕ is to the critical angle. If this angle ϕ is very close to the critical angle then even a small bending will cause the bend loss. If this angle is close to critical angle it means that the mode is close to cut off and the field spreads more and more in the cladding. So, I can also understand that if in this way that if the mode is close to cut off, then also fiber is more sensitive to bending. So, the bend loss of the fiber depends upon various parameters like core radius, a relative index difference between the core and the cladding, Δ and wave length λ naught.

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Bend loss depends on

- core radius, a
- relative index difference, Δ
- wavelength, λ_0

For a single-mode step-index fiber the bend loss is given by

$$\alpha(\text{dB/m}) = 4.343 \left(\frac{\pi}{4aR_c} \right)^{1/2} \left[\frac{U}{VK_1(W)} \right]^2 \frac{1}{W^{3/2}} \exp\left(-\frac{3W^3}{3k_0^2 a^3 n_1^2 R_c} \right)$$

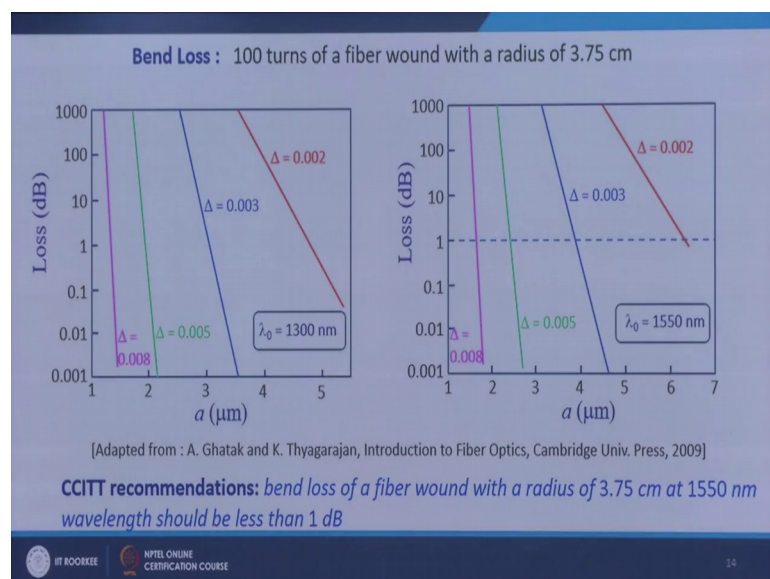
$R_c \rightarrow$ radius of curvature

[A. W. Snyder and J. D. Love, Optical Waveguide Theory, Chapman and Hall, 1983]

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For a single mode step index fiber the bend loss can be given by this formula, which involves R_c which is the radius of curvature, core radius a parameter U , which is given by this which contains a wave length n_1 and the mode effective index W . So, if I know all these parameters then for a step index fiber a step index single mode fiber I can find out the bend loss in dB per meter using this formula. Let us now look at the bend loss of a fiber as calculated by the formula shown in the previous slide.

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Here I have plotted the loss of 100 turns of a fiber, quite with the radius of 3.75 centimeter and this loss I have plotted as a function of core radius, for different values of core cladding index difference relative index difference, and in this figure I have plotted it at wave length 1300 nanometer.

Let us examine this and see what information do we get from these plots. If I fix the core radius a and change the relative index difference. So, if I go from here to here I increase the index contrast, I see that the bend loss reduces quite a lot if I increase the index contrast. So, at about 3.5 micrometer core radius, Δ is equal to 0.2 percent gives you a loss which is more than 1000 dB while a fiber with Δ is equal to my Δ is equal to 0.3 percent gives you a loss which is close to 0.001 dB. So, there is a huge difference. So, by changing the Δ a little, this Δ causes a huge impact on the bend loss of the fiber and it is understandable that if this Δ basically changes the changes the core cladding index difference and it changes the critical angle, the value of critical angle is directly affected by this Δ . So, if you increase Δ you increase Δ , your tolerance towards bending increases a lot.

Now if I take one particular value of Δ and then change the core radius, then what happens that if I increase the core radius then bend loss decreases and it is understandable, it when I increase the core radius the power is confined more and more in the core and less power extends in the cladding, that is my mode is going away from the cut off. If the mode is moving away from the cut off then it is less sensitive to bends. So, the bend loss decreases.

Next is how wave length affects the bend loss. So, for that I have plotted these curves these variations at wave length λ is equal to 1550 nanometer also, and what do I see that if I pick up one particular value of Δ and one particular value of a , then I find that when I increase the wave length then the bend loss increases tremendously. For example, here if I take the value of a as 3 micrometer then the bend loss is somewhere between 1 and 10 dB for Δ is equal to 0.003, but here if I take value of a as three micrometer then corresponding to Δ is equal to 0.003 the bend loss would be much larger than 1000 dB.

So, if I increase the wave length what happens the mode modal field spreads in the cladding and that is why it is more sensitive to bending the bend loss increases.

According to CCITT recommendations, the bend loss of a fiber quilled with the radius of 3.75 centimeter at 1550 nanometer wave length with 100 turns should be less than 1 dB. So, so CCITT recommendations say that at 1550 nanometer wave length I should operate below this line of 1 dB, this loss should be less than 1 dB for 100 turns. So, I should choose the combination of delta and a in such a way that I always remain below this line.

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Example

Q. Consider a step-index optical fiber with $n_1 = 1.447$, $n_2 = 1.444$, and $a = 4.5 \mu\text{m}$. Calculate the bend loss at $\lambda_0 = 1.3 \mu\text{m}$ for 100 turns of the fiber wound with a radius of 3.75 cm

$$V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} = 2$$

$$b = 0.4162, \quad U = V\sqrt{1-b} = 1.5282, \quad W = V\sqrt{b} = 1.2902$$

$$\alpha(\text{dB/m}) = 4.343 \left(\frac{\pi}{4aR_c} \right)^{1/2} \left[\frac{U}{VK_1(W)} \right]^2 \frac{1}{W^{3/2}} \exp\left(-\frac{3W^3}{3k_0^3 a^3 n_1^2} R_c \right) = 0.205 \text{ dB/m}$$

$$L = 2\pi R_c \times 100 = 23.56 \text{ m}$$

Bend Loss = 4.83 dB

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Let us work out a numerical example, for this if I consider a step index optical fiber with parameters n_1 is equal to 1.447, n_2 is equal to 1.444 and a is equal to 4.5 micrometer and calculate the bend loss at 1.3 micrometer for 100 turns of the fiber wound with a radius of 3.7 centimeter. So, I know that V can be calculated for these parameters by this formula, and if I calculate V it comes out to be 2. Next what I do for this value of V I calculate the value of normalized propagation constant b , I can find it from the table or I can calculate it from the empirical relation. So, b for this value of V is 0.4162, correspondingly u is equal to 1.5282 and w is equal to 1.2902. Now R_c here is 3.75. So, if I plug in these values in to this formula, I get α as 0.205 dB per meter.

Now, what is the total length which is in the coil when I coil the fiber and put and take 100 turns with this radius, then the total length of the fiber in the coil would be $2\pi R_c$ times 100, which is 23.56 meters, and this is the loss in dB per meter. So, the total bend loss in the coil would be 4.83 dB.

So, in this lecture we had seen a few more spot sizes Peterman 1, Peterman 2 and Peterman three and also seen the bend loss of the fiber. In the next lecture we will study what happens if I join 2 fibers together, what kind of losses it causes and what are the sources of these losses.

Thank you.