Fiber Optics
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Lecture – 16
Electromagnetic Analysis of Waveguides- VI

After having evaluated the modal fields and propagation constants of the modes of asymmetric planar dielectric waveguide, now in this lecture let us find out how much power is associated with a mode, how much energy these modal fields carry as they propagate along the waveguide.

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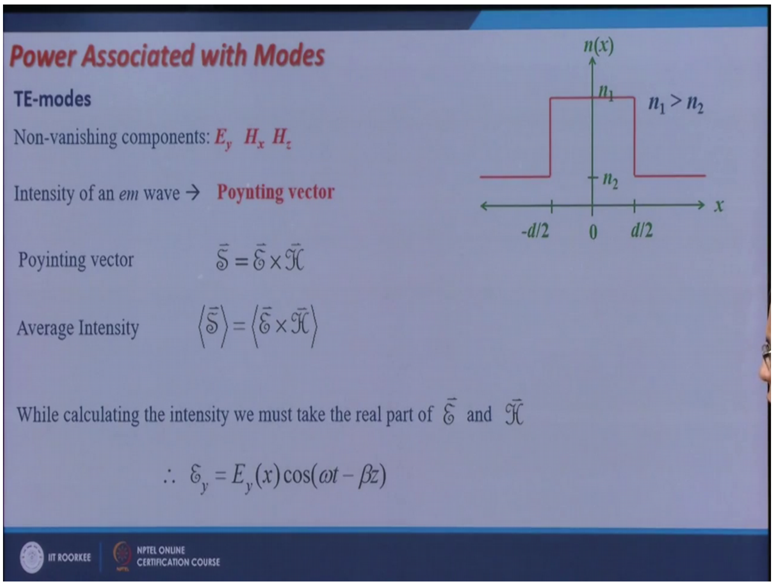

So, we will do the analysis for TE-modes we are talking about. AAnd for this waveguide, this refractive index profile and propagation direction is Z the non,-vanishing components of electric and magnetic field for TE-modes are so E y, H x and H z.

We know that the intensity of an em wave is given by pointing vector. So, we now need to find out what is the pointing vector corresponding to these fields; the modal fields. The pointing vector is given by S is equal to E cross H. And since we are talking about electromagnetic waves in optical frequency range, so E is fluctuating with a frequency of something like 10 to the power 15 hertz and so the magnetic field that is. Therefore, S is also fluctuating at a very rapid rate. Any detector even though it is very fast detector cannot record such rapid fluctuations and so our eye. So, what we record is basically the average value- time averaged value.

So, we will find out what is the average intensity by taking the average of E cross H; time average of E cross H. And since intensity is a real quantity, so while calculating intensity we must take the real parts of E and H. So, for TE-modes I know that E y is defined as E y of x e to the power i omega t minus beta Z. So, it is real part would be E y of x cosine omega t minus beta Z.

So, I have E y what is left is H x and H Z so that I can find out the pointing vector and therefore, the intensity.

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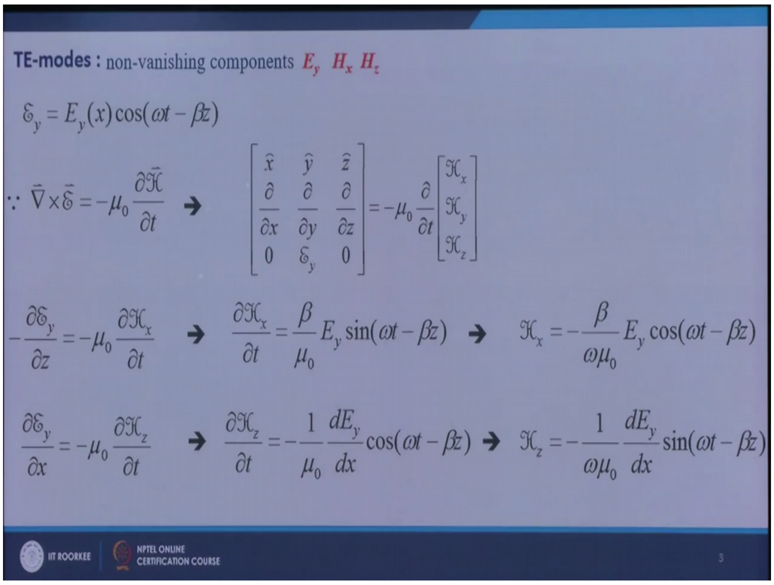

So E y is this, how do I find out corresponding H x and H z? Well, I know how H and E are related through Maxwell's equations. It is del cross E is equal to minus mu naught del H over del t. So, if I expand this in matrix form it would look like this. And in E I know E x is equal to 0 and E Z is equal to 0 only E y is non-vanishing for TE-modes. So, from here I can find out what is H x H y and H z. H y is not there of course, in case of TE-modes so H x and H Z I can get from here. So, from here I get if I take the x component del E y over del Z is equal to with a negative sign is equal to minus mu naught del H x over del t, and E y is given by this. This gives me del H x over del t is equal to beta over mu naught E y sin omega t minus beta Z.

If I integrate this I can get H x; integrate this with respect to time so I get H x is equal to minus beta over omega mu naught E y cosine of omega t minus beta Z. Similarly, if I take Z component from here then I get del E y over del x is equal to minus mu naught del H Z over del t. And if I differentiate this with respect to x I get del H Z over del t is equal to minus 1 over mu naught d E y over d x cosine omega t minus beta Z integrating it over time will give me H z; which comes out to be minus 1 over omega mu naught d E y over d x sin omega t minus beta Z.

So, I have all the three components in place corresponding to TE-modes. So now, we are ready to calculate the pointing vector.

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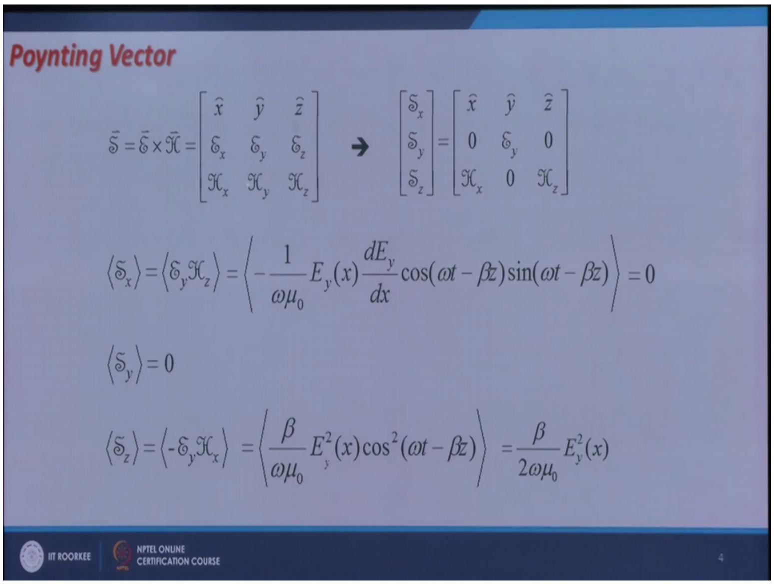

Pointing vector is given as E cross H, so I fill in the values here now. So, S x S y S Z in matrix form I can write it like this; these are the non-vanishing components of electric and magnetic fields for TE-modes E y H x and H z. And from here I can find out what is S x S y and S z; that is x y and Z components of the pointing vector. From here I get S x is equal to E y H Z minus 0. So, average value of S x would be average value of E y H z. And if I put E y H Z as I had calculated in the previous slide so this comes out to be like this. And in this, what I see of vector here cosine omega t minus beta Z multiplied by sin omega t minus beta Z, ok.

So, this is something like half of sin 2 omega t minus 2 beta Z. So, this is fluctuating and if I average it over a complete cycle for any value of Z the average value would be 0; the time average would be 0. So, this gives me 0. So, x component of pointing vector; the average value of x component of pointing vector comes out to be 0.

Now let me evaluate S y: S y is clearly 0 from here itself and S Z if I calculate S z, S Z will give you minus E y H x. So, now I substitute for E y and H x. So, I get beta over omega mu naught E square of x cosine square omega t minus beta Z. I know for any value of Z the time average of cosine square omega t would be equal to half. So, this gives me beta over 2 omega mu naught E y square of x.

So, what do I see? I see that for TE-modes the average value of S x and S y is 0 and I get the average value of only Z component. And this is obvious also, this is understandable, because my mode is propagation propagating in Z direction so it should carry energy along Z direction. So, this is the intensity. So, if I know the modal field that is E y of x then I can find out the intensity. If I integrated over the entire area then I can get the power associated with the mode.

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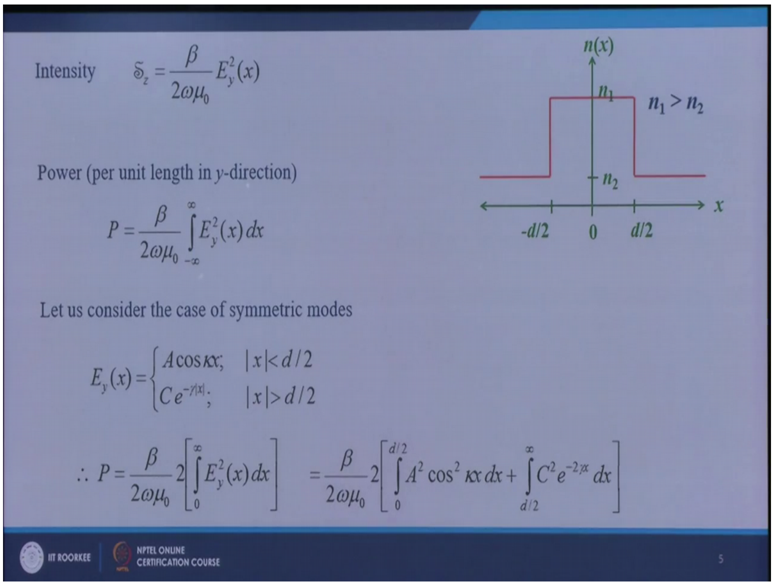

So, intensity is this, ok. To find out power I should integrate it over the area, over the transverse cross section. The mode is propagating in Z direction so the transverse plane is x y plane, but why is also extended to infinity. So I cannot integrate it over y, I can integrate it only over x. So, in the case of planar waveguide I cannot have power in terms of watts, but I can have only power per unit length in y direction, because I can integrate it only over x.

So, I get P is equal to beta over 2 omega mu naught integration E y square x d x from minus infinity to plus infinity. And this will give me power per unit length in y direction in the units of watts per meter. So, now, if I know E y of x in the entire region I can find this out. So, let us consider the case of symmetric modes to evaluate this. The modal field for symmetric modes is given by A cosine kappa x for mode x less than d by 2 and C e to the power minus gamma mode x for mode x greater than d by 2.

So, let me substitute this into this expression. So I get, and I also make use of the fact that this is symmetric mode. So, this integral from minus infinity to plus infinity can be written as 2 times 0 to infinity. So, I make use of that and then substitute E y of x, then I get beta over 2 omega mu naught 2 times 0 to d by 2 A square cosine square kappa x d x plus d by 2 to infinity C square e to the power minus 2 gamma x d x.

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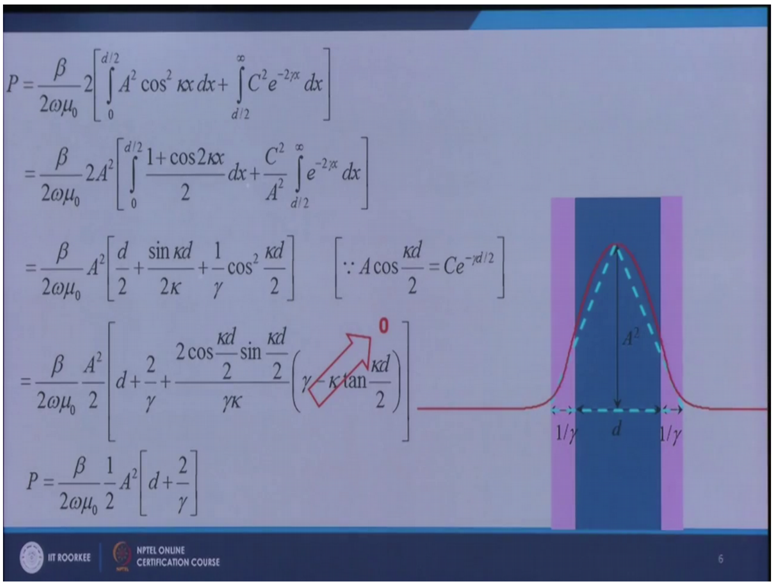

So, let me evaluate this integral while using the boundary conditions also, because I need to relate C to A. So, here I simplify this as- I take A square outside and cosine square kappa x can be written as 1 plus cosine 2 kappa x by 2 and the C square comes out to C square over A square integral d by 2 to infinity e to the power minus 2 gamma x d x.

And since, from boundary conditions I know A cosine kappa d by 2 would be equal to C times e to the power minus gamma d by 2. So, from here I will get C over A which I substitute here and evaluate this integral which is very simple e to the power minus 2 gamma x divided by minus 2 gamma and then I put the limit. So, when I simplify this what I get; I get d by 2 from here and sin kappa d over 2 kappa from here and 1 over gamma cosine square kappa d by 2 from this term. This I can further simplify. So, I take this vector to outside, so this becomes d and this becomes 2 over gamma; cosine square kappa d by 2 can be written as 1 minus sin square kappa d by 2. So, this 2 over gamma which is associated with 1 comes out here and the rest of the terms I can write as sin kappa d as 2 cosine kappa d by 2 sin kappa d by 2, and this I take common.

So, in the bracket inside I will be left with the term which goes as gamma minus kappa 10 kappa d by 2. It would be clear if you do this little mathematics. And why I have done in this fashion because I can see this is nothing but the transcendental equation. So this has to be 0, because gamma is equal to kappa tan kappa d by 2. So, if this is 0 this whole thing goes out and I get a very neat expression for power associated with the symmetric modes. And it comes out to be beta over 2 omega mu naught half A square times d plus 2 over gamma.

I can remember it in a very interesting way. And it is interesting to see that this term comes out to be the area of triangle under this curve. So, how you see that P is equal to beta over 2 omega mu naught times integral E y square of x d x.

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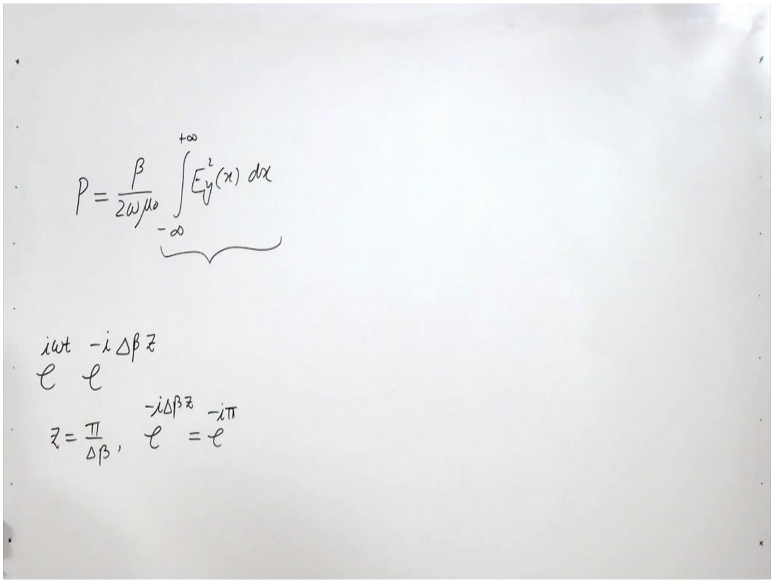

So, P is beta over 2 omega mu naught integral minus infinity to plus infinity E y square of x d x. So, if I find out the area under this E y square of curve then let us see what do I get. So, if this is E y square as a function of x then this is nothing but A square, this is waveguide width. And field extends into n 2 regions by distance 1 over gamma on either side. So, if I make a triangle which has height A square and base as d plus 2 over gamma, then the area of this triangle is simply half A square times d plus 2 over gamma. So, this is interesting that this comes out to be like this.

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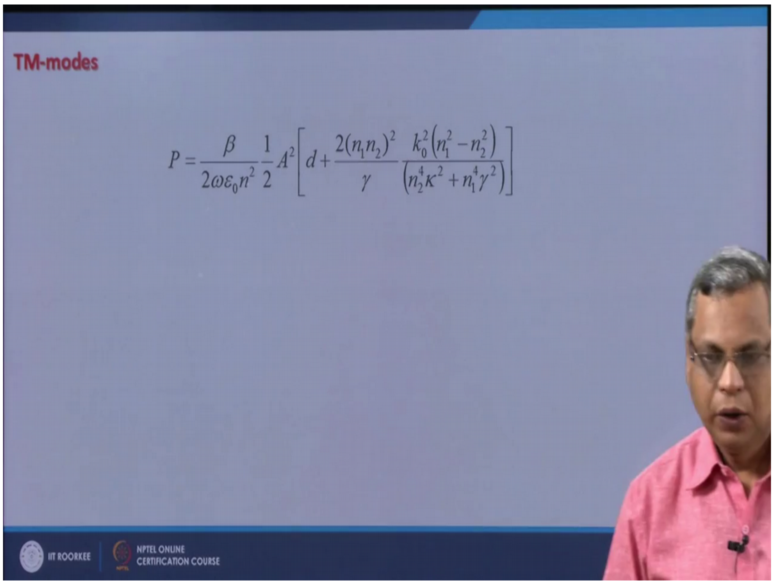

For TM-modes I can do the same analysis and find out the power associated with TM-modes and it is given by this. Although, I had found out the power by taking the example of symmetric modes this expression, this expression, and this expression, these expressions are valid for antisymmetric modes also. This can be proved and this can be evaluated. So, this is how I can get power associated with the mode and these powers are in watts per meter; power per unit length in y direction.

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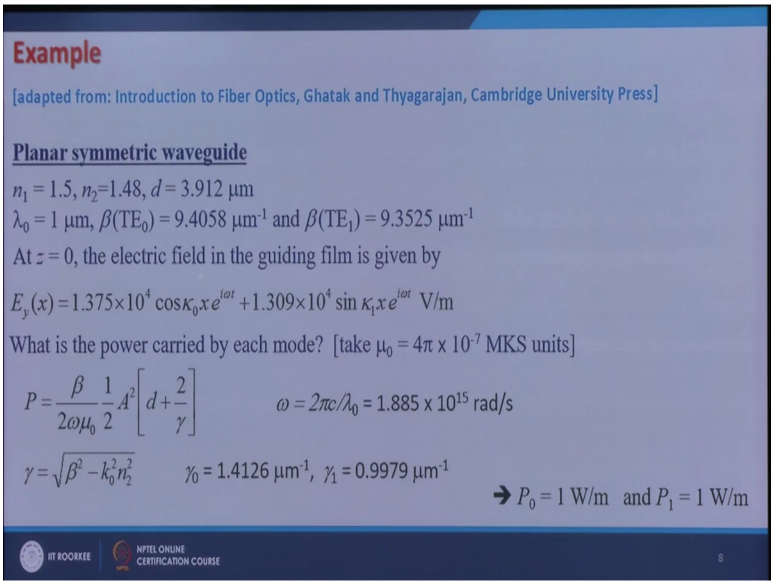

Let us work out some examples. This is the example adopted from introduction to fiber optics by Ghatak and Thyagarajan. Where I have a planar symmetric waveguide with n 1 is equal to 1.5, n 2 is equal to 1.48, and d is equal to 3.912 micrometer. At lambda naught is equal to 1 micrometer beta for TE 0 mode is this and beta for TE 1 mode is this; it supports two modes at 1 micrometer wavelength. If at Z is equal to 0 the electric field in the guiding film is given by this.

So, you see I have TE 0 mode and TE 1 mode, added Z is equal to 0 I excide both the modes with different amplitudes. This mode is excited with this amplitude, TE 0 mode is excited with this amplitude and TE 1 mode is excited with this amplitude. So, the total field at Z is equal to 0 is this much volts per meter.

Then what is the power carried by each mode? You can take mu naught is equal to this. So, I know the power carried by a mode is given by this. So, what I need to know; I need to know what is the beta for that mode, what is the amplitude of that mode, what is gamma for that mode. And of course, I need to know what is omega and omega is nothing but 2 pi C over lambda naught, since lambda naught is given to you so you can immediately calculate the value of omega.

Then you find out gamma beta is already given to you, A is already given to you. Gamma you can find out from square root of beta square minus k naught square n 2 square. So, for TE 0 mode this gamma comes out to be 1.4126 micrometer inwards, for TE 1 mode gamma comes out to be 0.9979 micrometer inwards. And if you put these values into this expression you will find that for TE 0 mode the power associated comes out to be 1 watt per meter and for TE 1 mode also you find out that the power comes out to be 1 watt per meter. In fact, these amplitudes have been adjusted in such a way that both the modes carry unity power.

When the amplitudes are adjusted in such a way then they are power normalized. If you remember that when we found out the modal fields we had retained A only and v related C to A and said that A can be found out by normalization. This is one way of normalization that you find out the value of A in such a way that that the modal field carries unity power. So, these are power normalized modes.

Let me take another interesting example of again a symmetric planar waveguide which supports two modes: TE 0 and TE 1. There propagation constants are beta 0 and beta 1 and electric field amplitudes are A 0 and A 1.

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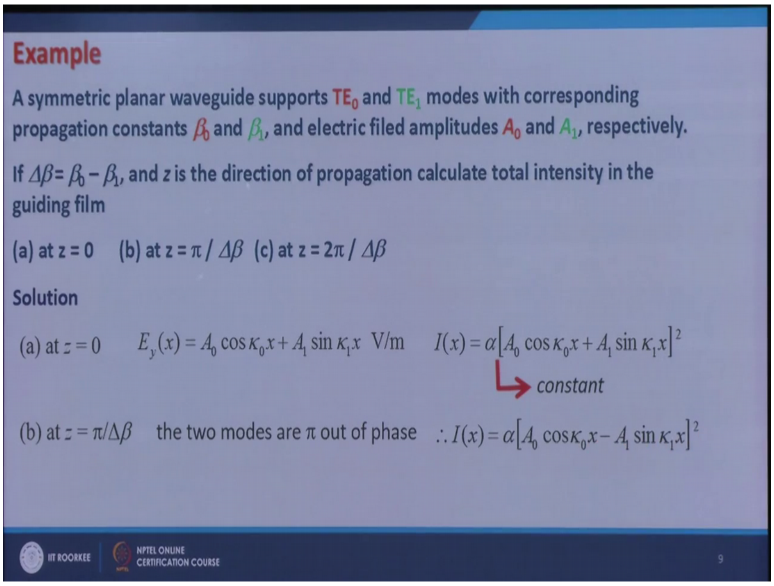

If the difference in their propagation constants is defined by delta beta and Z is the direction of propagation, then what would be the total intensity in the guiding film at different values of z. First one at Z is equal to 0, second Z is equal to pi over delta beta, and third Z is equal to 2 pi over delta beta. So, what do I see here at Z is equal to 0 the total field would be A 0 cosine kappa 0 x, where kappa 0 you can find from beta 0 plus A

1 sin kappa 1 x where kappa 1 can be found out from beta 1. So, at Z is equal to 0 this would be the total field.

As these fields propagate it will go with propagation constant e to the power i beta 0 z, this will go as e to the power i beta 1 z. So, as they propagate in Z direction they would be a phase shift accumulated between them; a phase difference accumulated between them. And that phase difference would be delta beta Z. So, you will have e to the power i delta beta Z. And I know that at Z is equal to pi over delta beta then this e to the power i delta beta Z would simply be e to the power i pi; which means that these two modes would be pi out of phase. And if we are doing in the way e to the power i omega t plus then it should be minus, so we can have this form of expression. So, instead of plus i beta Z, because I am doing it in such fashion so I can retain the same convention.

So, the thing is that the two modal fields will be pi out of phase when they traverse this distance. The intensity is nothing but the field square, so intensity would be some alpha times E y square. So, this would be at Z is equal to 0, but at Z is equal to pi over delta beta the total field would be this minus this, this minus this because they are pi out of phase. So, the intensity would be this much. While at Z is equal to 2 pi over delta beta it would be 2 pi phase shifted 2 pi phase shifted means there is no phase shift. So, your intensity would again be this.

And the analysis of guided modes of a planar symmetric waveguide with food for thought I had said that.

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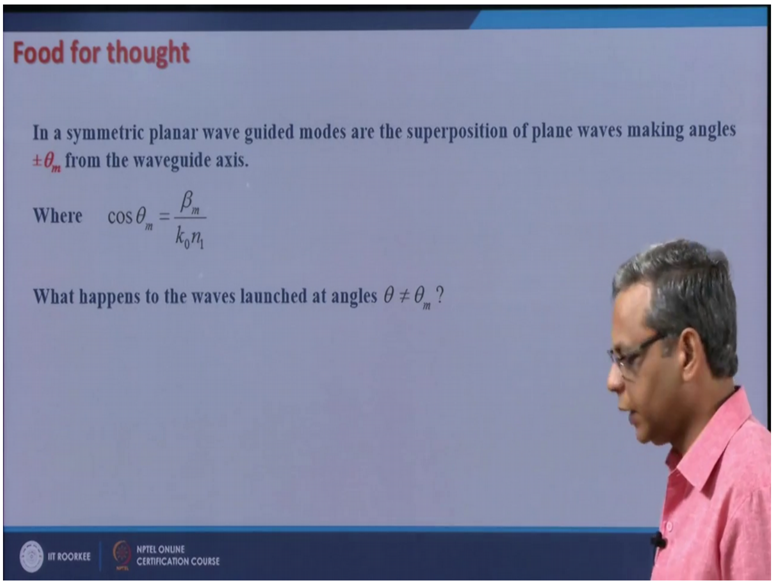

In a symmetric planar waveguide the guided modes are the superposition of plane waves making angles plus minus theta m from the waveguide axis. Where the angles are given by cos theta m is equal to beta m over k naught n 1. So, if I launched two plane waves at angles plus minus theta 0 then I excide the TE 0 mode; then TE 0 mode is excited and the pattern corresponding to TE 0 mode is found and it goes along the waveguide and it sustains its shape.

Similarly, if I exited plus minus theta 1 TE 1 mode is there. The question is what happens if the waves are launched at angles which do not correspond to these guided modes; the angles corresponding to these guided modes please think about it.

And in the end I complete this analysis of planar symmetric waveguide by briefly mentioning the radiation modes.

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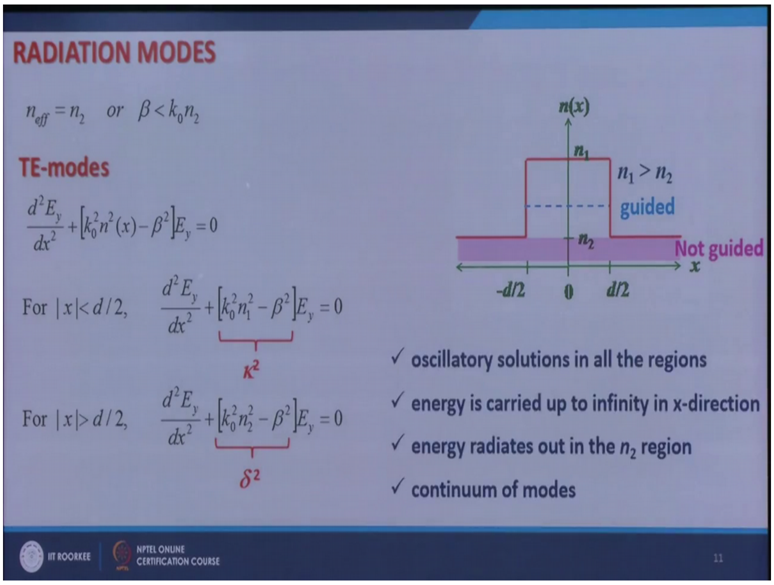

I have seen that if beta lies between k naught n 2 and k naught n 1 or n effective lies between n 2 and n 1 then I have guided modes. If beta is less than k naught n 2 they should be less than n 2. So, n effective is less than n 2 or beta is less than k naught n 2 then what will happen. If I write down the equation wave equation for TE-modes and then I write it down in both the regions, in this region and in this region then for mode x less than d by 2 I have this equation beta is less than k naught n 2 and hence beta is also less than k naught n 1. So, kappa square is positive.

Now, in the region for mode x greater than d by 2 I would have d 2 E y over d x square plus k naught square n 2 square minus beta square is equal to 0. And since beta is smaller than k naught n 2 then if I define this as delta square and delta square would be positive. And what I will have? I will have oscillatory solutions here as well as here. So, everywhere I will have oscillatory solutions; which means that energy is carried upto infinity in x direction that is energy radiates out in the n 2 region. And these kinds of modes are known as radiation modes because the energy radiates out corresponding to these modes, and they form continuum they are not discrete modes. So, they form continuum of modes.

So, with this finish the analysis of planar waveguide, symmetric planar waveguide, whatever we have learned about the modes, modal fields, and the procedure of finding out the modes would be very useful when we will do a more complicated structure such as optical fiber.

Thank you.