Fiber Optics
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Lecture – 22
Optical Fiber Waveguide – IV

After having understood the modal field patterns of a step index optical fiber now let us have a look at what is the fractional power in the core, how the power is distributed between the core and the cladding.

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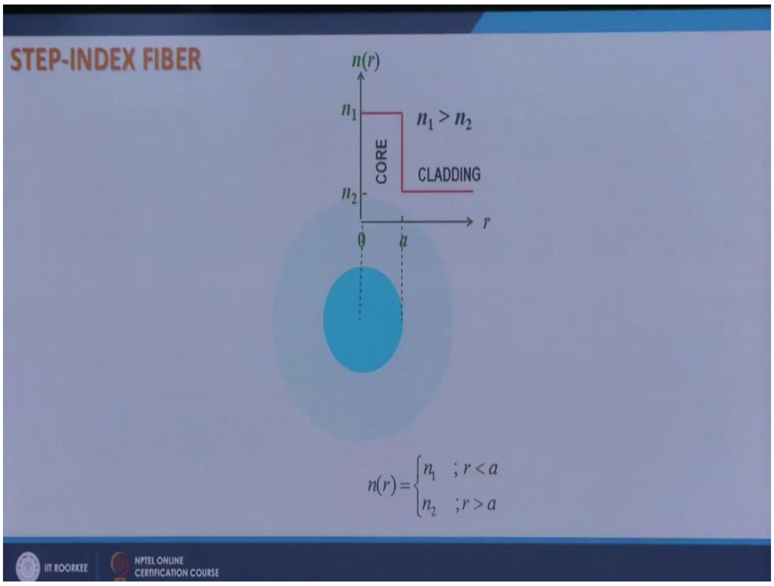

So, again we are analyzing this kind of step index optical fiber, and we know that the power is proportional to if psi is the modal field which is basically the electric field, then the power is proportional to the electric field square integrated over the entire transverse cross section.

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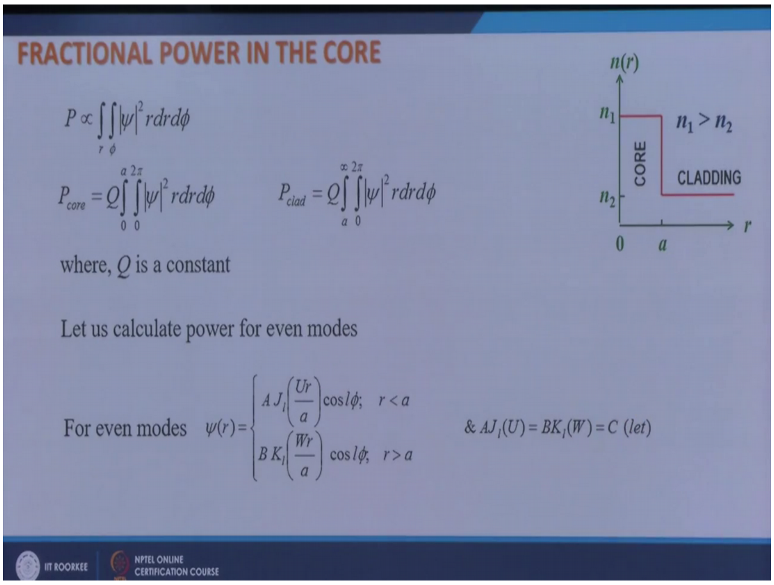

So, if psi is the modal field then the power would be proportional to integral over r phi mod psi square r d r d phi.

So, in the core it would be given by some constant Q and this integration over r would be from 0 to a, and if I want to calculate the power in the cladding then this integration over r will go from a to infinity because I am considering a case where the cladding is infinitely extended. So, let us calculate the power for even modes, for even modes the solutions psi are in the region's core and the cladding are given by this, where A and B are related to each other by boundary conditions, here I have used the boundary condition that psi r is continuous at r is equal to a. So, A J i U is equal to B K l W and let me equate two some other constant C, so I can get these A and B in terms of a single constant C.

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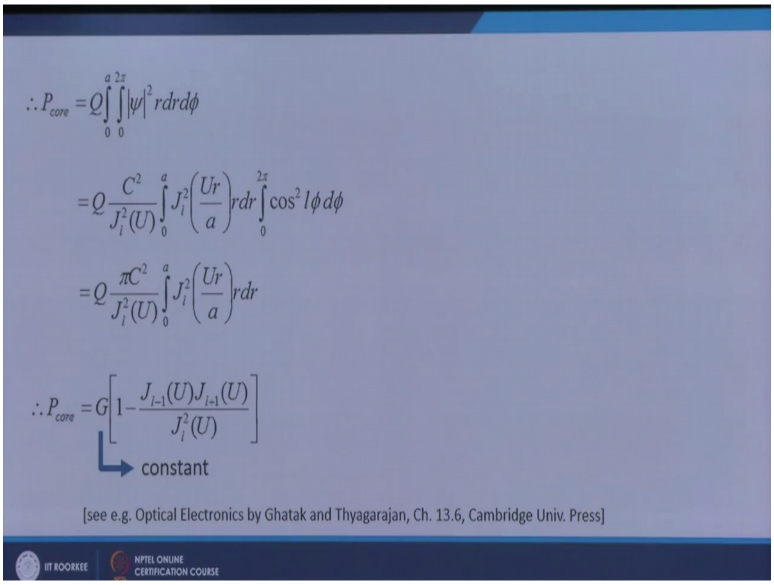

So, P core is this and if I now substitute the modal field of the core, then it would be of the form J l square U r over a r d r and then phi. So, phi part would be cosine square l phi t phi, and this would be the constant which comes out.

C square times Q and then this there would be J l square U for a given mode even J l square U would be constant because beta is constant for a given mode. So, this I can simplify like this coz phi solution the phi integral is a straight forward, and then it can be written in this particular form by doing some mathematical manipulations if you are interested in the details of this, then you can refer to the book optical electronics by Ghatak and Thyagarajan or to introduction to fiber optics by Ghatak and Thyagarajan any of these books. So, there are the details.

Similarly, in the cladding I can get the power as this where G is a constant then I can find out the fractional power as power in the core divided by total power and I can express this in this particular form and again the details can be seen in this book.

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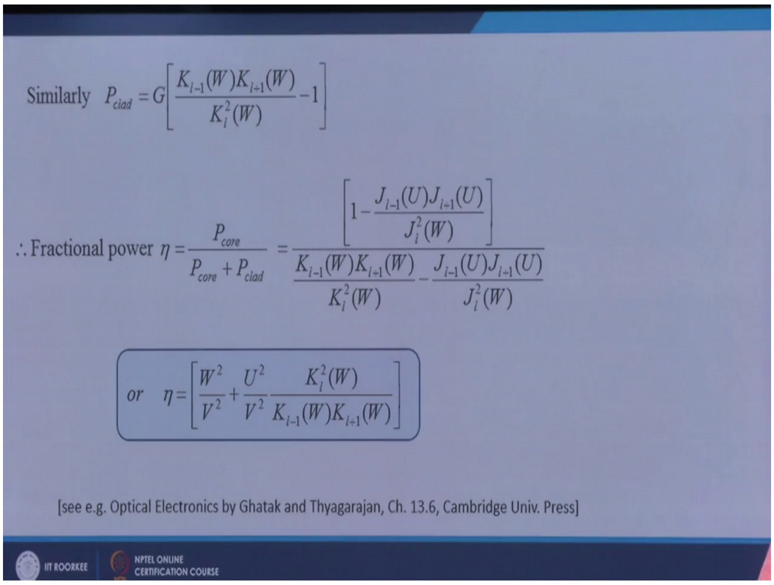

So, I have the fractional power in the core given by this expression. Now I am interested in how does this look like when I vary V, when I vary the parameters of the fiber and in this way I vary the value of V normalized frequency, then how the power in the core and the cladding is distributed how this distribution changes. So, for that let me plot this eta frictional power as a function of V and this has been taken from this paper. I have adopted this. So, it is slightly modified.

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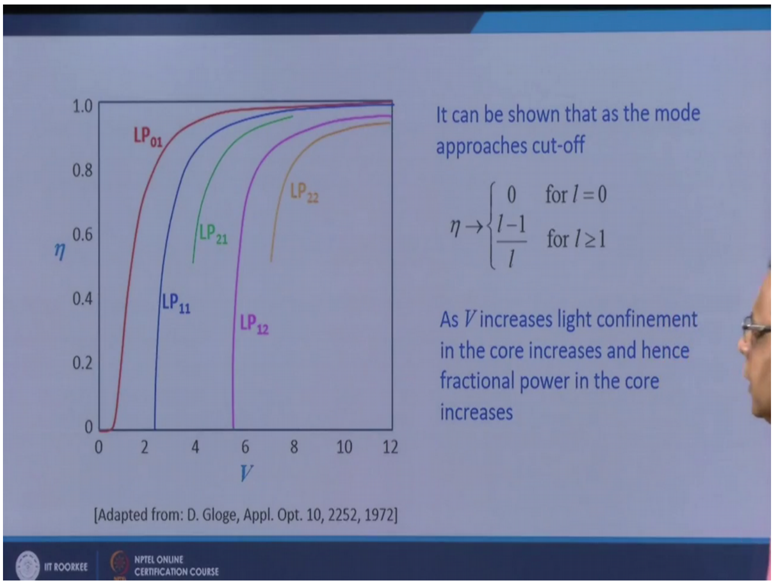

So, what I see that if I increase the value of V then the fractional power in the core increases. This is a very simple observation here. Another thing that I see is that for LP 01 mode and LP 11 mode LP 12 mode this starts from 0, while for these modes this starts from 0.5 and what I see that if I do a little mathematics I can show. I can show that for l is equal to 0 mode, l is equal to 0 mode as the mode approaches to cut off then the fractional power can be given by eta tending to 0 for l is equal to 0, and eta tends to l minus 1 over l, for l greater than or equal to 1.

So, I can see for LP 01 more the cut off is 0 LP 01 mode the cut off is 0 and l is equal to 0. So, as I approach to cut off the power is 0. And if I take LP 11 mode then again eta is 0 LP 11 mode cut off is 2.4048. So, as I approach to cut off at 2.4048 it will start from 0; however, for l is equal to 2 mode this would be half. So, l is equal to 2 mode LP 21 mode the cut off is around 3.8. So, around 3.8 it will start from 50 percent. So, this is how the power is power of different ports is distributed among between the core and the cladding. As and I can see that as we increases as we increases there is more and more power confined in the core and therefore, the fractional power in the core increases as we increases this, this I know from the theory of planar wave guide also.

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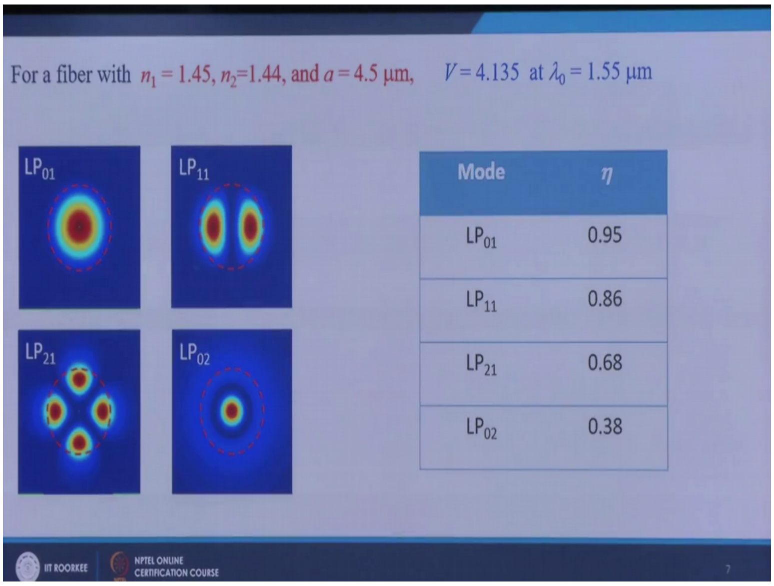

Mode	η
LP_{01}	0.95
LP_{11}	0.86
LP_{21}	0.68
LP_{02}	0.38

We have seen that now let us see some examples for a fiber with n 1 is equal to one point four five n 2 is equal to 1.44 and a is equal to 4.5 micrometer, if I calculate the value of V then it comes out to be 4.135 at lambda is equal to 1.55 micrometer and then if I see the fractional power in the core for various supported modes then it is given like this. For LP 01 mode 95 percent power is in the core. So, if I look at the mode intensity plots then 95 percent power is in the core, this dashed this dashed line corresponds to the core cladding interface. So, most of the power is inside the core for LP 11 mode about 86 percent power is in the core, and 14 percent goes out. For LP 21 mode 68 percent power is in the core and for LP 02 mode only 38 percent power is in the core.

So, I see that as I go towards higher order modes the fractional power in the core decreases. Because higher order modes are closer to cut off; if I look at how the power at different wave lengths is distributed between the core and the cladding. So, for that I consider a fiber with n 1 is equal to 1.45, n 2 is equal to 1.44 and a is equal to 4 micrometer and then here.

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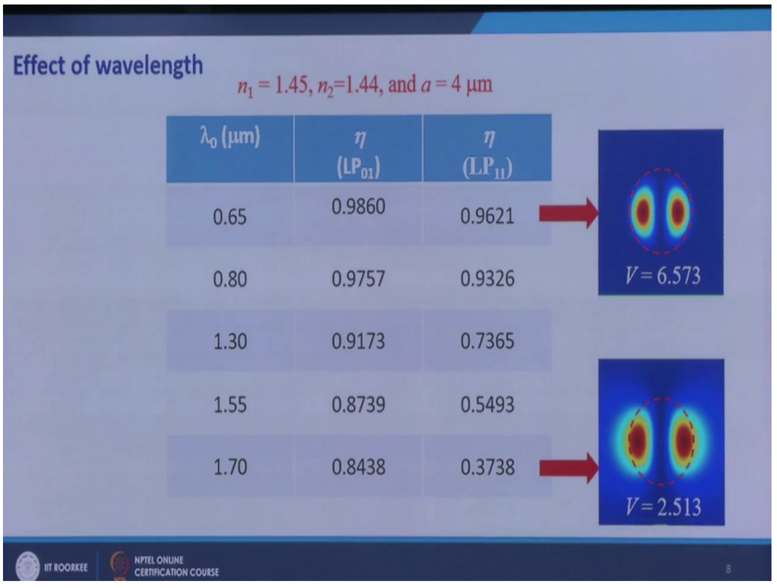

I have tabulated the fractional power in the core for LP 01 which is the fundamental mode and LP 11 which is the first higher order mode. So, I see that at 0.65 micrometer wave length, 90 more than 98 percent power for LP 01 mode is located in the core and 96 percent power is located in the core for LP 11 mode. So, as I increase the wave length I see the general trend that the power in the core for a given mode decreases.

So, as I increase the wave length the mode spreads out, because I know if I increase the wave length which means I am decreasing the value of V and the mode approaches to cut off and when a mode approaches to cut off then the field spreads out. So, the fractional power in the core would decrease. If you look at the modal fields of LP 11 mode at 0.65 micrometer wave length then it looks like this, we have the value of V at this wave length is 6.57. So, high value of V. So, very, so most of the power is in the core. But if you go to 1.7 micrometer wave length the value of V is 2.5 which is very close to cut off the cut off is 2.4048, and then I see that the field spreads out and there is only 37percent power in the core.

What happens if I change the numerical aperture of the fiber? So, for that I take a fiber with cladding refractive index 1.444, core radius four micrometer and I analyze it at 1.55 micrometer wave length. So, now, I change the value of numerical aperture.

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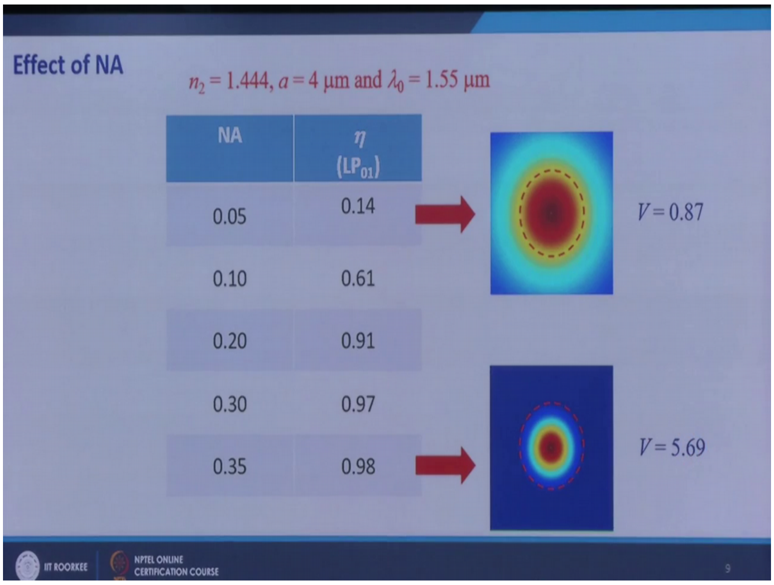

Then what I see as I increase the value of numerical aperture, then the fractional power in the core increases and this is intuitively correct I expect this intuitively that. As the index difference between the core and cladding increases, then the then more and more power is pushed inside the core. The index contrast between the core and the cladding increases. So, intuitively I think that the core cladding interface this boundary is hard to penetrate for the field and so, less field extends in the cladding region and there is more and more field in the core region.

So, that is why the power in the core region increases the fractional power in the core is very high if the value of numerical aperture is high. When the numerical aperture is low very low it is 0.05 just 0.05 it is very weekly guiding fiber. So, it hardly guides slight only 14 percent of light is in the core and 86 percent light spreads out in the cladding. So, it is really very poor guidance the value of V is very small 0.87 quite close to 0, and when the numerical aperture is high then of course, the entire field is pushed in the core V the value of V is very large about 5.7. Now after this I would like to focus on single mode fiber and because single mode fiber is the fiber which is used in long haul telecommunication system, and I would like to first look at some very important parameters of a single mode fiber what are these parameters.

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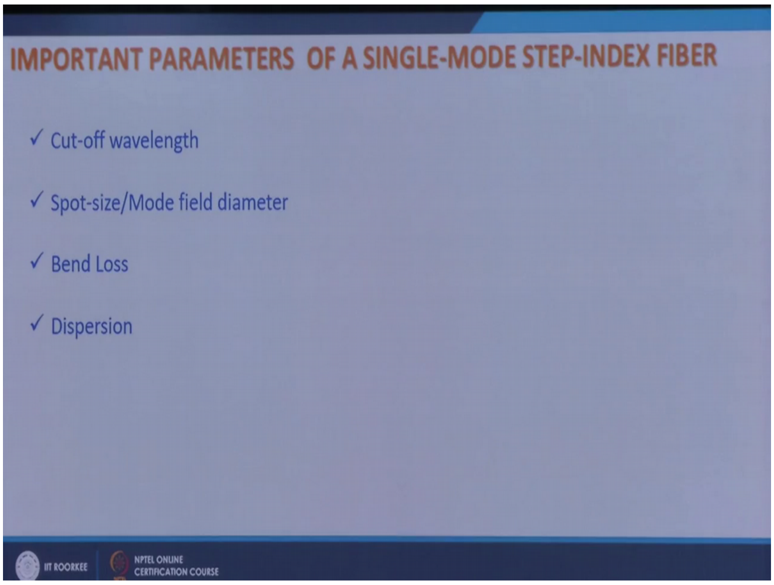

One is cut off wave length what is the cut off wave length of the fiber, what is the spot size or mode field diameter or effective area of the fiber, what is bend loss and what are the dispersion properties of this fiber. So, we would look in to these in subsequent lectures as we go along. So, let us first look at cut off wave length, what do I mean by cut off wave length.

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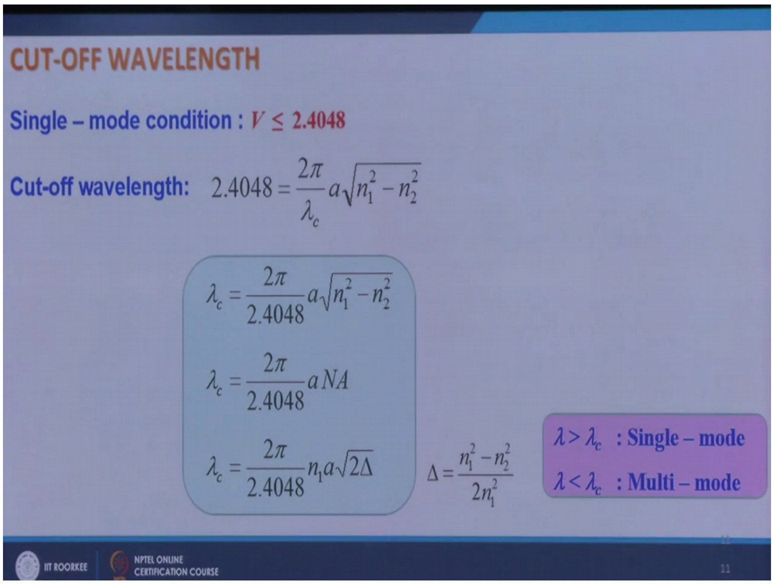

If you look at a fiber, then the single mode condition is given by V is less than equal to 2.4048 equal to sin corresponds to the limiting case, and V is given by V is given by 2 pi over lambda naught times a times square root of n 1 square minus n 2 square.

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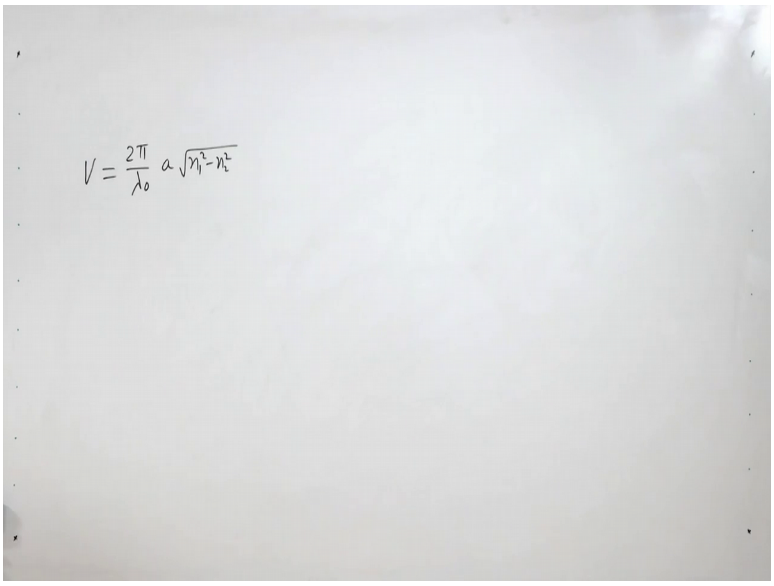

So, if I find out the wave length corresponding to this value of V 2.4048, then this wave length will come out to be 2 pi divided by 2.4048 times a times square root of n 1 square minus n 2 square, or I can also express it in terms of numerical aperture because the square root of n 1 square minus n 2 square is nothing, but numerical aperture or I can also represent it in terms of relative index difference delta, because delta is n 1 square minus n 2 square over two n 1 square. So, in this way I can calculate the value of lambda C and what happens that for all the wave lengths greater than lambda C the fiber is single mode, and for wave lengths is smaller than lambda c or shorter than lambda c the fiber is multimode or few moded. This wave length lambda C give you a demarcation between the single mode operation and multimode operation although this wave length corresponds to the cutoff of LP 1 1 mode, but in general we call it the cut off wave length of the fiber. So, when I say cut off wave length of the fiber then it means the cut off wave length of LP 1 1 mode of the fiber.

Next thing is propagation constant if I look at the b V curve of LP 0 1 mode, because it is a single mode fiber. So, only LP 0 1 mode is guided, if I look at the b V curve of LP 0 1 mode it looks like this. This I have obtained by solving the transcendental equation corresponding to l is equal to 0 mode and extracting the first root of that which corresponds to LP 0 1 mode. So, this exact means by numerically solving the transcendental equation or Eigen value equation.

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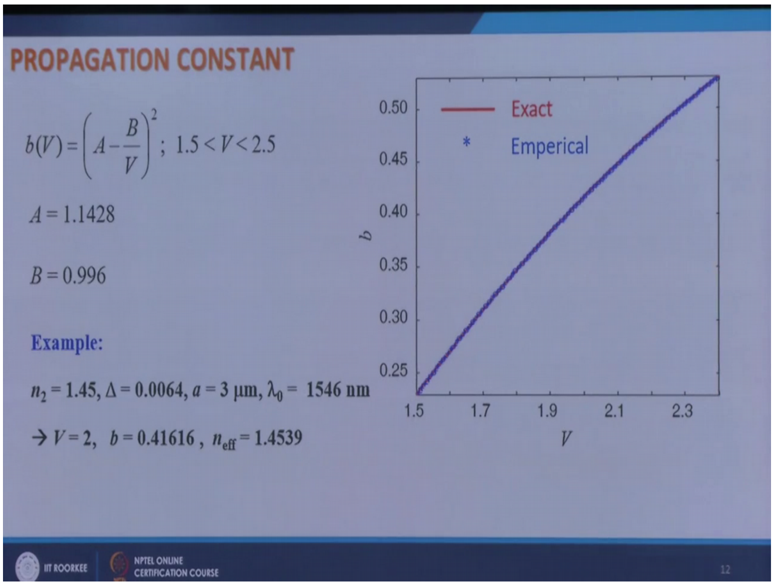

So, it goes like this. So, do I need to refer back always to this curve if I am given a fiber and wave length then I know the value of V, and I want to find out the value of b or propagation constant then I need to refer back to this curve always can I do something that I can fit some equation to this. So, that I just use that equation and extract the value of b if I know the value of V. And it can be done what is done what is seen that if you fit this equation to this curve, then it fits very well for a is equal to this and b is equal to this in the range 1.5 to 2.5 in the range of V which goes from 1.5 to 2.5.

So, this is an empirical relation between b and V, which is given by a minus b over V whole square where a is equal to 1.1428 and b is close to 1. So, I can always use this empirical relation if the value of V lies in this range to obtain the propagation constant of LP 01 mode of the fiber, and it becomes very handy while doing calculations. How good this approximation is, how good this empirical relation is for that I have superposed the values of b obtained by this empirical relation, on the values of V which I have obtained by solving the transcendental equation, and I find that the agreement is quite good if I look at this graph I can also have the quantitative estimate of this fitting how good this fitting is.

For that I will do in the next slide, but if I know now if I now know this empirical relation, then my life becomes very easy I need not to go to solve the transcendental equation again and again for example, if I have a fiber with n 2 is equal to 1.45, delta is equal to 0.64 percent, a is equal to 3 micrometer and I consider a wave length of 1546 nanometer. So, these values correspond to a value of V which is 2 and if I put it here then I find out the value of b immediately as 0.41616 and from there I can extract the value of n effective.

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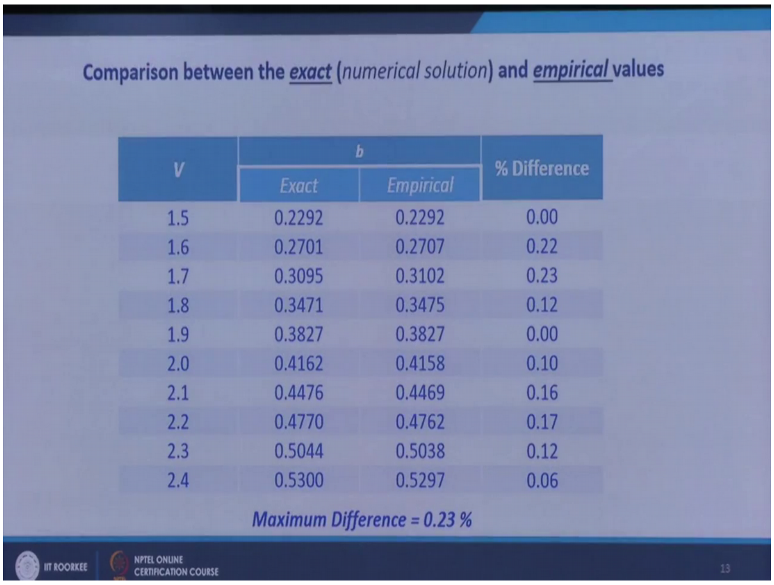

V	b (Exact)	b (Empirical)	% Difference
1.5	0.2292	0.2292	0.00
1.6	0.2701	0.2707	0.22
1.7	0.3095	0.3102	0.23
1.8	0.3471	0.3475	0.12
1.9	0.3827	0.3827	0.00
2.0	0.4162	0.4158	0.10
2.1	0.4476	0.4469	0.16
2.2	0.4770	0.4762	0.17
2.3	0.5044	0.5038	0.12
2.4	0.5300	0.5297	0.06

Now, coming back to how accurate it is, I have tabulated the values of b obtained by solving the transcendental equation and obtained by the empirical relation given in the previous slide, and I have looked at the percentage difference between the two; and I see that the maximum difference is 0.23 percent in the range of V going from 1.5 to 2.4. So, in this range it is a very good approximation I can use this empirical relation without any problem. Next important parameter of a single mode optical fiber is the spot size, when power exits from a single mode fiber, and if I capture the field or the intensity.

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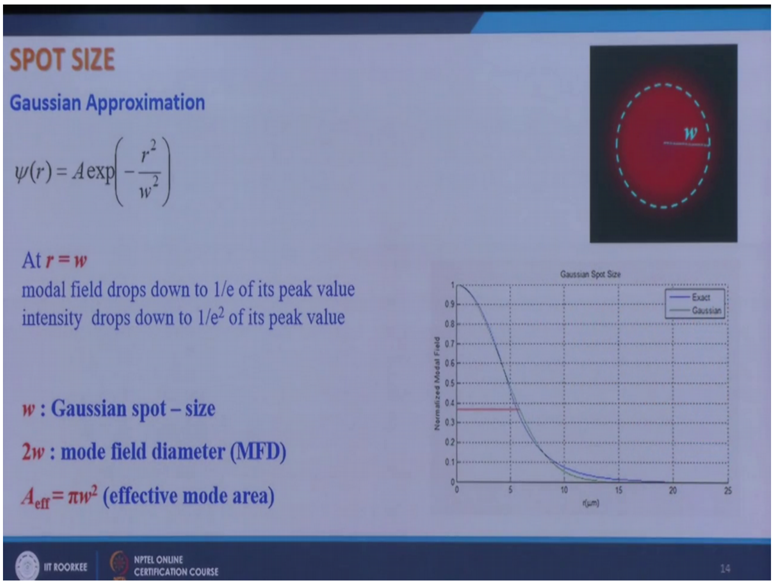

Which comes out of the fiber just at the output end of the fiber as it exits from the fiber, then it looks like this. When I look at it then it very much resembles to with the spot which comes out of a laser beam. So, it immediately tends me to think that weather can I weather can I approximate it by a Gaussian because the output of a Gaussian is output of a laser is Gaussian of a single mode laser.

So, what I try; I try to fit a Gaussian to this kind of distribution and a Gaussian I represent as sy r is equal to a e to the power minus r square over w square, where a represents what is the value at the center and w is the width of the Gaussian. So, what I do? I plot the exact field in blue line, which I obtained in the form of Bessel functions and then I plot in green color the best fitted Gaussian. How do I fit I will explain in the next slide and what I see that this fitting is very good, this fitting comes out to be very good at least for this case.

So, I can very well use this Gaussian approximation to represent this field, and why I want to do this? Because using Bessel functions all the time is not a convenient thing and working with Gaussian is very easy. So, I express my modal field with a Gaussian and then with the help of this I can estimate or define the spot size the size of this spot as you know that the field goes down to 0 when r tends to infinity, but what is the size, what is the region in which the maximum field is there. So, that we can define by r is equal to w because it r is equal to w the modal field drops down to 1 over e of its peak value, and intensity drops down to 1 over e square of its peak value. So, it can give me a good estimate of the size of the spot. So, this w is known as Gaussian spot size, and two w twice of this value is basically the diameter mode field diameter.

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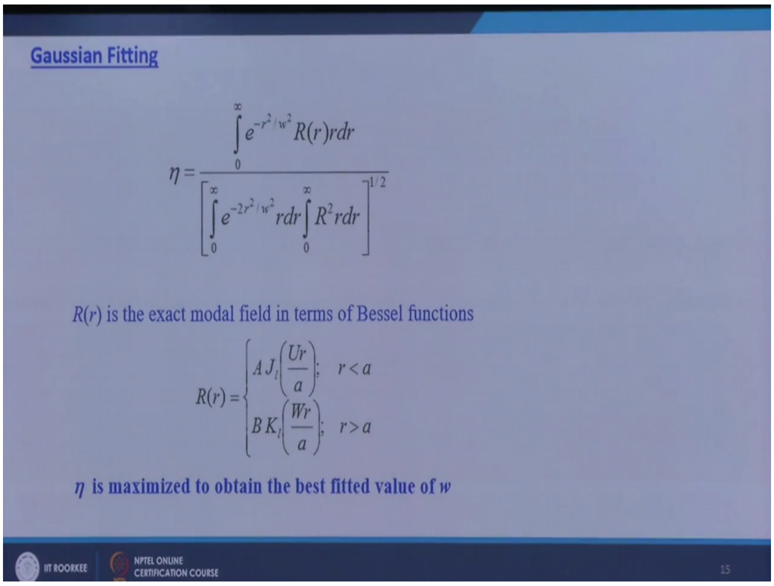

And I can find out the effective mode area by pi w square. How do I find out the best fitted Gaussian? The best fitted Gaussian has two parameters. So, I need to optimize 2 parameters a and w, a is easy to do because it can be found out what is the maximum value of the field, and to find out w what I do I take the overlap of this Gaussian with the exact field which is defined in terms of Bessel function. So, I take this overlap of the Gaussian with this field and I vary the value of the w, in order to obtain the maximum value of eta. The value of w which gives me which gives me the maximum value of overlap that gives me the best fitted Gaussian.

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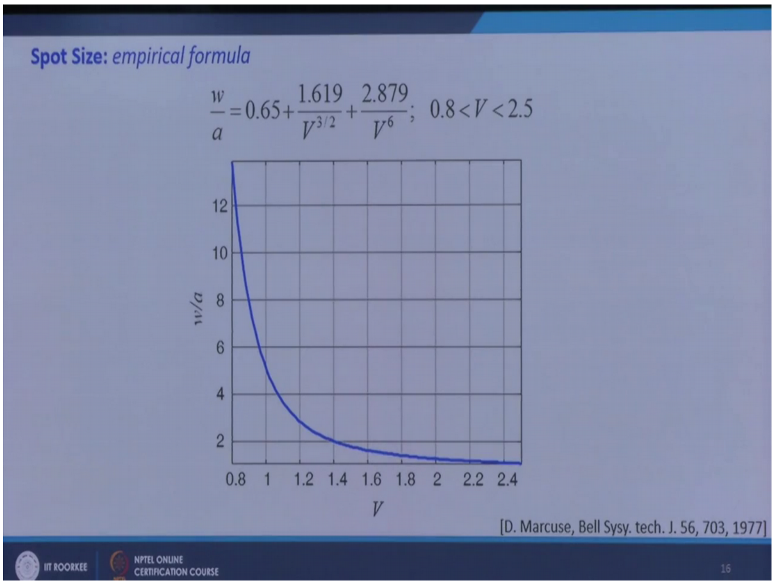

Then I tend to think just in the same way I have defined an empirical relation between the propagation constant and V, can I do the same for the spot size also. So, that I need not to all that time fit this Gaussian. So, there is an empirical relation which relates the spot size with normalized frequency V, and it has been given by d marquis. So, with this relation I can find out the spot size of a given fiber if I know the value of V. And this empirical relation is quite accurate in the range of V from 0.8 to 2.5. So, if I plot it. So, it goes like this and this is an obvious result it as I increase the value of V as I know the confinement in the core increases and therefore, the value of w over a would decrease. Now let me find out how accurate this approximation is how accurate this empirical relation is. So, for that I take two representative points.

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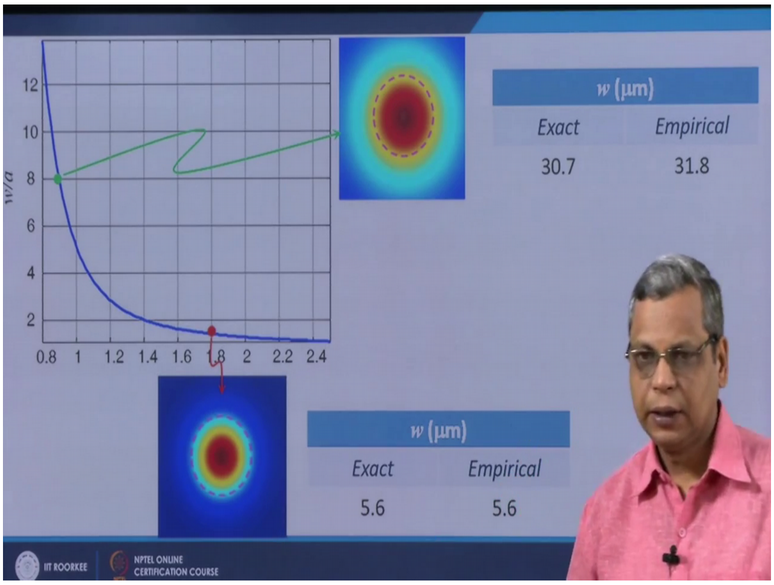

One is for smaller values of V that is at V is equal to 0.9, and if I see the modal filled here it looks like this, and I find out the value of w by two methods one is by Bessel functions the exact value, and another is from empirical relation. So, I find that at V is equal to 0.9 the exact value of w is about 30.7 micrometer, while empirical relation gives me a value of 31.8. I take another representative point at V is equal to 1.8 this is the model field and the value of w obtained from exact and empirical formula give me almost the same value they are in good agreement. So, I find that for lower value for smaller values of V there is some discrepancy, but this empirical formula works very well when I go towards higher values of V.

Now, let me look at the fractional power in the core and see how good the Gaussian approximation and empirical fitting is.

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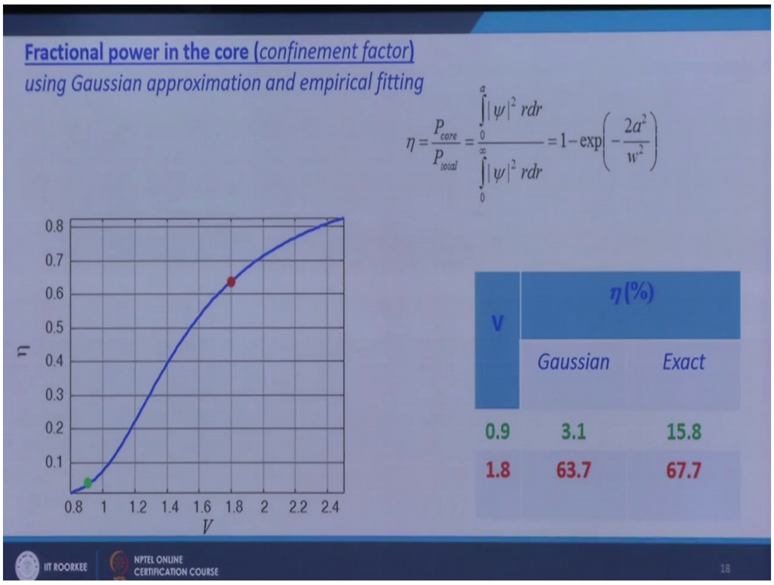

So, what I have done well I have calculated now, the fractional power in the core which is given as P core over P total, while considering the Gaussian spot size. So, as I have said that it is very easy to work with Gaussians.

So, now, calculating these integrals would not be very difficult if you use sy as a Gaussian. So, when I do this then it comes out to be 1 minus e to the power minus 2 a square over w square, and when I plot it goes like this. To find out how accurate it is I again take two representative point that V is equal to 0.9 and at b is equal to 1.8, and I find that at smaller values of V there is a huge discrepancy, it is because at lower value of V there is a small discrepancy between the exact value and the and the Gaussian and empirically fitted value, but this small discrepancy is amplified because this appears as the it appears in the form of e to the power minus something. So, this gets amplified a lot; however, at the higher values of V the discrepancies is small and therefore, the discrepancy in fractional power is also very small. So, the agreement is good.

So, in the next lecture I would look in to some other ways of defining the spot size, and I would also look into various examples to understand the cut off and spots sizes.

Thank you.