Fiber Optics
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Lecture - 20
Optical Fiber Waveguide- II

So, let us continue our analysis on step index optical fiber. So, we were doing the analysis of this step index optical fiber whose core has a radius a and refractive index n 1 and the cladding has refractive index n 2.

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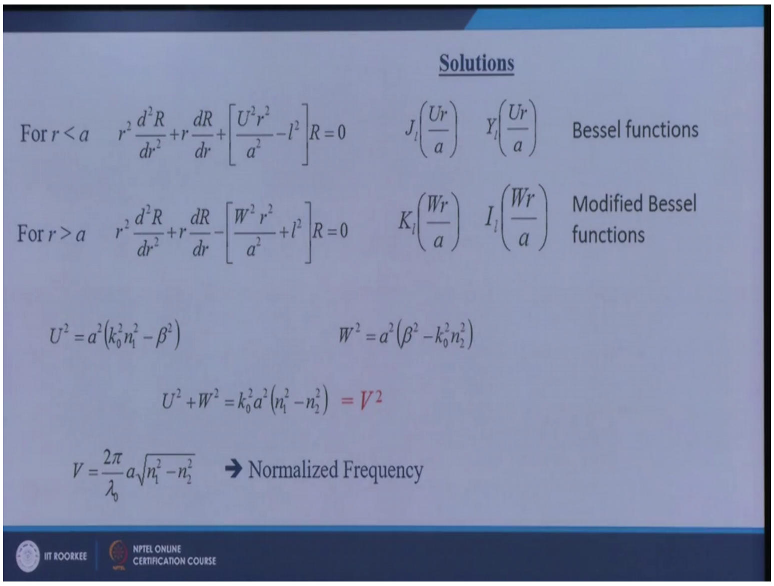

So, what we had done we had formed equations in capital R in the region r less than a and r greater than a and these equations have the solutions which are in the form of Bessel functions.

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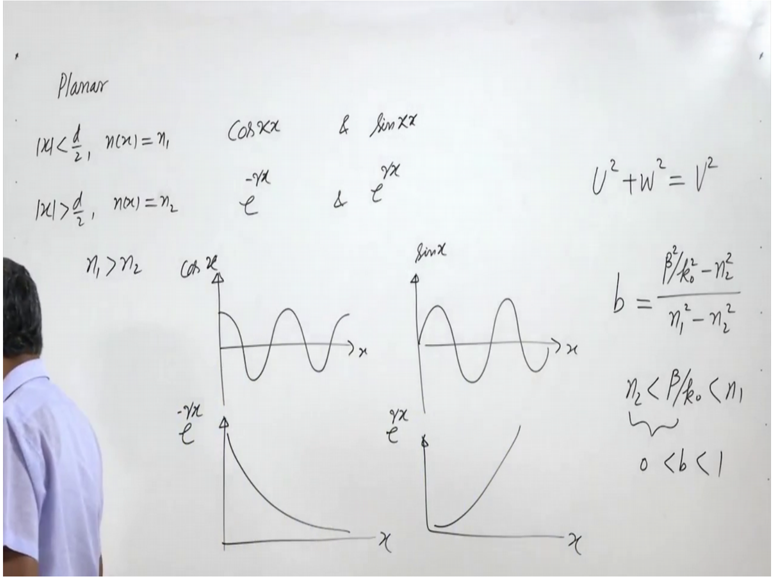

So, as we were discussing that in case of planar wave guide for mod x less than d by 2 where n x was equal to n 1, I had the solutions cosine kappa x and sin kappa x. While in the region mod x greater than d by 2 where n x is equal to n 2 I had solutions e to the power minus gamma x and e to the power gamma x. So, I had oscillatory solutions in the high index region because n 1 is greater than n 2. So, in the high index region I had oscillatory solutions and exponentially decaying solutions in the low index region so that the energies confined in the high index region.

Here I have sets of equations; you see this l can take several values l is equal to 0 1 2 3 and so on. So, for each of these equations I have each of these functions for l is equal to 0 I will have J 0, Y 0 l is equal to 1 J 1 Y 1 and so on. So, these are a kind of series of functions of family of functions J l, Y l, K l, I l; however, in this case in planar wave guide I did not have this kind of situation because the confinement was only in 1 direction here the confinement is on also in phi direction, so from there this l is coming.

If I look at this cosine function, it is simply if I have a function something like this then I call it cosine function similarly a function like this I labelled as sin function. Similarly if there is a function like this then I defined it as some exponentially decaying and a function which goes like this I define as exponentially amplifying function. So, let me treat them just labels, if there is this kind of variation I label this as cosine x if it is let me put it cosine x itself and if the variation is like this then let me label it as sin x. Similarly these Bessel functions I will see that what kind of variations they have. So, let me plot these functions let me look at these functions and just treat them as some particular variation of x.

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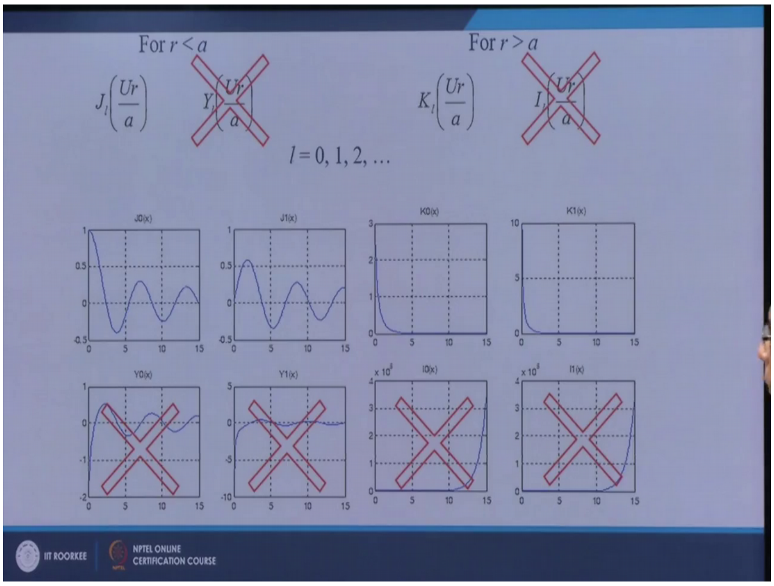

So, if now for different values of l I plot these J 0 Y 0 and J 1 Y 1 and so on. Then I find that I find that J 0 has this kind of variation. So, let me just label a function which has this kind of variation as J 0 just like the function which has this kind of variation I label as cosine x. J 1 x has this kind of variation.

So, what I see that these functions these functions have oscillatory behavior just like the cosine function. However, as I go in x direction the amplitude of oscillations it changes it is not constant as you have in cosine and in cosine functions you know that this is pi by 2, this is pi, this is 3 pi by 2, this is 2 pi and so on, but here it is not as simple as that. So, it is some complicated function, but let me say that if this is oscillatory function of this kind then it is J 0, this is J 1 if I look at y function then y function is also oscillatory it is also oscillatory, but the only thing is that it blows up at x is equal to 0, Y 0 Y 1 Y 2 and so on.

So, these are Bessel J and Bessel Y functions and if I now look at Bessel K and Bessel I functions then they look like this. So, Bessel K functions they have asymptotically decaying behavior and Bessel I functions they are exponent they are asymptotically amplifying behavior. So, now, remember that I want to find out the guided modes, which functions to choose in which region that I will have to take care of.

In the core I need oscillatory functions. So, of course, the solutions are oscillatory here, but I should pay attention that in the core I have r is equal to 0 and if I have r is equal to 0 then these functions will blow up at r is equal to 0. So, I cannot take these functions. So, I cannot include Bessel Y functions in the core in the cladding because r can go up to infinity. So, at infinity these 2 functions these I functions will blow up. So, I will have to discard I functions. So, in the core I will discard this function, in the cladding I will discard this function. So, now, I have the solutions I have the solution in the core I have the solution in the cladding.

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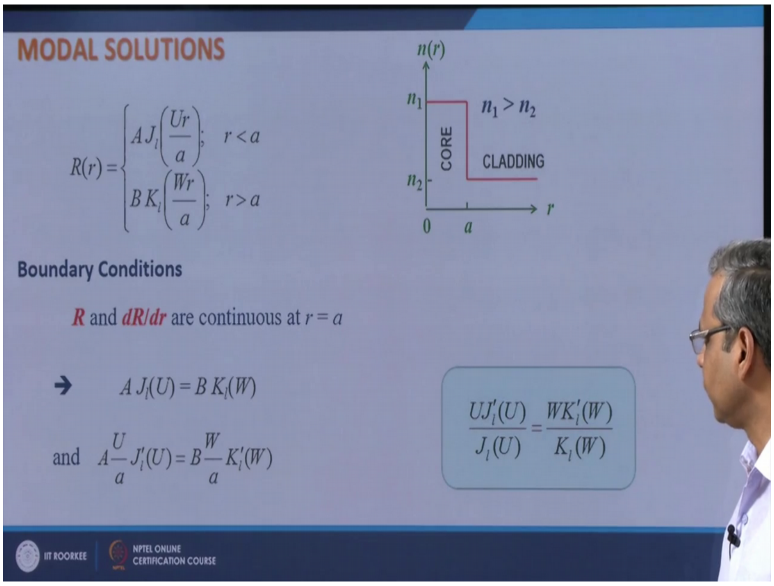

So, what are the solutions? So, if this is the fiber, then the radial part has the solution some constant a times J l Ur over a in the core and in the cladding some constant b times K l Wr over a. So, these are the solutions just like in case of planar wave guide in high index region I had A cosine kappa x plus B sin kappa x and here I had Ce to the power minus gamma x. So, similar are the solutions here also.

Next thing is now to find out the relationship between B and A and to form a transcendental equation. So, for that I apply boundary conditions what are the boundary conditions, boundary conditions are radial part R and dR over d r are continuous at r is equal to a and since it is weekly guiding fiber. So, I can take these as the boundary conditions. So, even though they are they are not exactly tangential at r is equal to a, but these are approximate, but they are quite valid quite valid for weekly guiding fiber.

So if I put these here the continuity of capital R at r is equal to a will give me A J l U is equal to B K l W and the continuity of its derivative at r is equal to a will give me A U over A J l prime U is equal to B W over a K l prime W. So, from here I can relate B to A and then I can eliminate A and B and get an Eigen value equations in beta. So, this Eigen value equation comes out to be if I divide this by this U J l prime U divided by J l U is equal to W K l prime W divided by K l W.

So, by solving this equation I can find out the modes of the fiber, but this equation has derivative in derivatives in Bessel functions. So, so either I numerically solve them using the derivatives or I can represent them in terms of Bessel functions itself non-prime functions for that I can make use of identities of Bessel functions, the Bessel functions satisfy these identities these four identities and we also make note that J minus 1 U is equal to minus J 1 U and K minus 1 W is equal to K 1 W.

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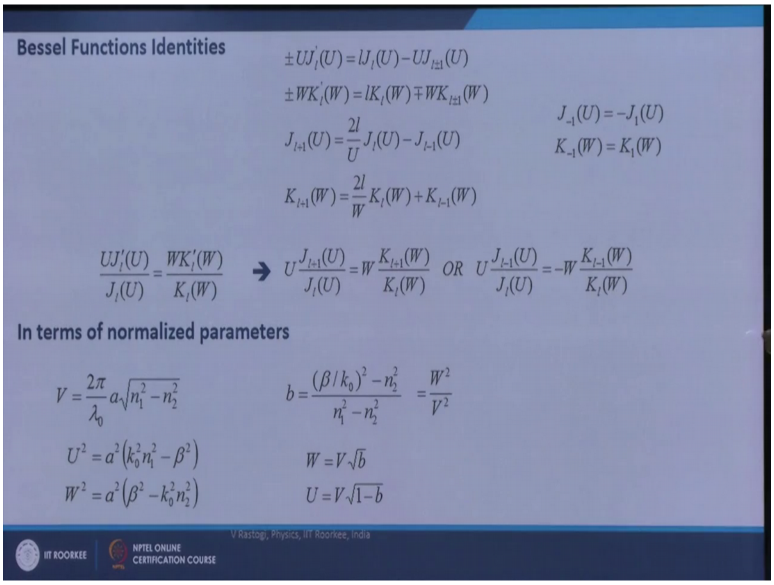

So, if I use these then this transcendental equation can now be written in 2 forms - one is U J l plus 1 U over J l U is equal to W K l plus 1 W over K l W or U J l minus 1 U over J l U is equal to minus W K l minus 1 W over K l W. So, I can get rid of these primed functions here. So, as we have been doing the representation of all these equations in a normalized parameter. So, in fiber also it is worth representing this in normalized parameters and do the analysis in normalized parameter so as to get rid of any fiber parameters and wave length in depth. So, we can have independent of fiber parameters and wave length.

So, we have already the normalized frequency which is 2 pi over lambda naught times a times the square root of n 1 square minus n 2 square, you can see that this is square root of n 1 square minus n 2 square is nothing, but n a numerical aperture. We define the normalized propagation constant exactly in the same way as we had done for planar wave guide.

So, it is beta over K naught square minus n 2 square over n 1 square minus n 2 square. And now in order to in order to represent these equations in terms of V and B I will have to I will have to find out U and W in terms of V and B. So, what is U? U square is a square times K naught square n 1 square minus beta square and W square is a square times beta square minus K naught square n 2 square which are similar to kappa square and gamma square of planar wave guide. So, from here I can immediately see that this B is nothing, but W square over V square if I take K naught square downstairs and multiply the numerator and denominator by A square then it becomes W square over V square.

So, I can immediately get W is equal to V times square root of B and I already have from previous slide that U square plus W square is equal to V square since U square plus W square is equal to V square then I immediately get that U is equal to V times square root of 1 minus B.

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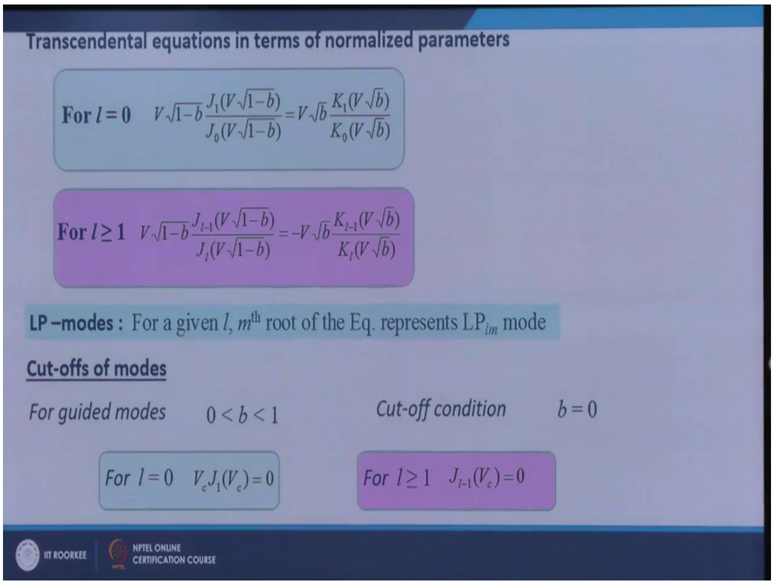

So, with the help of this I can get the transcendental equation or Eigen value equation in terms of normalized parameters and this equation I can put in 2 categories 1 is for l is equal to 0 and 1 for l greater than or equal to 1. So, for l is equal to 0 the equation becomes this V times square root of 1 minus b, J 1 V square root of 1 minus b divided by J 0 V square root of 1 minus b is equal to V square root of b K 1 V square root of b divided by K 0 of V square root of b. So, you can see on this side I will have K functions and with K functions I have associated V times square root of b and with on this side I have J function and with that is associated V times square root of 1 minus b because this is U this is W for l greater than or equal to 1 the equation becomes like this.

So, here I have J l minus 1 upstairs and J l downstairs while I have J 1 upstairs and J 0 downstairs. So, this is the only difference we should take care of and also here I have positive sign here I have negative sign. So, for any given value of l I can have the equation the Eigen value equation and I can solve it for different values of V and generate what are known as b V curves which are universal curves which are universal curves.

So, now, for any given value of l if I start solving this equation then depending upon the value of V I can have various routes and mth root of any equation for a given l will represent LP l m mode LP l m, mode l is the solution l represents the solution in phi direction, while m represents the solution in radial direction. What are the cut offs of the modes what are the cut offs of the modes well if you look back our b is defined as beta square over K naught square minus n 2 square over n 1 square minus n 2 square and for guided modes beta over K naught lies between n 2 and n 1 and beta over K naught is equal to n 2 defines the cut off. So, which means that b lies between 0 and 1 and b is equal to 0 corresponds to the cut off.

So, the cut offs of the modes are defined by b is equal to 0. So, if I put b is equal to 0 here I will get the cut offs for different values of l. So, for l is equal to 0 this is the equation and if I put b is equal to 0 then the cut offs are defined by V c J 1 V c is equal to 0 where V c are the cut offs, V c J 1 V c is equal to 0. For l is all l greater than or equal to 1 the cut offs will be defined from here which are J l minus 1 V c is equal to 0. So, from here I can get the cut offs of various modes.

If you remember in case of planners symmetric wave guide the cut offs were very simple cut offs for V c is equal to m pi by 2. So, for mth mode you have the cut off m pi by 2. So, TE 0 mode no cut off TE 1 mode pi by 2 TE 2 mode pi and so on, but here I will have to find out the 0s of these Bessel functions to find the cut offs. There, they were the 0s of ten V c and caught V c, but here they are the 0s of J 1 V c and J l minus 1 V c.

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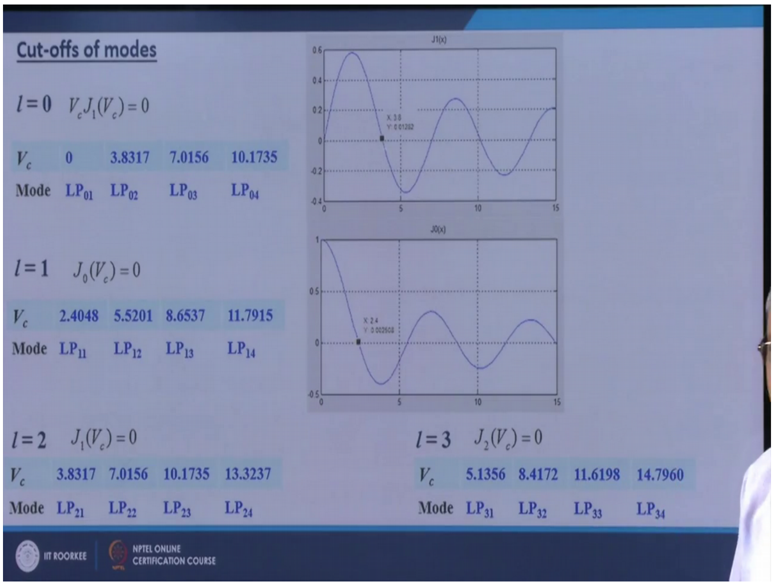

So, let us first find out the cut offs of l is equal to 0 modes cut off condition is V c J 1 V c is equal to 0. So, of course, the first root is V c is equal to 0 itself and then let us look at how J 1 varies with x. So, if I plot J 1 x then J 1 x looks like this its first 0 is at x is equal to 0 itself and that is coming from here also. So, first root is of course, V c is equal to 0. Now let us look at the second 0, second 0 appear somewhere here near 3.8 and then the third 0, four 0 and so on. So, if I list the 0s of J 1 x then they are 0, 3.8317, then here it is 7.0156 and then here it is 10.1735.

So, this is the cutoff of LP 01 more this is the first mode l is equal to 0 and first mode is m is equal to 1, so LP 01. This is m is equal to 2, so LP 02; m is equal to 3 LP 03 and so on. So, I have the cut offs of various l is equal to 0 modes here. Let us look at the cut offs of l is equal to 1 mode, l is equal to 1 mode has cut off condition J 0 V c is equal to 0 because it is J l minus 1 V c is equal to 0 and l is equal to 1. So, it becomes J 0 V c is equal to 0, you should remember that V c is equal to 0 can be the cutoff of only 1 mode and that is the fundamental mode. So, this cut off cannot be shared with any other mode; however, non 0 cut offs can be shared with other modes.

So, what are the 0s of J 0 now? Let us plot J 0 x and J 0 x goes like this and we find that the first 0 is around 2.4 and then and so on. Let us list the 0es of J 1 x J 0 x, first 0 is 2.4048, second is 5.5201, third is 8.6537 and 11.7915 and so on. So, you have l is equal to 1 m is equal to 1, LP 11, 1 is equal to 1 m is equal to 2 LP 12 and so on. So, you have the cutoff of LP 11 more this LP 12 mode, LP 13, LP 14. So, all LP 1 m modes you have here.

Now let us look at l is equal to 2 l is equal to 2 since the cut offs are J l minus 1 V c is equal to 0. So, this becomes J 1 V c is equal to 0 for l is equal to 2. So, it is the same as this one, but I will have to exclude this V c is equal to 0 here because this cut off cannot be shared with any other mode. So, now the cut offs are 3.8317, 7.0156 and so on. For l is equal to 2 modes, so this will correspond to m is equal to 1, so LP 21 mode, LP 22, LP 23, LP 24 and so on. l is equal to 3 will give you J 2 V c is equal to 0 and these are the 0s of J 2. So, you will have these cut offs for various l is equal to 3 modes. So, in this way for whatever value of l I want then I can have the cut offs.

How to arrange these modes? I have so many mode 01, 02, 03, 11, 12, 13, 21, 22, 23, 31, 32, 33 and so on. In what order their propagation constants are arranged increase or decrease in case of planar wave guide it was very simple TE 0 has the highest propagation constant then TE 1 then TE 2 then TE 3 and so on. So, the propagation constant they decrease with mode number, but here it is not obvious and also the cut offs you can see the numbers the numbers are weird. So, I cannot fit any pattern there in planar wave guide it was very simple TE 0 mode 0 TE 1 mode pi by 2 TE 2 mod pi. So, m pi by 2 as more number increases. So, the cut offs are every pi by 2 there is a cut off of the higher order mode the next mode, but here I will have to really look into these numbers and then put them together. So, I will have to arrange them in order of their cut offs.

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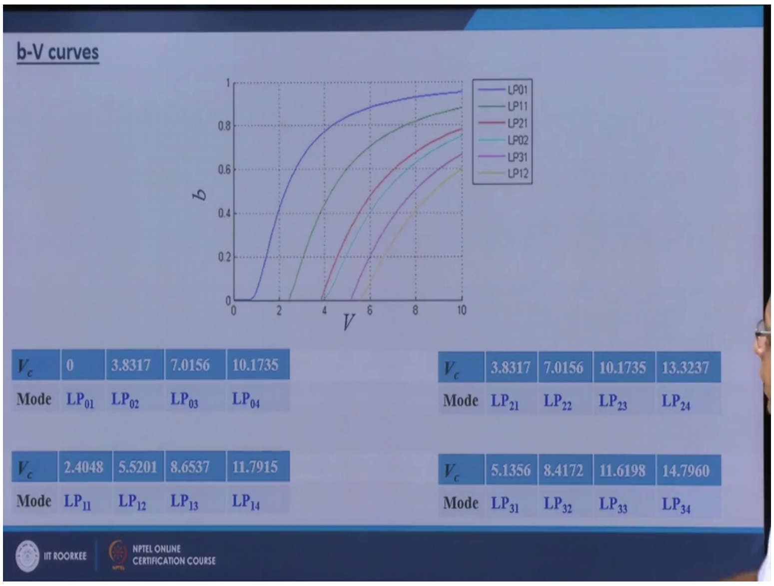

Then we can generate we can generate b V curves. So, so I choose l is equal to 0 and solve the transcendental equation corresponding to l is equal to 0 and list the highest root always list the value of highest root and plot the value of highest root as a function of V then it will go like this, this is LP 01 mode the highest root. The first one corresponding to l is equal to 0. So, this is LP 0 1 mode and this is the fundamental mode this is the fundamental mode. It is cut off is 0, so this is always guided this is always guided and remember that we are doing the scalar analysis under weekly guiding approximation. So, there they are LP modes in terms of vector mode they are different in terms of vector mode they are different. So, we are not going into the details of vector modes we will confine ourselves to the scalar modes in this course.

Now what I see that the cut off the next cut off appears for LP 11 mode. So, this is 0 the next number is 2.4048 which corresponds to LP 11 mode. So, now I plot the propagation constant of LP 11 mode that is the highest root corresponding to l is equal to 1 which is

LP 11. So, I plot the propagation constant of that with respect to V and it goes like this. So, this is LP 11 mode and it is cut off at 2.4048.

If I look for the next number I find the next number is 3.8317 and it appears twice here for LP 02 and LP 21. So, both will start from here at 3.8317 and then if I solve the transcendental equation corresponding to these modes then the propagation constant vary like this. So, LP 21 mode goes like this and LP 02 mode go goes like this similarly the next 1 is LP 31 and the next one is LP 12. So, I can see that a fiber would never guide 3 modes either there will there would be 1 2 or 4, 1 2 or 4 because these 2 modes share the cut off, these 2 modes share the cut off. So, these are b versus V curves and the cut offs of various modes.

Now if I want to find out how many modes are guided then it is not as easy as it was in case of planar wave guide that I just calculate the value of V divided by pi by 2 get the number and find the closest, but greater than the integer for corresponding to that number, but it is not that easy here. Here what I will have to do? If I have a fiber and wave length I calculate the value of V and then find out what modes are guided and what modes are cut off looking in to these numbers, looking in to these numbers and then only I shall be able to find out the find out the number of modes.

So, in the next lecture I will look into the modal fields and I will work out some examples also on number of modes and what happens if the value of V is very large then what are the approximate number of modes.

Thank you.