Fiber Optics
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Lecture - 19
Optical Fiber Waveguide- I

After having understood the light propagation in planar waveguides in the previous section; now in this section we will analyze light propagation in an optical fiber- the modes of light propagation in an optical fiber.

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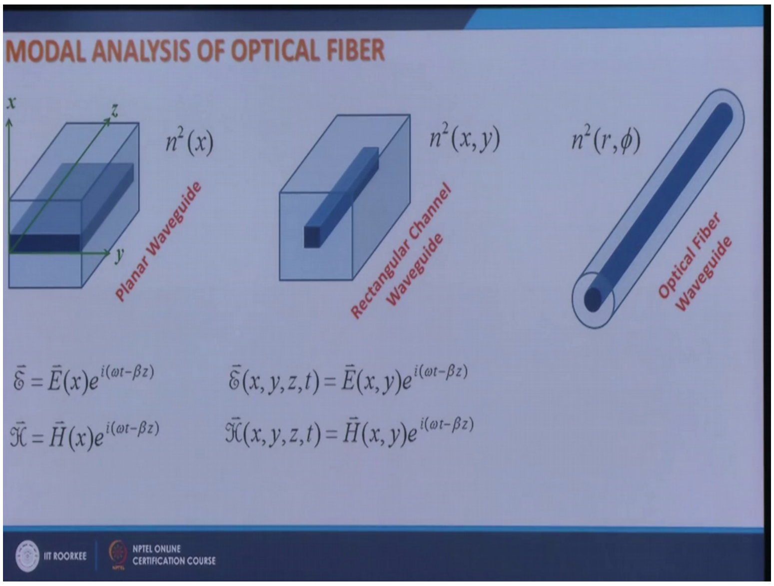

So, if I look back to planar waveguide where the refractive index variation is only in one direction x and the propagation direction is z, then I had seen that the modes are given by these E and H E is equal to E of x e to the power i omega t minus beta z. And H is equal to H of x e to the power i omega t minus beta z.

You remember that these E and H are not constants, if they are constants then they are plane waves. But here E is a function of x and H is a function of x, so they are not plane waves propagating in z direction. But we had seen that these are the superposition of two plane waves: one going in plus x z direction and another going in minus x z direction. And that gives us a standing wave pattern in x direction which flows in z direction. So, they are the modes.

If I look at rectangular channel waveguide, so now if I have confinement in x as well as in y, so refractive index variation in x and y both. So, this is the kind of profile. Then the modes would be given by E is equal to E of x y e to the power i omega t minus beta z and H is equal to H of x y e to the power i omega t minus beta z. So, these E and H would now be the functions of x and y both. So, the confinement is in both the directions x and y, and those fields will propagate in z direction with certain propagation constant beta.

If now I convert this into a cylindrical geometry then it becomes an optical fiber waveguide. I can represent the refractive index profile in x and y; n s square x and y, but because of cylindrical geometry it is much more easier to analyze the structure if I represent this refractive index profile in cylindrical coordinates: so r phi. So, r phi are the transfers coordinates and z is the longitudinal along which the light is propagating.

So, now what would be the modes of this kind of a structure? Well, they can be given by E r phi e to the power i omega t minus beta z and H are phi e to the power i omega t minus beta z; so this E which is a function of r and phi, so this particular function propagates in z direction with certain propagation constant beta. So these are the modes.

In all the three cases what I observed is the direction of propagation z is common. So, I would like to make use of this to analyze optical fiber. Let me go back to the modes of the planner waveguide.

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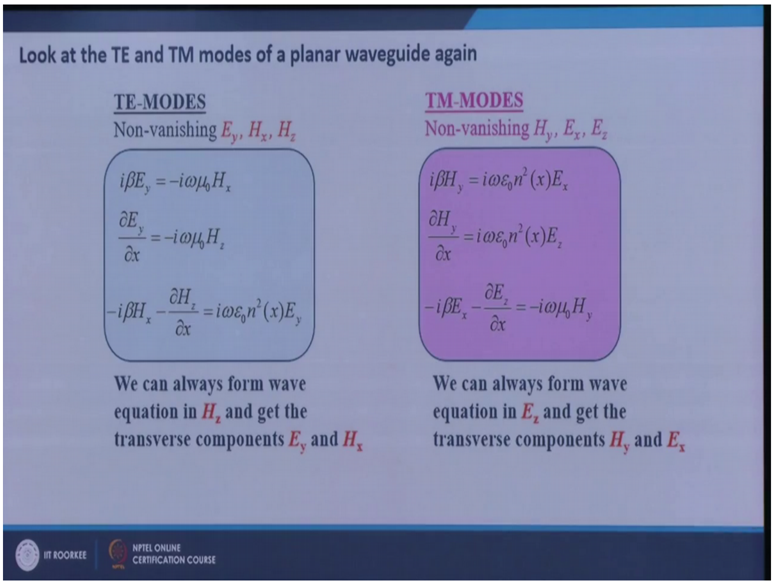

So, I had TE-modes with non-vanishing components E y H x and H z and TM-modes with non-vanishing components H y E x and E z. And these three equations relate these three non-vanishing components here, and these three equations relate the non-vanishing components of TM-modes. So what I have done actually, I found out E y and then I can get H x and H z from these equations. So, once I know E y I had formed the differential equation in E y solved the equation for E y got the modal fields. So, once I get E y then from there I could get H x and H z.

Similarly here I get H y first and then E x and E z and that is how I have the complete solution. But, as I have seen in the previous slide that the common thing in all the three structure is the longitudinal direction z. Can I utilize this here? Well, if instead of solving for E y and then getting H x and H z from here I can also do another thing that I first form the equation in H z solve it for H z and then get E y and H x.

Here what I notice is that the only longitudinal component here is H z, and E z is 0. And in this case I can do the same thing, and here I notice that the only longitudinal component is E z and H z is 0. So, I can define my TE-modes in such a way that the fields for which E z is equal to 0 our TE-modes and if I have H z is equal to 0 then they are TM-modes. If E z is equal to 0 then they are TE-modes and if H z is equal to 0 then they are TM-modes.

Now let us look at an optical fiber.

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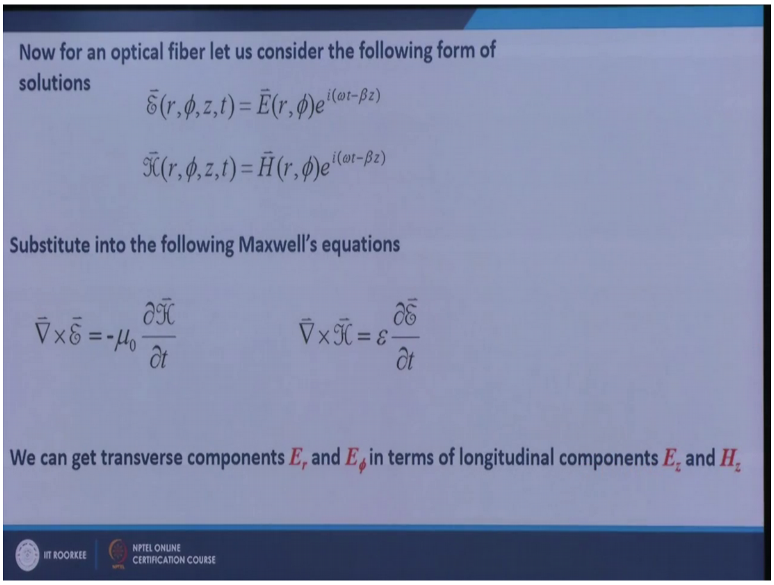

So, in optical fiber I have these E and H, if I substitute these solutions; if I substitute these into Maxwell's equations del cross E is equal to minus mu naught del H over del t and del cross H is equal to epsilon del E over del t then of course I will get 6 equations.

The only thing is that I should now use cylindrical polar coordinate system r phi and z and see what these 6 equations are. So, these 6 equations will give me the transverse components E r and E phi in terms of longitudinal components E z and H z.

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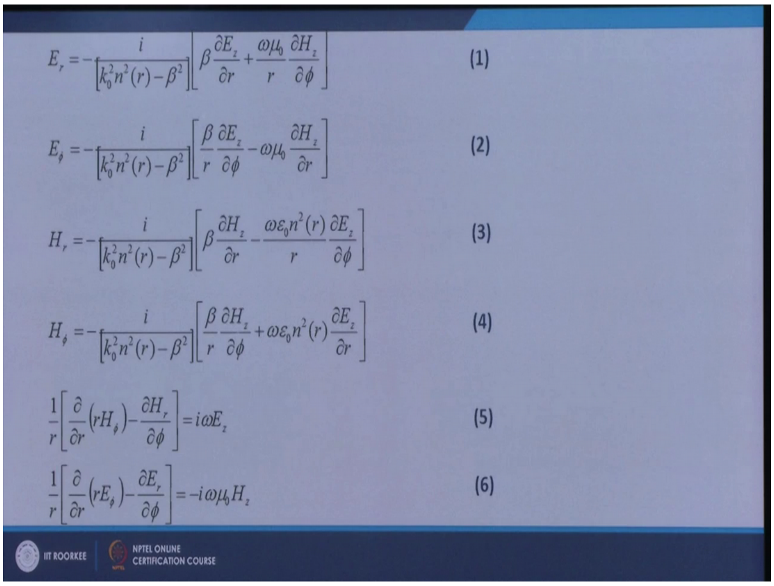

So, these are the 6 equations which I get from those to Maxwell's equations.

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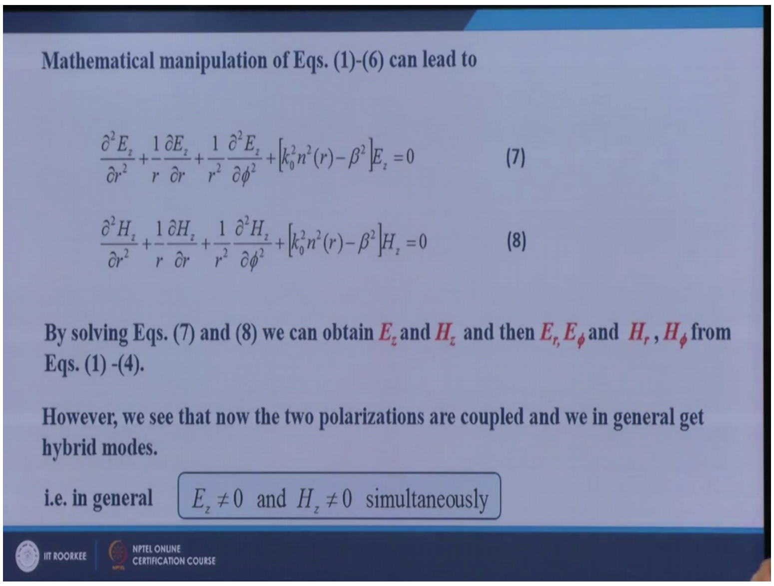

What I see here is that these mathematical equations from 1 to 6 these equations if I do mathematical manipulation of these 6 equations then they can lead to these two equations: one in E z and one in H z. So, I can form a differential equation in E z or H z. Solve these equations and then get E r, E phi, H r, H phi from these E z and H z using those 6 equations.

However, what I see that if you look back to those 6 equations; let me look back to those 6 equations. I cannot find an instance where I can have only E z or only H z; that is I am not able to find out where E z is not equal to 0 and H z is not equal to 0 at one time. I have that simultaneously both are nonzero; which means that now the polarizations are coupled, I cannot separate two orthogonal polarizations. So, two polarization are coupled and in general what I will get I will get hybrid modes. I cannot get TE polarization and TM polarization in general.

That I could be able to do in case of planar waveguide, that this is TE polarization and this is TM polarization, but here in general it is difficult. So, E z naught is equal to 0 and H z naught is equal to 0 simultaneously are there, so I cannot decouple to polarizations.

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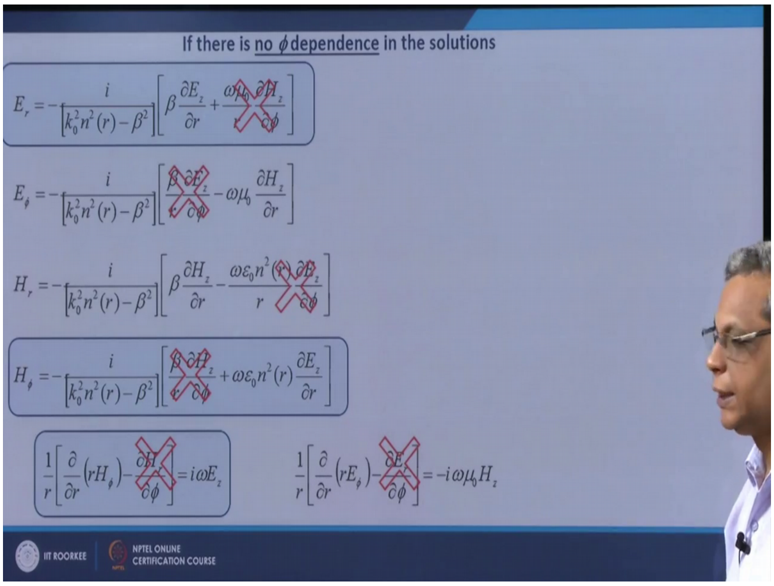

However there is one particular case where I can still have this de coupling. And this case is when there is no phi dependence in the solutions; when there is no phi dependence in the solution then let us see what happens. So, if there is no phi dependence then I put these del phi terms 0 everywhere. So, these del phi terms are not there then I can see that in these three equations I have only E z: E z, E z, E z, and there is no H z. So, in these three equations if I considered these three equations then H z is equal to 0. And in these three equations there is no E z, so E z is equal to 0.

So, only for this case I can separate out the two polarizations; I can de couple two polarizations TE and TM.

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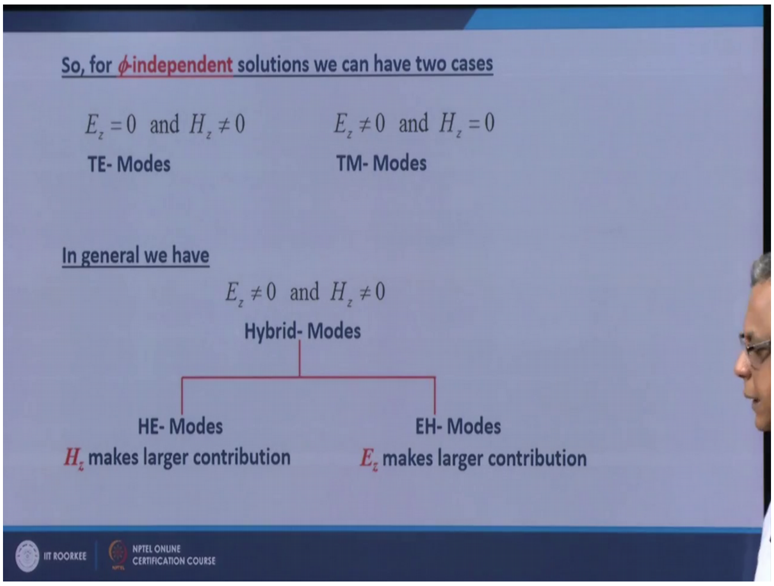

So now, for phi independent solutions we can have two cases. One is E z is equal to 0 and H z is not equal to 0, which is nothing but the case of TE-modes. And another is when E z is not equal to 0 and H z is equal to 0, which is nothing but the case of TM-modes. So, in this way I can separate out two polarizations. But in general what I will have? I will have E z not equal to 0 and H z not equal to 0 simultaneously.

So, E z not equal to 0 H z not equal to 0 simultaneously it means in these kinds of modes two polarizations will be there and they are called hybrid modes. In these hybrid modes there can be two instances: one is when H z makes larger contribution than E z, then we label them as HE-modes, and when E z component makes larger contribution than H z then we label them as EH-modes. So, in fiber I can have TE-modes, TM-modes HE-modes, EH-modes, ok.

And these modes are also known as vector modes of the fiber.

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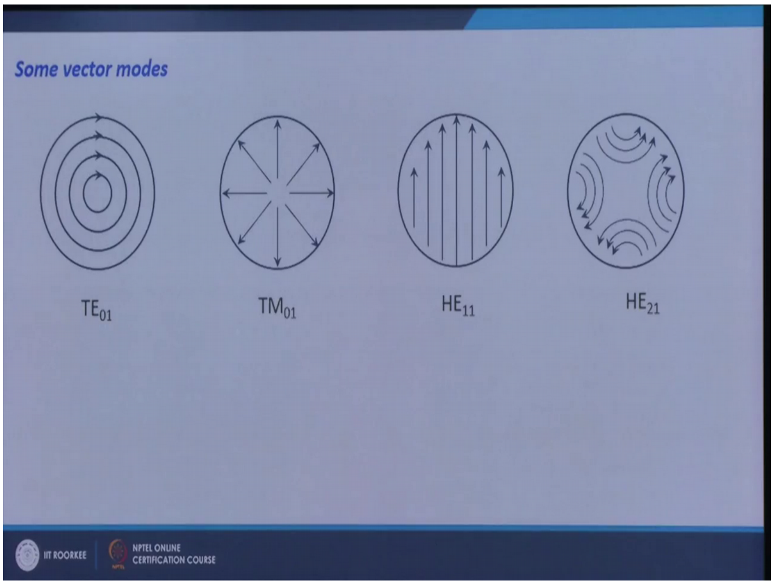

Here I have plotted some typical vector modes of the fiber and their electric fields. So, this is a typical TE 01 mode, this is TM 0 mode, this is HE 11 mode, this is HE 21 mode.

But the fiber that we practically use has a small index contrast. The index contrast between the core and cladding.

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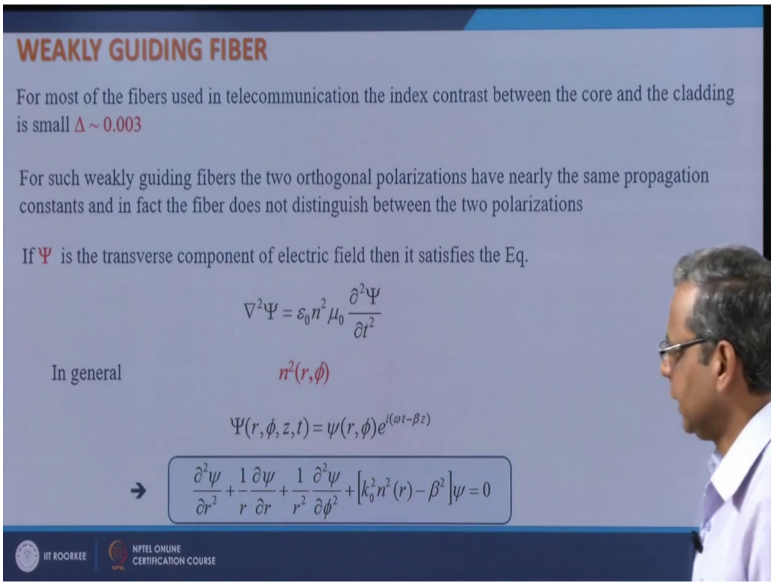

The relative index difference between the core and the cladding is typically 0.3 percent which is very small. And if you look back to the analysis of your symmetric planar waveguide or even asymmetric planar waveguide: we see that if the index contrast between the high and low index regions is small then the propagation constants of TE and TM polarizations they merge. It is very difficult to distinguish between TE and TM polarization in terms of their propagation constant if index contrast is a small.

So, here we use that fact that if index contrast is a small then the fiber will not distinguish between this polarization and this polarization. So, in such kind of fibers we can still have the modes which are linearly polarized. That is if one is polarized like this then another has to be like this or if there is any arbitrary polarization then that arbitrary polarization can always be represented as the superposition of this and this two orthogonal polarizations: horizontal and vertical.

So, in case of weekly guiding fiber what we have are linearly polarized modes and we can have two orthogonal polarizations which have nearly the same propagation constants. And now if I say that psi is the transverse component of electric field so you have a fiber and psi is the transverse component of the electric field then it satisfies the equation: del square psi is equal to epsilon naught n square mu naught del 2 psi over del TE square, this psi is capital psi so it is a function of all the spatial coordinates as well as time.

In general this n s square is a function of r and phi. And if this is a function of r and phi then I can in general write this capital psi which is a function r phi z and t as psi small psi of r phi e to the power i omega t minus beta z, because n square is not a function of z. So, z part can be separated out. And this is how the solution of z part and t part will come out. We have seen this several times.

So now, we want to find out this psi r phi. This function, this function, and see how this propagates. This function psi r phi represents the mode which travels with propagation constant beta. So, the problem reduces to find out this now.

For that what I do? I put this capital psi back into this wave equation and when I do this then this equation becomes this: del 2 psi over del r s square plus 1 over r del psi over del r plus 1 over r s square del 2 psi over del phi square plus k naught square n square of r or r phi minus beta square psi is equal to 0.

In this course we will confine, we will limit our discussion only to refractive index profiles which do not depend upon phi. So, we will consider only n square of r and not n square of r phi.

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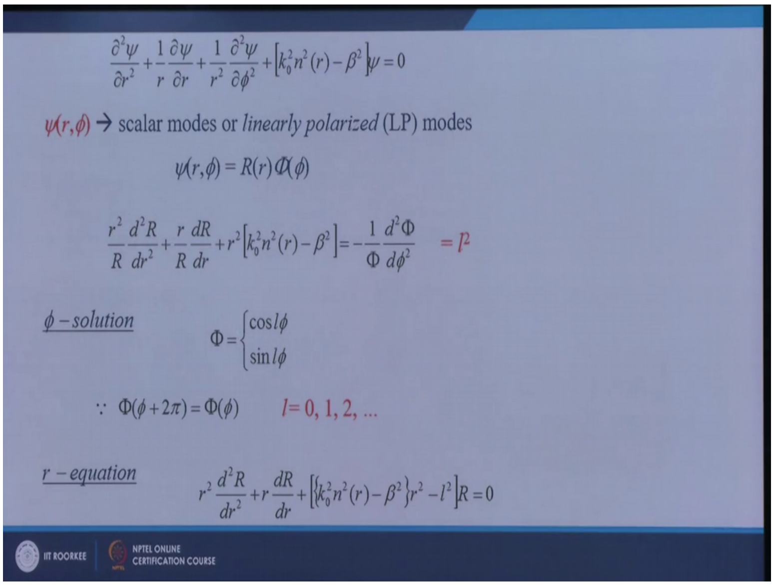

So, now this is the equation. And if n square is a function of r only then this solution psi r phi can be separated out in r and phi. I can separate out of r part, and I can separate out phi part; and this psi r phi as I have said earlier that they are now the scalar modes or linearly polarized modes.

So, since n square is a function of r only naught of phi, so psi r phi can be written as R of r and capital phi of phi. And if I now put this back into this equation then I can separate out r apart from phi part. So, I get r square over capital R d 2 R over dr square plus small r over capital R d capital R over d small r plus r square k naught square n square r minus beta square is equal to minus 1 over phi d 2 phi over d phi square. So, I have separated them out, the usual procedure now is to equate it to some constant. And since this is second order equation, so I will take the constant in the form of the square.

So, I take it as l square. So, first let me solve this phi part and the phi solution comes out to be cosine l phi and sin l phi. I make the use of the fact that in phi direction the solutions are periodic. So, you start at some phi and then you make a round of 2 phi you come back to the same point. So, capital phi at phi plus 2 pi is same as phi at phi. So, that there is only single solution, then this restricts the values of l to only integer values. So, l can only be 0 1 2 and so on.

So, this is the phi solution. Now for each value; and we see that in phi direction now the solutions are discrete, the solutions are discrete because of this condition. So, not all the values of l are allowed, only certain discrete values of l are allowed which are integer values. Now for each of these integer values of l I can find out the solutions r. So, let me now look at r part.

So, the r equation is r squared d 2 R over dr square plus r dR over dr plus k naught square n square of r minus beta square times R square minus l square times R is equal to 0. So, this is the r equation.

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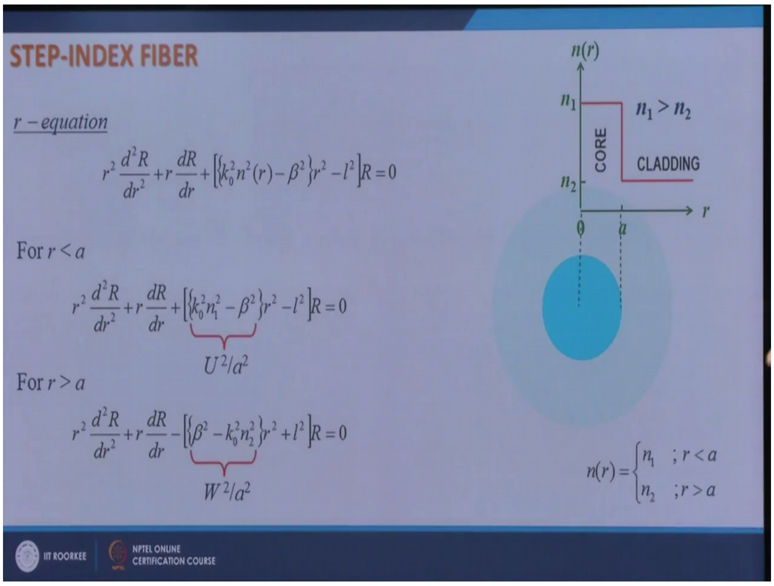

And I can solve this r equation for a given n square of r. What is the n square of r? I am going to take in this course, I will again limit myself to step index fiber and that to only two layer fiber; so which has only two regions. So, n r is equal to n 1 when r is less than a, and it is n 2 when r is greater than a. So, this is the core, this is the cladding: core has refractive index n 1 and radius a. And this is infinitely extended cladding, ok. So, this is the r equation and this is n of r.

The procedure to solve this equation is exactly the same as we have been doing for planar waveguides. What we will have to do? I will have to write this equation in this region, in the core, and in the cladding. So, in the core which is defined by the region r less than a this equation becomes this. And for r greater than a which is the cladding region the equation becomes this.

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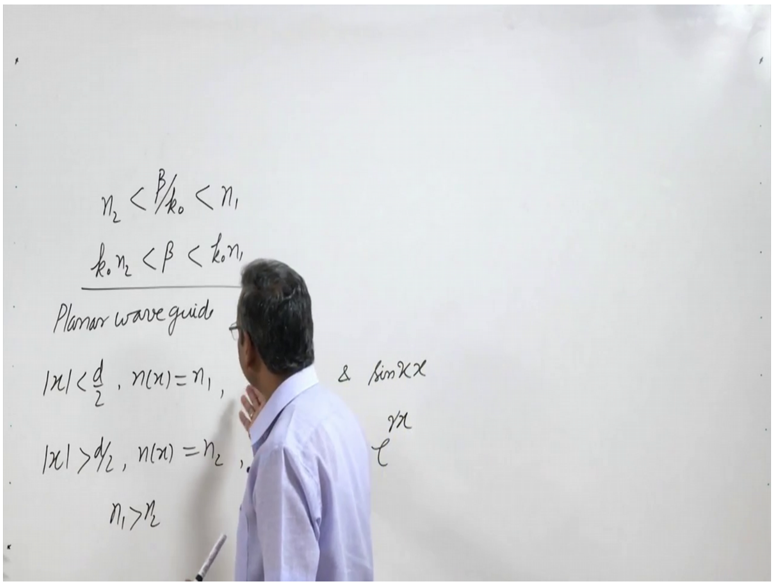

We should pay attention again that for guided modes our beta over k naught should lie between n 2 and n 1. So, beta lies between k naught n 2 and k naught n 1.

So, that is how I have arranged the terms here in order to have this quantity positive. So, I have plus sign here and k naught square n 1 square minus beta square, and a negative sign here and beta square minus k naught square n 2 square here. So now, I have these two equations which I have to solve to find out the values of beta for any given value of l: l can be 0 1 2 3 and so on.

So, let me define this k naught square n minus square minus beta square as some U square over a square and this as some W square over a square.

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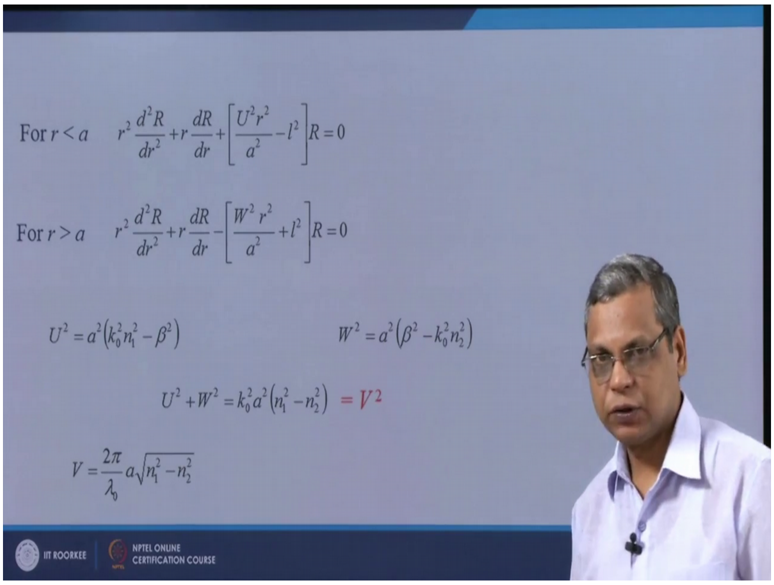

So, these equations now become r square d 2 R over dr square plus r dR over dr plus U square r square over a square minus l square times R is equal to 0 in the core. And in the cladding r square d 2 R over dr square plus r dR over dr minus W square r square over a square p l square times R is equal to 0. So, now I have these two equations, I have to solve them.

If I have a little background of mathematical physics then I can immediately see that these are Bessel's equations and solutions of these equations are Bessel functions. So, where U square is this and W square this. So, I am going to get the solutions of these, but before that I observe the fact that if I do U square plus W square then it becomes k naught square a square times n 1 square minus n 2 square. And by this time I am able to recognise this kind of term very well, this is nothing but the normalised frequency V. So, this is V square.

So, in case of optical fiber I have v is equal to 2 pi over lambda naught times a is the radius which is half the diameter times n 1 square minus n 2 squares square root. This is normalised frequency. And again it contains all the fiber parameters and the wavelength.

Let us come back to the solutions of this.

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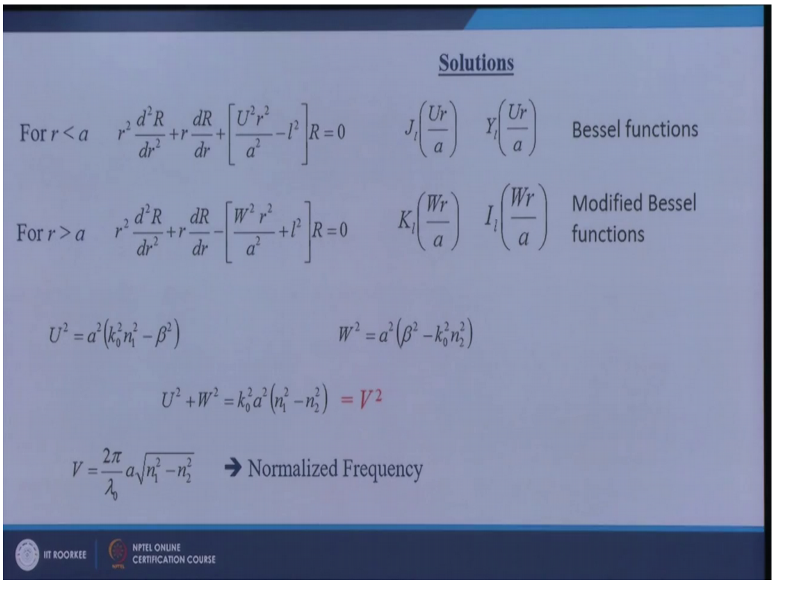

The solution of this equation is given by J l Ur over a; in terms of J l Ur over a and Y l Ur over a which are Bessel functions. And of this equation K l Wr over a and I l Wr over a which are modified Bessel functions, ok.

So, if you go back to your planar waveguides I had in the region mod x less than d by 2 where n x was n 1, I had the solutions cosine kappa x and sin kappa x. And in the region mod x greater than d by 2 where n x was n 2. And of course, n 1 was greater than n 2. I had solutions in the form e to the power minus gamma x and e to the power plus gamma x.

So, I had oscillatory solutions in mod x less than d by 2 and exponential amplifying decaying solutions in this. Here instead of sin cosine I have some J l Y l, instead of e to the power minus gamma x and e to the power plus gamma x I have some K l I l. So, what these solutions are, what these functions are, and how do I understand them and then how do I proceed further I will see in the next lecture.

Thank you.