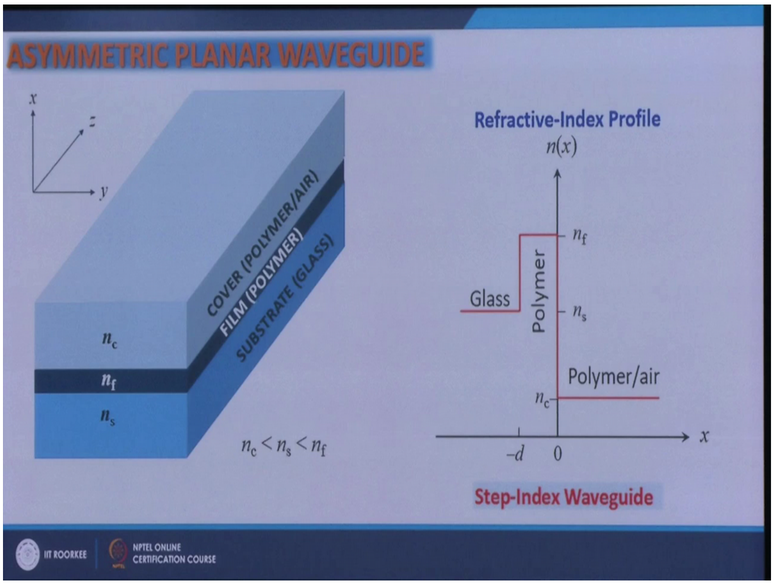
Fiber Optics
Dr. Vipul Rastogi
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 18
Electromagnetic Analysis of Waveguides- VIII

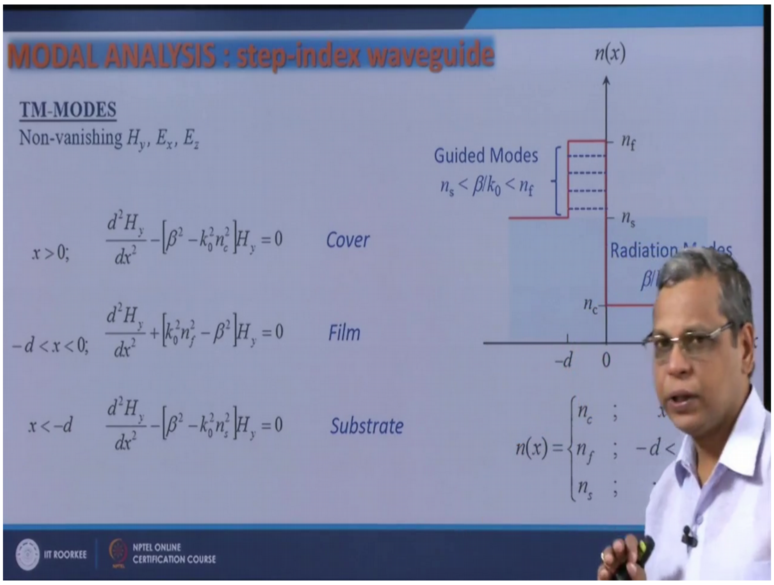
In the last lecture we had carried out the analysis of asymmetric planar waveguide for TE-modes. In this lecture we will look into the TM-modes of the waveguide.

(Refer Slide Time: 00:33)



So this is the waveguide again, and we will confine our analysis only to step index waveguides. So, this is the step index asymmetric planar waveguide.

(Refer Slide Time: 00:46)

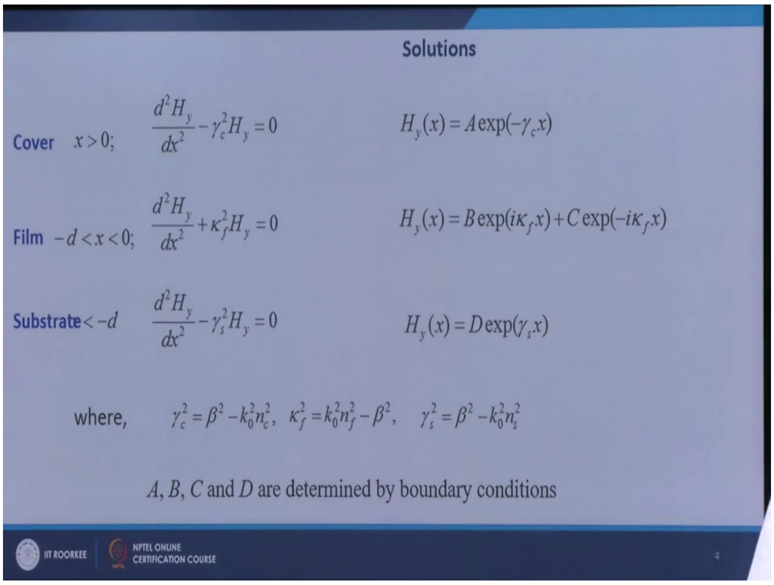


So, for TM-modes the non-vanishing components of electric and magnetic fields for this refractive index profile and z propagation are H y, E x and E z.

Again for guided modes the beta over k naught should lie between n s and n f, and if the beta over k naught is below n s then field radiates out. So, they are radiation mode. So, I again write down the wave equation in three different regions: the cover, the film, and the substrate. And now the equations are in the form d 2 H y over dx square minus beta square minus k naught square n c square H y is equal to 0 for the cover. And similarly in the film this is the equation, and in the substrate this is the equation.

What I see that these equations are exactly the same. These equations are exactly the same in TE case these equations were in E y and now these equations are in H y. Again the definitions of gamma c kappa f and gamma s are the same.

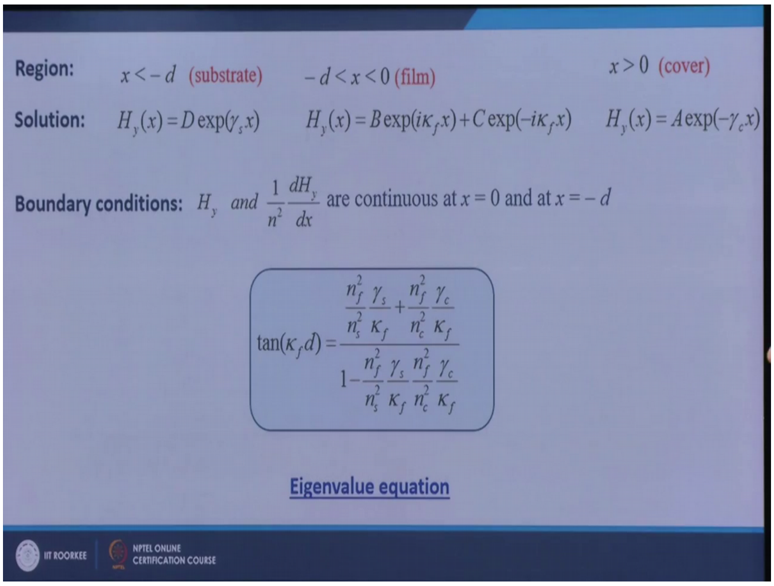
(Refer Slide Time: 02:07)



And if I write the solutions the solutions in the cover region is H y of x is equal to A E to the power minus gamma c x. In the film it is B E to the power i kappa f x plus C E to the power minus i kappa f x. In the substrate it is D E to the power gamma s x, where gamma c kappa f and gamma s are defined by these expressions.

Again I will have to apply the boundary conditions at the interfaces x is equal to 0 and x is equal to minus d; to obtain the relationships between A B C and D, and to obtain the eigenvalue equation.

(Refer Slide Time: 02:52)

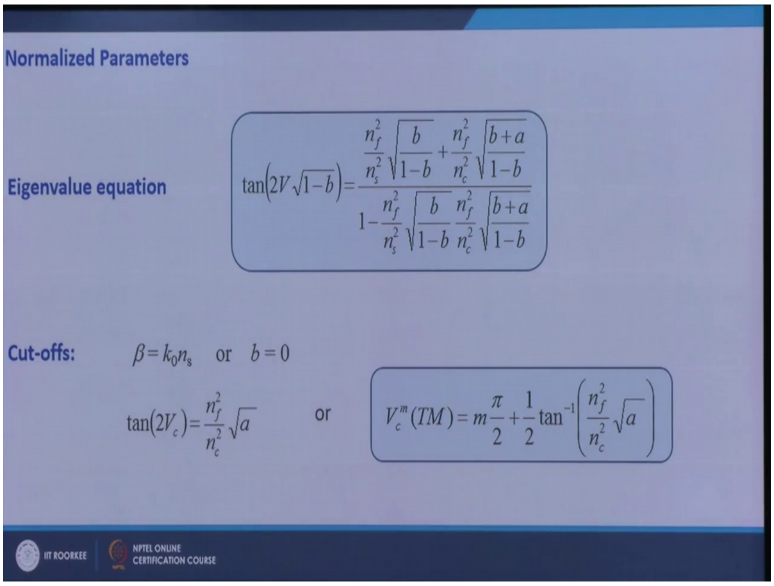


So, if I do this I again write down the solutions in different regions. And now apply the boundary conditions, remember again the boundary conditions are the tangential components of E and H are continuous. So, these boundary conditions now become H y and 1 over n square d H y over dx are continuous at the interfaces x is equal to 0 and at x is equal to minus d

So, when I now apply these boundary conditions to these fields and do mathematical manipulations similar to what we had done in the case of TE-modes I get the eigenvalue equation as this. The only difference is now these factors of n f square over n s square associated with the substrate term, n f square over n c square associated with the cover term. So, these extra factors are there otherwise the equation is similar.

So, this is the eigenvalue equation, after solving this I can find out the modes, the propagation constants of the modes and later on the fields.

(Refer Slide Time: 04:17)

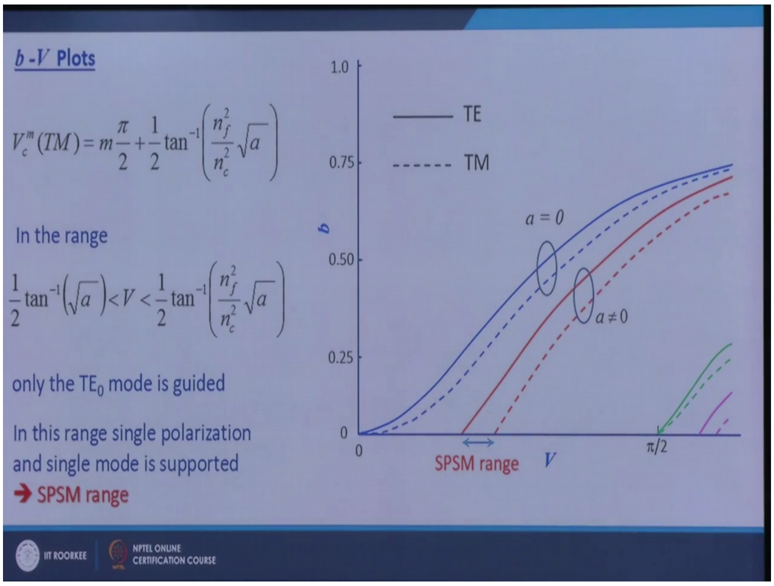


In normalized parameters if I define this equation then it becomes tan 2 V square of 1 minus b n f square over n s square square root of b divided by 1 minus b plus n f square over n c square square root of b plus a over 1 minus b divided by 1 minus n f square over n s square times square root of b over 1 minus b and n f square over n c square times square root of b plus a over 1 minus b.

What are the cut offs? The cut offs are again defined by b is equal to 0. So, if I put b is equal to 0 here the equation define the cut offs for TM-modes are now tan 2 V c is equal to n f square over n c square times square root of a. So, if I compare it with the cut offs of TE-modes then I have this extra factor of n f square over n c square here. So, my cut offs for m-th TM-modes are now given by m pi by 2 plus half tan inverse of n f square over n c square square root of a.

I can see from here that since n f is larger than n c then these cut offs of TM-modes are larger than the cut offs of corresponding TE-modes.

(Refer Slide Time: 05:52)



So, let me now plot b V curves here by solving the transcendental equation for different values of V and for the given value of a.

So here, I have plotted for two cases: one is for symmetric waveguide and another is for asymmetric waveguide. For comparison I have also plotted the b V curves for TE-modes. So, the b V curves for TE-modes are given by solid line, for TM-modes they are given by dashed line. If I look at these curves now, for a is equal to 0 this is TE 0 mode, this is TM 0 mode. Solid line is TE 0 mode dashed line is TM 0 mode, and I see that both the modes have the same cut off.

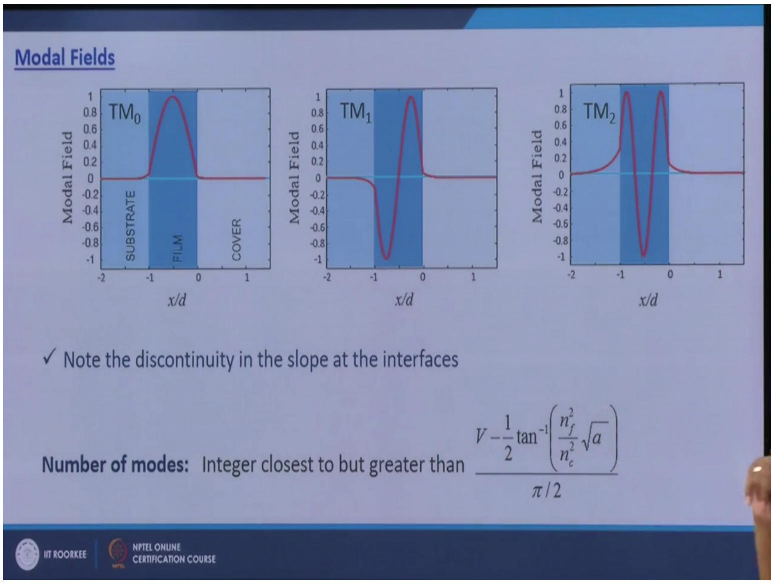
But if I take a naught equal to 0 that is I consider the case of asymmetric waveguide then the TE 0 mode has cut off somewhere here, and TM 0 mode has cut off somewhere here.

And I can see that in this range, so this cut off point is defined by half tan inverse of square root a and this cut off point is defined by half tan inverse n f square over n c square square root of a.

So, in this region if the value of V lies in this region then only TE 0 mode is guided and all the other modes included in TM 0 are cut off. So, in this range I guide strictly one mode and that mode is TE 0 mode. Or rigorously speaking in this range I guide only one mode and one polarization only TE polarization. So, this range is also called SPSM range- Single Polarization Single Mode range.

Let us look at the modal fields.

(Refer Slide Time: 08:11)

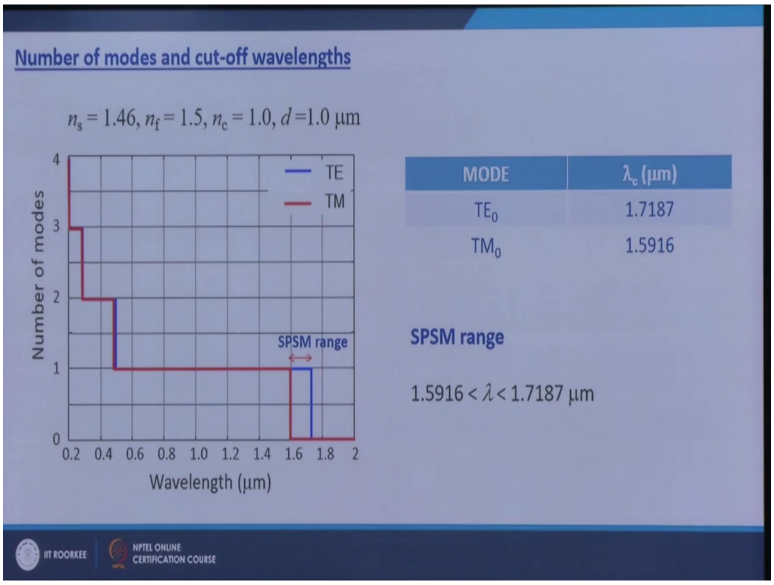


Modal fields are similar but there is one difference we should mark, and that difference is discontinuity at on the slope at the interfaces. Because H y is continuous so there is no discontinuity in the field, but d H y over dx is not continuous at the interface. So, there is discontinuity in the slope at the interfaces.

The number of modes: how many modes are guided? Well, now all the modes are shifted by this much amount half tan inverse of n f square over n c square square root of a. So, the number of modes would now be an integer closest to but greater than this number.

Let us look at how the number of TM-modes vary when we change wavelength and compare it with the TE case also.

(Refer Slide Time: 09:19)

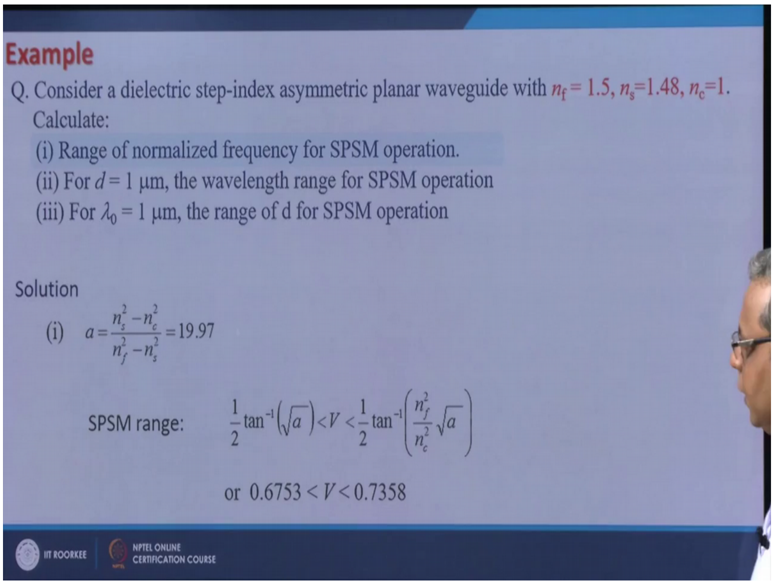


So, here I have plotted the number of modes as a function of wavelength, ok. If I find out the cut off wavelength of TE 0 mode then for these waveguide parameters, then it comes out to be 1.7187 micrometre. If I now find out the cut off wavelength for the same waveguide for TM 0 mode then it comes out to be 1.5916. So now, if I again start from a wavelength two micron and start decreasing the wavelength, then as I cross 1.7187 as I cross 1.7187 TE 0 mode starts appearing this blue one; TE 0 mode starts appearing but TM 0 is still not there. And as soon as I go below 1.5916 then TM 0 also starts appearing.

So, in this range only T 0 mode is there and this is SPSM range, while if I go below this value then I have both TE 0 and TM 0. Similarly if I go below this then TM 1 will also start appearing. So, this is the range of wavelength 1.5916 to 1.7187, in this wavelength range I have single polarization single mode operation of the waveguide.

Let us workout few examples here.

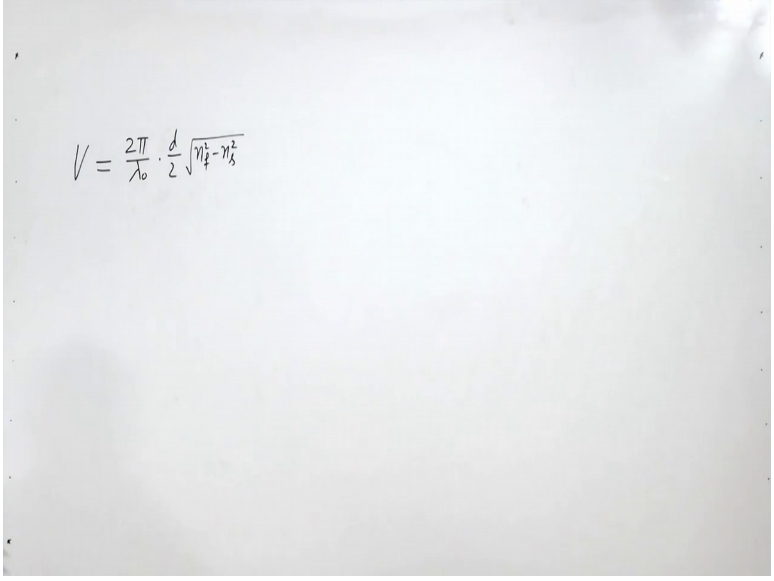
(Refer Slide Time: 11:09)



Let me consider a dielectric asymmetric planar waveguide with n f is equal to1.5 and n s is equal to 1.48 and n c is equal to 1.

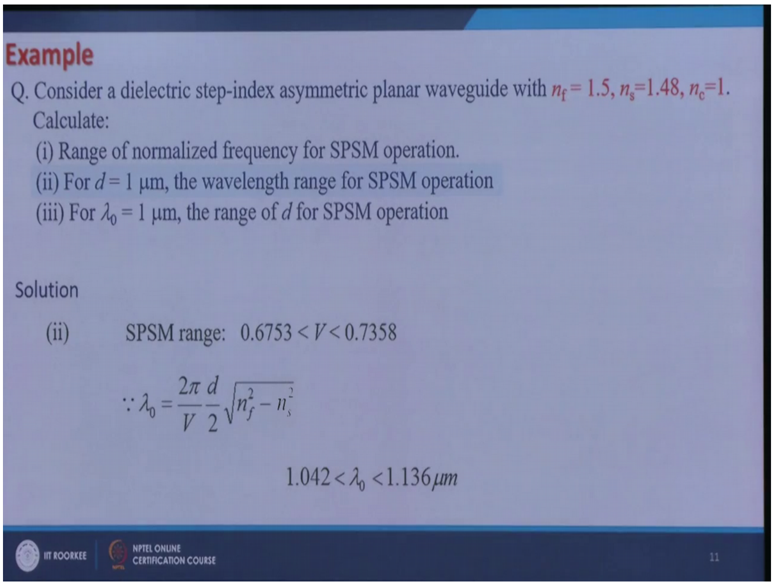
Now, I want to calculate the range of normalized frequency for SPSM operation. So, I know that SPSM operation involves asymmetry parameter a, so first I calculate a for this waveguide and a comes out to be about 20. Then SPSM range is given by half tan inverse square root a smaller than V smaller than half tan inverse n f square over n c square square root of a. So now, if I calculate these then this range comes out to be 0.6753 less than V less than 0.7358. So, this is the range of normalized frequency V for SPSM operation.

(Refer Slide Time: 12:18)



And remember that the definition of v which I have used is 2 pi over lambda naught times d by 2 times square root of n f square minus n s square. And I would like to bring out that in the text books in several textbooks the definition of V is 2 pi over lambda naught times d times n f square minus n s square, while I use d by 2. So, there would be a factor of 2.

(Refer Slide Time: 12:55)

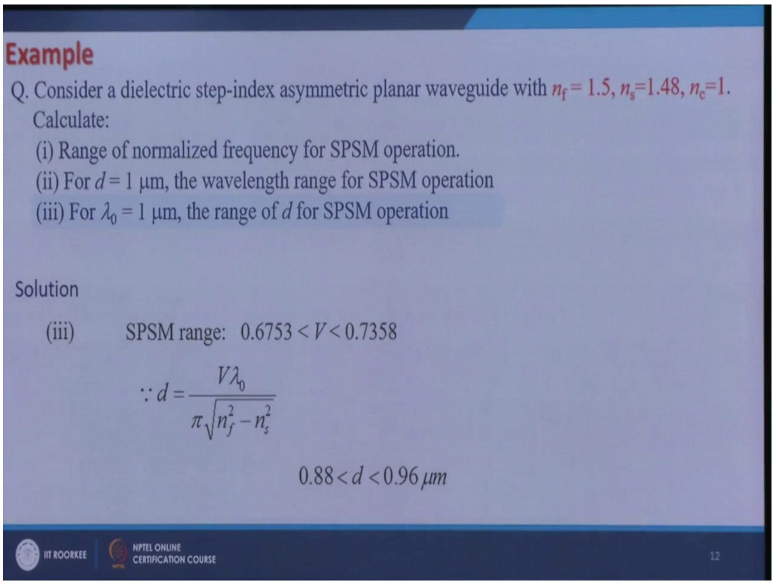


Second is for d is equal to 1 micrometre what is the wavelength range for SPSM operation? So, I know that from the previous problem I know that V should lie between

0.6753 to 0.7358 for SPSM operation. Now for this range of V I can find out the corresponding range of lambda if d is given. So, lambda in terms of V and d and n f n s is given by 2 pi over V d by 2 square root of n f square minus m c square. So, I simply find out the value of lambda naught corresponding to these values of V.

So, it comes out to be lambda line between 1.042 micrometre and 1.136 micrometer.

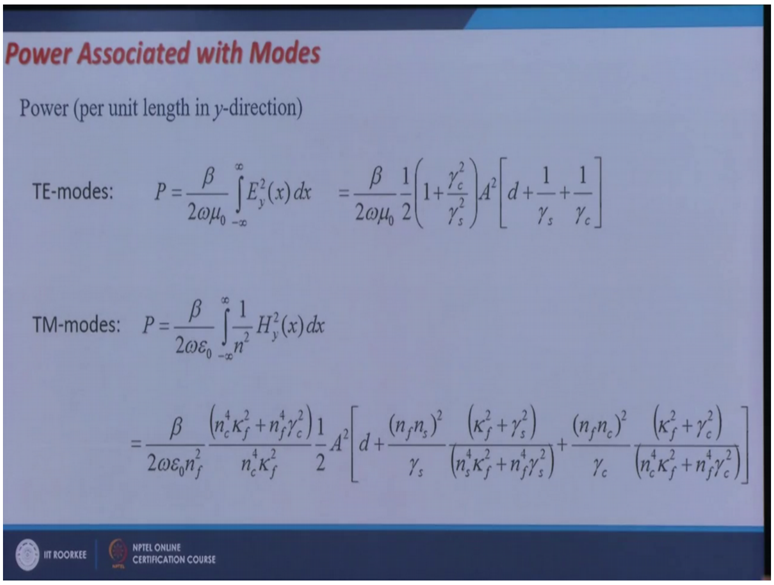
(Refer Slide Time: 13:59)



Third is if I choose a light source of wavelength lambda naught is equal to 1 micrometre then in what range of film thickness there would be SPSM operation. So, again I have the SPSM range of V from 0.6753 to 0.7358 and I simply now find out the corresponding values of d if lambda naught is given to me.

So, this comes out to be between 0.88 micrometre and 0.96 micrometre. So, in this range in this very small range of d I will have SPSM operation.

(Refer Slide Time: 14:47)

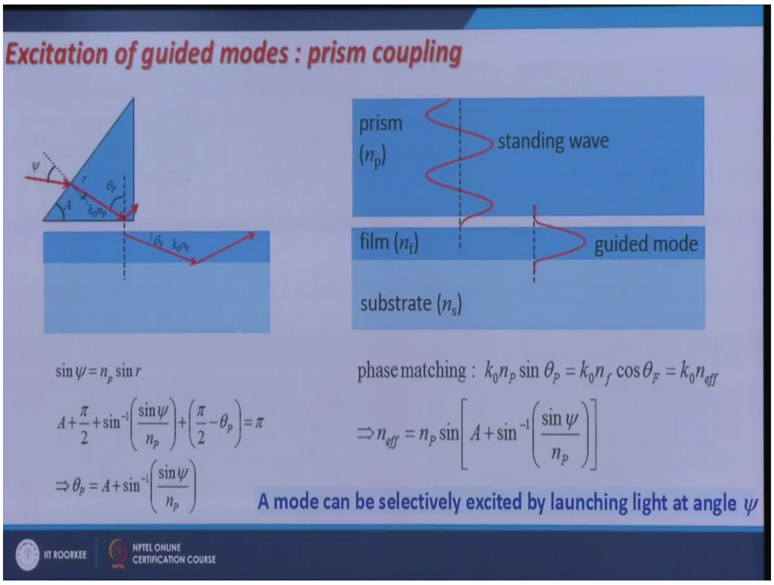


How much is the power associated with the modes. So, to find out the power associated with the mode I will have to calculate the pointing vector and then integrate it over the entire cross section. Since it is a planar waveguide, so I cannot integrate it over y, because in y direction it is infinitely extended. So, what I can have is power per unit length in y direction.

So, for TE-mode I have already seen in case of symmetric waveguides that P is given by beta over 2 omega mu naught times integral minus infinity to plus infinity E y square of x dx. So, if I now find out the integration E y square of x dx for the modes; for the modal fields of asymmetric planar waveguide for TE-mode then I can find out the power associated with TE-modes and it is given by this.

Similarly for TM-modes the power per unit length is given by beta over 2 omega epsilon naught integral minus infinity to plus infinity 1 over n square H y square x dx. And if I do the same then the power corresponding to TM-modes comes out to be like this.

(Refer Slide Time: 16:26)



The last thing that I would like to do in this lecture is- how do I excite a particular mode. Can I excite a particular mode? Selectively excite; I know if I have waveguide and if I launched the light from end then all the modes would be excited in different proportion depending upon what is the intensity profile or what is the amplitude profile of the incident light.

But, can I selectively excite one particular mode? So for what is used is prism coupling technique. So, what you do? You have a waveguide this is an asymmetric planar waveguide whose cover is air. So, this can be glass and this can be polymer film for example, and this is air. Now what I do I put a prism on top of this and press it, clamp it, then what I have here even though it is pressed hard there is some air gap between the prism and the film. And what happens is now I launched light from here into the prism, this light beam gets reflected into the prism.

And then depending upon this angle psi which can be translated, which can be related to this angle theta P with this beam makes with the normal to the prism base then depending upon this angle theta P this would get totally internally reflected. So, I can have total internal reflection at the prism base. I know that a total internal reflection is always associated with an evanescent tail. So, here what basically I have a standing wave here whose tail evanescent tail extends into the film.

So, I have a standing wave here whose evanescent tail extends into the field and it is this evanescent field which excites the mode of the waveguide. So, this is the standing wave. And this is one typical guided mode. This is you can see that it is TE 0 mode if launched polarization is TE. This guided mode is nothing but a superposition of two plane waves and these plane waves make angle plus minus theta f from the waveguide axis. So, if this angle is theta ft then the propagation constant of this plane wave is k naught n f, then the propagation constant of the mode is k naught n f cos theta f.

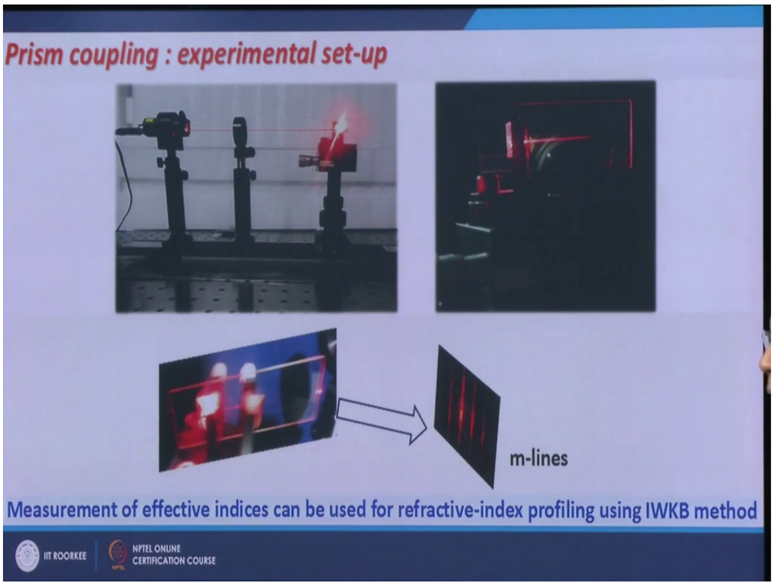
So, propagation constant of the mode is k naught n f cos theta f, which is the horizontal component of this. While if I look at this standing wave pattern then, what is the horizontal component of this plane wave. The horizontal component of this plane wave is k naught n P sin theta p. So, if the horizontal component of this standing wave is the same as the horizontal component of this guided mode then well horizontal component of the plane wave corresponding to guided board then there would be phase matching, because these two horizontal components are now able to catch up each other. And that is how this would be able to resonantly couple energy into this mode.

So, if this condition is satisfied I have excitation of that particular guided mode which corresponds to a particular angle theta f. And I know that this k naught n f cos theta f is nothing but k naught n effective. So, from here I can even find out the effective index of the mode if I know angle psi. How I have got this? Well, this angle psi can be related to this theta P. If I look at this triangle then A plus pi by 2 plus r where r is the angle of reflection plus pi by 2 minus theta P should be equal to pi. Or I get r in terms of psi from a Snell's law sin psi is equal to n P sin r and if I put it here then I get theta P is equal to a plus sin inverse sin psi over n P.

And this gives me n effective in terms of the incident angle, the prism refractive index, prism angle A, and again prism reflective index. So, if I know this then by just measuring these psi I can find out the effective index of the mode. And you can see that this resonant excitation can take place when this condition is satisfied and theta f is discrete; theta f is discrete for different modes. For t 0 mode it is different for TE 1 mode it is different for TE 2 mode it is different.

So, for different values of psi I will have different values of theta P. And if they match to these discrete theta f then I will excite those particular modes. So, in this way I can selectively excite, I can selectively excite the modes of a planar waveguide.

(Refer Slide Time: 22:50)



This is the experimental setup. So, you have a laser beam and this is a lens through which you focus this onto the prism coupling arrangement. This is a typical prism coupling arrangement, this is the waveguide, and this is the prism. And you can see that when I tune because this assembly can be rotated with respect to the beam, so I can change the angle psi. And I can see that for a particular value of psi the other input angle I see a mode is excited and I see a streak going down the length of the waveguide.

So, this represents the propagation of mode. And I can see that as it goes then this becomes feeble and feeble it becomes week because of the losses. And in fact, you can see this because there are scattering losses. There are scattering losses that is how you can see. So, more strong the streak is bad is the quality of the waveguide. I can also do, I can put another prism here and I can de couple the modes in the same way as I have coupled them. So, I put another prism here.

So, you can see that from this side I am coupling light this is the streak and then I decouple, then I decouple and when I put it here on the screen then I see these lines which correspond to the modes different modes. For example, in this I can see 1, 2, 3, 4, 5, 6 modes, and by changing the angle I can put light into one particular mode. For example, here it is coupled to third mode from the left. These are known as m lines. So, by measuring angles psi I can find out the effective index of the mode n effective. And once I have the values of n effective then using a technique called inverse WKB method I can do the refractive index profiling of the waveguide.

So, by measuring the refractive indices of the modes I can find out the refractive index profile of the waveguide. The only thing is that I should have sufficient values of n effective, I should have sufficient values of n effective. For a single mode waveguide it would not work, if you have only one value of n effective it will not work. So, if it is a single mode waveguide at one particular wavelength then it will not work, you will have to use different wavelengths to have the values of n effective at different wavelengths and then you can do it; otherwise, at a single wavelength if it is highly multimode waveguide then it is more accurate.

So, this is all in planar waveguides. And in the next lecture we will go into cylindrical geometry and find out how the modes are formed in an optical fiber, and how do we analyze the modes of an optical fiber, how do we find out the modes of an optical fiber. I have spent a lot of time in the analysis of planar waveguide. And in the optical fiber I would adopt all these results to cylindrical geometry, because the physics is now clear.

And it was easier to understand all these in planar geometry, because it was one dimensional problem. And also the functions involved were very simple: sin cosine functions and exponentially amplifying and decaying functions. But in case of cylindrical geometry these functions would be different. However, the physics which we have understood from the analysis of planar waveguide would be applicable there also.

Thank you.